Covid 19 Seen by Differential Equations:Model I (Covid 19 Vu par des Equations Différentielles: Modèle I)

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- Since differential equations with delay include ordinary ones, can we generalize the results established for ordinary differential equations to those with delay?

(Comme les equations differentielles avec retard englobent celles ordinaire , peut-on generaliser les resultats etablis pour les equations differentielles ordinaires à celles avec retard ?)

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  4- How can we define an equilibrium of the equation(3)?
  5- How can we iterate the equation (3)?
  6- You can imagine other questions.
  7- First of, for each question take an example and try to do as it is done for ordinary differential equation and note the difficulties

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7- Tout d'abord, pour chaque question, prenez un exemple et essayez de faire comme pour les équations différentielles ordinaires et notez les

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- Concerning the nature of the equation

$$N(t) + \int_{t-T}^{t} f(N(s))ds = k$$

it is an integral equation.

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Which can be written as

$$\dot{N}(t) = F(N(t), N(t-T)) \tag{2}$$

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- In most cases yes, but by imposing the right assumptions...

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b) Then the chauchy problem is defined as follow

$$(CP)\begin{cases} \dot{N}(t) & = F(N(t), N(t-T)) \text{ if } t \geq t_0 \\ x(t) & = \phi(t - t_0) \text{ if } t \in [t_0 - T, t_0] \end{cases} \quad (3)$$

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c) Then the phase space for the equation (1) is $C([-T, 0], \mathbb{R})$ which is a Banach space with the norme $\|\phi\| = \sup_{s\in[-T,0]} |\phi(s)|$.

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Sur if we take $T = 0$, one has

$F(N(t), N(t-T)) = F(N(t), N(t)) := G(N(t))$ and

$x(t_0) = \phi(t_0 - t_0) = \phi(0) := x_0$ then the equation (3) becomes

$$(CP)\begin{cases} \dot{N}(t) & = G(N(t)) \text{ if } t \geq t_0 \\ x(t_0) & = x_0 \text{ if } t \in [t_0, t_0] = \{t_0\} \end{cases}$$

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In the same way as ordinary differential equations, an equilibrium is an initial condition which gives a constant solution $N^*(t) = N^*$ and from there one has

$$0 = \dot{N}(t) = F(N(t), N(t-T)) = F(N^*, N^*)$$

Then the equilibriums are the solutions of $F(N^*, N^*) = 0$.

5- How can we iterate the equation (3)?

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As in the example seen in the classroom, we use the steps method in the following manner: Let us consider the integrale form

$$N(t) = \phi(0) - \int_{t_0}^{t} F(N(s), N(s-T))ds$$

with $N(t) = \phi(t - t_0)$ on $[t_0 - T, t_0]$ and we take steps of lenght T.

- On the step $[t_0, t_0 + T]$. so if $t_0 \leq s \leq t_0 + T$ then $t_0 - T \leq s - T \leq t_0$ and from there one has $N(s - T) = \phi(s - T - t_0)$ and the integral form becomes

$$N(t) = \phi(0) - \int_{t_0}^{t} F(N(s), N(s-T))ds$$
$$= \phi(0) - \int_{t_0}^{t} F(N(s), \phi(s-T-t_0))ds$$
$$= \phi(0) - \int_{t_0}^{t} G(N(s))ds$$

where $G(N(s)) = F(N(s), \phi(s-T-t_0))$ and in this integral the delay disappear since ϕ is known. We calculate the piece of the solution on $[t_0, t_0 + T]$ and we denote it as $N_1(t)$

- On the step $[t_0 + T, t_0 + 2T]$. If $t_0 + T \leq s \leq t_0 + 2T$ then $t_0 \leq s - T \leq t_0 + T$ and from there one has $N(s-T) = N_1(s-T)$ and the integral form becomes

$$\begin{aligned} N(t) = & \quad N_1(t_0+T) + \int_{t_0+T}^{t} F(N(s), N(s-T))ds \\ = & \quad N_1(t_0+T) + \int_{t_0}^{t} F(N(s), N_1(s-T))ds \\ = & \quad N_1(t_0+T) + \int_{t_0}^{t} G_1(N(s))ds \end{aligned}$$

where $G_1(N(s)) = F(N(s), N_1(s-T))$ and in this integral the delay disappear since N_1 is known. We calculate the piece of the solution on $[t_0+T, t_0+2T]$ and we denote it as $N_2(t)$

- One repeat the process on the step $[t_0+2T, t_0+3T]$ to obtain the piece of the solution on this intervall and so on.

And so on we obtain the solution

$$N(t) = \begin{cases} \phi(t - t_0) & \text{if } t \in [t_0 - T, t_0] \\ N_1(t) & \text{if } t \in [t_0, t_0 + T] \\ \dots & \\ N_k(t) & \text{if } t \in [t_0 + (k-1)T, t_0 + kT] \end{cases}$$

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