Chapter 1: Numbers and arithmetic Operations

I. Introduction

Two kinds of activity made our ancestors develop numbers. The first for counting their things (cardinal numbers) and the second for comparing them and then creating order (ordinal numbers).

A. Cardinal Numbers (Counting Numbers)

Example:
1 one, 2 two, 3 three, 4 four, 5 five, 6 six, 7 seven, 8 eight, 9 nine, 10 ten,
11 eleven, 12 twelve, 13 thirteen, 14 fourteen, 15 fifteen, 16 sixteen, 17 seventeen, 18 eighteen, 19 nineteen
20 twenty, 21 twenty-one..., 30 thirty, 40 forty, 50 fifty, 60 sixty, 70 seventy, 80 eighty, 90 ninety
100 a hundred; one hundred, 101 a hundred and one...
200 two hundred, 300 three hundred,..., 900 nine hundred
1 000 a thousand, one thousand, 1 001 a thousand and one...
1 100 one thousand, one hundred, 1 101 one thousand, one hundred and one...
9 999 nine thousand, nine hundred and ninetynine
10 000 ten thousand
15 356 fifteen thousand, three hundred and fifty six
100 000 a hundred housand
1 000 000 a million
1 000 000 000 a billion
1 000 000 000 000 a trillion

B. Ordinal Numbers (Place Numbers)

Example:
1st first, 2nd second, 3rd third, 4th fourth, 5th fifth,, 6th sixth, 7th seventh, 8th eighth, 9th ninth, 10^{th} tenth,
11th eleventh, 12th twelfth, 13th thirteenth, 14th fourtheenth, 15th fidteenth, 16th sixteenth, 17th seventeenth,
18th eighteenth, 19th nineteenth,
20th twentieth, 21st twenty-first, 22nd twenty-second, 23rd twenty-third, 24th twenty-fourth, 25th twenty-fifth,
26th twenty-sixth, 27th twenty-seventh, 28th twenty-eighth, 29th twenty-ninth
30th thirtieth, 40th fortieth, 50th fiftieth, 60th sixtieth, 70th seventieth, 80th eightieth, 90th ninetieth,
100th hundredth
1 000th thousandth
1 000 000th miilionth

C. Multiples

Sometimes numbers aren't expressed in cardinal or ordinal numbers. Here are a few examples:
half (1/2)
single (1)
double (x2)
triple (x3)
quadruple (x4)
a pair/couple (2)
a few (2 or 3)
several (more than 3 but not many)
a dozen (12)
a half dozen (6)

II. Numbers Sets

a) Natural Numbers: 1,2,3,… one, two, three, and so forth (without end). 1,2,3,…, 10 one, two, three, and so forth up to ten. Natural numbers can be divided into two sets: Odd Numbers 2,4,6,.... and Even Numbers 1,3,5,...
b) Whole Numbers: Natural Numbers + 0 (zero/o/nought)
c) Integers: -2, -1,0,1,.. .., negative two, negative one, zero, one, ...
d) Rational Numbers: are numbers that can be expressed as fraction.
e) Irrational Numbers: are numbers that cannot be expressed as fraction, such as $\sqrt{2}$,π .
f) Real Numbers: are made up of rational and irrational numbers.
g) Complex Numbers: Complex numbers are numbers that contain real and imaginary part. 2 + 3i : 2 is called the real part, 3 is called the imaginary part, and i is called imaginary unit of the complex number.

III. Digits

A Digit is any one of the ten numerals 0,1,2,3,4,5,6,7,8,9.

Example: 3 is a single-digit number, but 234 is a three-digit number. In 234, 4 is the units digit, 3 is the tens digit, and 2 is hundreds digit.

III. Consecutive Numbers

They are counting numbers that differ by 1. Examples: 83, 84, 85, 86, and 87 are 5 consecutive numbers. 84, 85, 86, … are successor of 83. 84 is the immediate successor of 83. 1, 2, …, and 82 are predecessor of 83. 82 is the immediate predecessor of 83. 36, 38, 40, and 42 are 4 consecutive even numbers.

