Exercice 1 : Soit $\vec{A} = 2xyz\vec{i} + (2x^2 - y)\vec{j} - yz^2\vec{k}$

et $\phi = x^2y + 2y^2z^3$

Donner au point (1,0,0): $\overrightarrow{grad}\,\phi$, $div\vec{A}$, $\overrightarrow{Rot}\vec{A}$.

Solution

- $\overrightarrow{grad}\,\phi(x,y,z) = \vec{\nabla}\,\phi(x,y,z)$

$= \frac{\partial}{\partial x}\phi(x,y,z)\,\vec{i} + \frac{\partial}{\partial y}\phi(x,y,z)\,\vec{j} + \frac{\partial}{\partial z}\phi(x,y,z)\,\vec{k}$

$$\phi = x^2y + 2y^2z^3$$

$$\frac{\partial}{\partial x}\phi(x,y,z) = 2xy$$

$$\frac{\partial}{\partial y}\phi(x,y,z) = x^2 + 4yz^3$$

$$\frac{\partial}{\partial z}\phi(x,y,z) = 6y^2z^2$$

$\overrightarrow{grad}\,\phi(x,y,z) = 2xy\,\vec{i} + (x^2 + 4yz^3)\,\vec{j} + 6y^2z^2\,\vec{k}$

au point (1,0,0): $\overrightarrow{grad}\,\phi(x,y,z) = \vec{j}$

- $div\,\vec{A} = \vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\vec{A} = 2xyz\vec{i} + (2x^2 - y)\vec{j} - yz^2\vec{k}$

$A_x = 2xyz,\quad A_y = (2x^2 - y),\quad A_z = -yz^2$

$\frac{\partial A_x}{\partial x} = 2yz,\quad \frac{\partial A_y}{\partial y} = -1,\quad \frac{\partial A_z}{\partial z} = -2yz$

$div\,\vec{A} = 2yz - 1 - 2yz = -1$

- $\overrightarrow{Rot}\,\vec{A} = \vec{\nabla}\wedge\vec{A} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xyz & (2x^2 - y) & -yz^2 \end{vmatrix}$

$$\vec{i}\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (2x^2 - y) & -yz^2 \end{vmatrix} - \vec{j}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ 2xyz & -yz^2 \end{vmatrix} + \vec{k}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ 2xyz & (2x^2 - y) \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}(-yz^2) - \frac{\partial}{\partial z}(2x^2 - y)\right)\vec{i} - \left(\frac{\partial}{\partial x}(-yz^2) - \frac{\partial}{\partial z}(2xyz)\right)\vec{j}$$

$+\left(\frac{\partial}{\partial x}(2x^2 - y) - \frac{\partial}{\partial y}(2xyz)\right)\vec{k}$

$$\overrightarrow{Rot}\,\vec{A} = (-z^2 - 0)\vec{\imath} - (0 - 2xy)\vec{\jmath} + (4x - 2xz)\vec{k}$$

au point (1,0,0): $\overrightarrow{Rot}\,\vec{A} = 4\,\vec{k}$