Corrigé de TD N° 01 d'Electricité

1ère partie :ELECTROSTATIQUE « charges ponctuelles »

Exercice 1 :

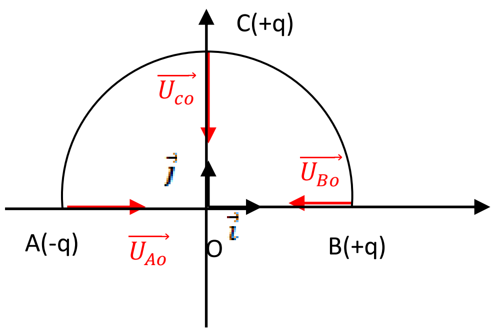
- Le potentiel au point O

$$V_O = V_A + V_B + V_C = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC}$$

OA=OB=OC=R

$$V_O = k\frac{(-q)}{R} + k\frac{(+q)}{R} + k\frac{(+q)}{R} \Rightarrow V_O = k\frac{q}{R}$$

- Le champ électrique au point O



$$\overrightarrow{E_O} = \overrightarrow{E_A} + \overrightarrow{E_B} + \overrightarrow{E_C}$$

Avec

$$\overrightarrow{E_A} = k\frac{q_A}{(OA)^2}\overrightarrow{u_{AO}}\ ,\ \overrightarrow{E_B} = k\frac{q_B}{(OB)^2}\overrightarrow{u_{BO}}$$

$$\overrightarrow{E_C} = k\frac{q_C}{(OC)^2}\overrightarrow{u_{CO}}$$

$$\overrightarrow{u_{AO}} = \vec{i},\ \overrightarrow{u_{BO}} = -\vec{i}, \qquad \overrightarrow{u_{CO}} = -\vec{j}$$

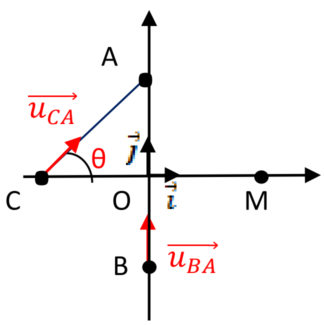
Donc $\overrightarrow{E_O} = k\frac{(-q)}{R^2}\vec{i} + k\frac{q}{R^2}(\overrightarrow{-i}) + k\frac{q}{R^2}(\overrightarrow{-j}) \Rightarrow \overrightarrow{E_0} = -k\frac{q}{R^2}(2\vec{i}+\vec{j})$

La force électrostatique au point O

$$\overrightarrow{F_0} = q'\,\overrightarrow{E_O} = q\overrightarrow{E_O} = -k\frac{q^2}{R^2}(2\vec{i}+\vec{j})$$

Exercice 2

- **Force électrostatique exercée sur la charge q_A**



$$\overrightarrow{F_A} = \overrightarrow{F_{BA}} + \overrightarrow{F_{CA}}$$

$$\overrightarrow{F_{BA}} = k\frac{q_Aq_B}{(BA)^2}\overrightarrow{u_{BA}},\ \ \overrightarrow{F_{CA}} = k\frac{q_Aq_C}{(CA)^2}\overrightarrow{u_{CA}}$$

Avec q_A=+q, q_B=+q, q_C= - 2q et BO=AO=a et CO=b

BA=a+a=2a et CA= ?

D'après la règle de Pitagort dans le triangle droit (ACO) ; $CA^2=CO^2+OA^2$

$$CA^2 = a^2 + b^2$$

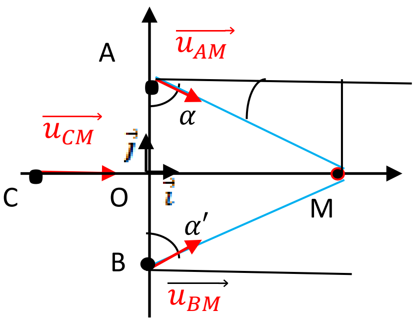
$\overrightarrow{u_{BA}} = \vec{j}$, $\overrightarrow{u_{CA}} = \cos\theta\,\vec{\imath} + \sin\theta\,\vec{j}$ avec $\cos\theta = \frac{CO}{CA} = \frac{b}{\sqrt{a^2+b^2}}$ $et \sin\theta = \frac{OA}{CA} = \frac{a}{\sqrt{a^2+b^2}}$