IV. Operations on Numbers

Addition (+) , Subtraction (-) , Multiplication (×) , Division(:)

a) Symbols in Numbers Operation:

+ : added by/plus/and

- : subtracted by/minus/take away

± : plus or minus

× multiplied by/times

: : divided by/over

b) Symbols for Comparing Numbers:

= : is equal to/equals/is

≠ : is not equal to/does not equal /iz not

< : is less than/is smaller than

> : is greater than/is more than

≤ : is less than or equal to

≥ : is more/greater than or equal to

≅ : is approximately equal to

Examples

➢ 2 + 3 = 5

two	is added by plus and	three	is equal to equals is	five.

2 and 3 are called addends or summands, and 5 is called sum.

➢ 10 – 4 = 6

Ten	is substracted by minus take away	four	is equal to equals is	six.

4 is the subtrahend, and 6 is the difference

➢ 7 × 8 = 56

Seven	is multiplied by times	eight	is equal to equals is	fifty-six

7 is the multiplicator, 8 is the multiplicand, and 56 is the product

➢ 45 : 5 = 9

forty-five	is divided by over	five	is equal to is	nine.

5 is the divisor /də‘vaizə(r)/, and 9 is the quotient/‘kwəuʃnt/.

V. Equations and Inequalities

The mathematical sentences that use symbols « = » are called equation, and the mathematical sentences that use symbols « < », « > », « ≤ », or « ≥ » are called inequalities.

Examples:

$ax+b=0$ is a linear equation.

$a\,x^2+bx+c=0$ is a quadratic equation.

$3x^3-2x^2+3=0$ is a cubic equation.

$\frac{a+b}{2}\geq\sqrt{ab}$ is called AM-GM inequality.

VI. Fractions

Definition : A common (or simple) fraction is a fraction of the form a/b where a is an integer and b is a counting number

Properties :

* In the fraction p/q, p is called the numerator of the fraction and q is called the denominator of the fraction.

*If the numerator < the denominator, then (p/q) is a proper fraction.
*If the numerator > the denominator, then (p/q) is an improper fraction.
*3 ¼ is a mixed number because it contains number part and fractional part.
*The fraction a/b is simplified ("in lowest terms") if a and b have no common factor other than 1.

Saying Fractions:

$\frac{1}{2}$: one half

$\frac{1}{3}$: one third

$\frac{1}{4}$: one quarter

$\frac{5}{6}$: Five sixths/Five over six

$\frac{22+x}{7}$: Twenty-two plus x all over seven

$13\frac{3}{4}$: Thirteen and three quarters

0.3 : Nought/zero/o point three
3.056 : Three point o five six
273.856 : Two hundred and seventy-three point eight five six

VI. Divisibility

Definition : The notation 4|12 means that 12 is divisible by 4 or 4 divides 12.
*We say that 12 is a multiple of 4 or 4 is a factor of 12.
*17 is not divisible by 4. If 17 is divided by 4 then the quotient is 3 and the remainder is 1.
*0 is divisible by all integers,

*Prime Numbers

Every numbers is divisible by 1 and itself. These factors (1 and itself) are called improper divisors.
Definition : Prime numbers are numbers that have only improper divisors.
Example: 5 is a prime number, but 9 is not a prime number we say that it is a composite number.

*Common Divisors

Example: 1,2,3,4,6, and 12 are divisors (factors) of 12.
1,3,5, and 15 are divisors of 15.
1 and 3 are common divisors of 12 and 15.
3 is the greatest common divisor (g.c.d.) of 12 and 15. We write gcd(12,15) = 3.

*Common Multiples

Example: 5,10,15,20,25, …are multiples of 5.
4,8,12,16,20,24,… are multiples of 4.
5,10,15,20 are four first multiples of 5.
4,8,12,16,20 are five first multiples of 4.
20,40,60, … are common multiples of 4 and 5.
20 is the least common multiple (l.c.m) of 4 and 5. We write lcm(4,5) = 20.