Donc $\overrightarrow{u_{CA}} = \frac{b}{\sqrt{a^2+b^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+b^2}}\,\vec{j}$

$$\overrightarrow{F_A} = k\frac{q^2}{4a^2}\vec{j} + k\frac{(-2q^2)}{a^2+b^2}\left(\frac{b}{\sqrt{a^2+b^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+b^2}}\,\vec{j}\right) \Rightarrow \overrightarrow{F_A} = kq^2\left[\frac{-2b}{(a^2+b^2)^{\frac{3}{2}}}\,\vec{\imath} + \left(\frac{1}{4a^2} - \frac{2a}{(a^2+b^2)^{\frac{3}{2}}}\right)\vec{j}\right]$$

- **Force électrostatique exercée sur la charge q_M**

$$\overrightarrow{F_M} = \overrightarrow{F_{BM}} + \overrightarrow{F_{CM}} + \overrightarrow{F_{AM}}$$



$\overrightarrow{F_{BM}} = k\frac{q_M q_B}{(MA)^2}\overrightarrow{u_{BM}}$, $\overrightarrow{F_{CM}} = k\frac{q_M q_C}{(CM)^2}\overrightarrow{u_{CM}}$, $\overrightarrow{F_{AM}} = k\frac{q_M q_A}{(AM)^2}\overrightarrow{u_{AM}}$

Avec q_A=+q, q_B=+q, q_C= - 2q et q_M=+q
BO=AO=a, CO=b et OM=x

CM= b + x$\Rightarrow CM^2 = (b + x)^2$ et AM= ?, BM= ?

D'après la règle de Pitagort dans le triangle droit (AMO) ;

$AM^2=MO^2+OA^2 \Rightarrow AM^2 = a^2 + x^2$

D'après la règle de Pitagort dans le triangle droit (BMO) ;

$BM^2=MO^2+OB^2 \Rightarrow AM^2 = a^2 + x^2$

$\overrightarrow{u_{CM}} = \vec{\imath}$, $\overrightarrow{u_{AM}} = \sin\alpha\,\vec{\imath} - \cos\alpha\,\vec{j}$ avec $\cos\alpha = \frac{OA}{AM} = \frac{a}{\sqrt{a^2+x^2}}$ $et \sin\alpha = \frac{OM}{AM} = \frac{x}{\sqrt{a^2+x^2}}$

Donc $\overrightarrow{u_{AM}} = \frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} - \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}$ et $\overrightarrow{F_{AM}} = k\frac{(q^2)}{a^2+x^2}\left(\frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} - \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}\right)$

$\overrightarrow{u_{BM}} = \sin\alpha'\vec{\imath} + \cos\alpha'\,\vec{j}$ avec $\cos\alpha' = \frac{OB}{BM} = \frac{a}{\sqrt{a^2+x^2}}$ $et \sin\alpha' = \frac{OM}{BM} = \frac{x}{\sqrt{a^2+x^2}}$

Donc $\overrightarrow{u_{BM}} = \frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}$ et $\overrightarrow{F_{BM}} = k\frac{(q^2)}{a^2+x^2}\left(\frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}\right)$

$$\overrightarrow{F_M} = k\frac{-2q^2}{(b+x)^2}\vec{\imath} + k\frac{(q^2)}{a^2+b^2}\left(\frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} - \frac{a}{\sqrt{a^2+x^2}}\,\vec{j} + \frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}\right)$$

$$\Rightarrow \overrightarrow{F_M} = kq^2\left[\left(\frac{2x}{(a^2+x^2)^{\frac{3}{2}}} - \frac{2}{(b+x)^2}\right)\vec{\imath}\right]$$

- **Le champ électrique qu point M**

$$\overrightarrow{F_M} = q_M\overrightarrow{E_M} \Rightarrow \overrightarrow{E_M} = \frac{\overrightarrow{F_M}}{q_M}$$

$$\Rightarrow \overrightarrow{E_M} = kq\left[\left(\frac{2x}{(a^2+x^2)^{\frac{3}{2}}} - \frac{2}{(b+x)^2}\right)\vec{\imath}\right]$$

- **Le potentiel au point M**

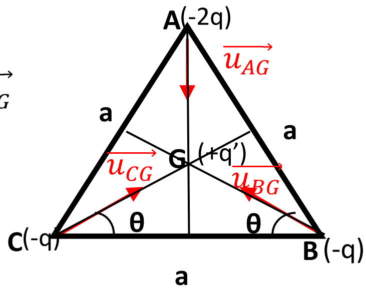
$$V_M = V_A + V_B + V_C = k\frac{q_A}{AM} + k\frac{q_B}{BM} + k\frac{q_C}{CM}$$

$$\Rightarrow V_M = k\frac{q}{\sqrt{a^2+x^2}} + k\frac{q}{\sqrt{a^2+x^2}} - 2k\frac{q}{b+x}$$

$$\Rightarrow V_M = 2kq\left(\frac{1}{\sqrt{a^2+x^2}} - \frac{1}{b+x}\right)$$

Exercice 3 :

- **La force électrostatique au point G :**



$$\overrightarrow{F_G} = \overrightarrow{F_{BG}} + \overrightarrow{F_{CG}} + \overrightarrow{F_{AG}}$$

$$\overrightarrow{F_{BG}} = k\frac{q_G q_B}{(GB)^2}\overrightarrow{u_{BG}},\quad \overrightarrow{F_{CG}} = k\frac{q_G q_C}{(CG)^2}\overrightarrow{u_{CG}},\quad \overrightarrow{F_{AG}} = k\frac{q_G q_A}{(AG)^2}\overrightarrow{u_{AG}}$$

Avec q_A= -2q, q_B= -q, q_C= - q et q_M= +q'
$AG^2= BG^2= CG^2=a^2/3$

$$\overrightarrow{u_{AG}} = -\vec{\jmath},\quad \overrightarrow{u_{BG}} = -\cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath} \text{ et } \overrightarrow{u_{CG}} = \cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath}$$

$$\theta = \frac{\pi}{6}(30°)\ donc\ \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\ et\ \sin\frac{\pi}{6} = \frac{1}{2}$$

$$\overrightarrow{u_{BG}} = -\frac{\sqrt{3}}{2}\vec{\imath} + \frac{1}{2}\vec{\jmath} \text{ et avec } \overrightarrow{u_{CG}} = \frac{\sqrt{3}}{2}\vec{\imath} + \frac{1}{2}\vec{\jmath}$$

$$\overrightarrow{F_{BG}} = k\frac{(-qq')}{a^2/3}\left(-\frac{\sqrt{3}}{2}\vec{\imath} + \frac{1}{2}\vec{\jmath}\right),\quad \overrightarrow{F_{CG}} = k\frac{(-qq')}{a^2/3}\left(\frac{\sqrt{3}}{2}\vec{\imath} + \frac{1}{2}\vec{\jmath}\right),\quad \overrightarrow{F_{AG}} = k\frac{(-2qq')}{a^2/3}(-\vec{\jmath})$$

$$\overrightarrow{F_G} = -3k\frac{qq'}{a^2}\left(\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\vec{\imath} + \left(\frac{1}{2} + \frac{1}{2} - 2\right)\vec{\jmath}\right)$$

$$\overrightarrow{F_G} = -3k\frac{qq'}{a^2}(-\vec{\jmath})$$

$$\overrightarrow{F_G} = 3k\frac{qq'}{a^2}\vec{\jmath}$$

- **Le champ électrique au point G**

$$\overrightarrow{F_G} = q_G\overrightarrow{E_G} \Rightarrow \overrightarrow{E_G} = \frac{\overrightarrow{F_G}}{q_G}$$
$$q_G = q'$$

$$\Rightarrow \overrightarrow{E_G} = \frac{\overrightarrow{F_G}}{q'} = 3k\frac{q}{a^2}\vec{j}$$

- **Le potentiel au point M**

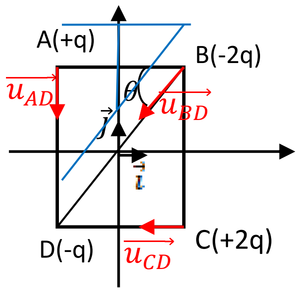
$$V_G = V_A + V_B + V_C = k\frac{q_A}{AG} + k\frac{q_B}{BG} + k\frac{q_C}{CG}$$
$$\Rightarrow V_G = k\frac{(-2q)}{a/\sqrt{3}} + k\frac{(-q)}{a/\sqrt{3}} + k\frac{(-q)}{a/\sqrt{3}}$$

$$\Rightarrow V_G = -4\sqrt{3}\,\frac{kq}{a}$$

Exercice 4 :

- **La force électrostatique au point D :**

$$\overrightarrow{F_D} = \overrightarrow{F_{BD}} + \overrightarrow{F_{CD}} + \overrightarrow{F_{AD}}$$



$$\overrightarrow{F_{BD}} = k\frac{q_D q_B}{(DB)^2}\overrightarrow{u_{BD}},\quad \overrightarrow{F_{CD}} = k\frac{q_D q_C}{(CD)^2}\overrightarrow{u_{CD}},\ \overrightarrow{F_{AD}} = k\frac{q_D q_A}{(AD)^2}\overrightarrow{u_{AD}}$$

Avec $q_A = +q$, $q_B = -2q$, $q_C = +2q$ et $q_D = -q$

AD=BD=a

$BD^2 = CD^2 + BC^2 = a^2 + a^2 = 2a^2$

$BD = a\sqrt{2}$

$\cos\theta = \frac{AB}{BD} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\sin\theta = \frac{AD}{BD} = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$ donc $\theta = \frac{\pi}{4}$

$$\cos\left(\frac{\pi}{2} - \varphi\right) = \sin\ \varphi$$
$$\sin\left(\frac{\pi}{2} - \varphi\right) = \cos\ \varphi$$

$\overrightarrow{u_{AD}} = -\vec{j}$, $\overrightarrow{u_{CD}} = -\ \vec{i}$ et $\overrightarrow{u_{BD}} = -\cos\ \theta\,\vec{i} - \sin\theta\,\vec{j}$

$$\theta = \frac{\pi}{4}\ donc\ \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\overrightarrow{u_{BD}} = -\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}$$

$$\overrightarrow{F_{BD}} = k\frac{(-2q)(-q)}{2a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) = k\frac{q^2}{a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),$$

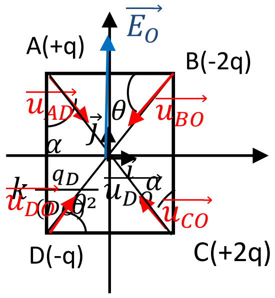
$$\overrightarrow{F_{CD}} = k\frac{(+2q)(-q)}{a^2}(-\vec{\imath}\,) = 2k\frac{q^2}{a^2}\vec{\imath},$$

$$\overrightarrow{F_{AD}} = k\frac{(+q)(-q)}{a^2}(-\vec{\jmath}\,) = k\frac{q^2}{a^2}\vec{\jmath}$$

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) + 2k\frac{q^2}{a^2}\vec{\imath} + k\frac{q^2}{a^2}\vec{\jmath}$$

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\left(\left(-\frac{\sqrt{2}}{2}+2\right)\vec{\imath} + \left(-\frac{\sqrt{2}}{2}+1\right)\vec{\jmath}\right)$$

- **Le champ électrique au point O :**



$$\overrightarrow{E_O} = \overrightarrow{E_{BO}} + \overrightarrow{E_{CO}} + \overrightarrow{E_{AO}} + \overrightarrow{E_{DO}}$$

$\overrightarrow{E_{BO}} = k\frac{q_B}{(oB)^2}\overrightarrow{u_{BO}}$, $\overrightarrow{E_{CO}} = k\frac{q_C}{(CO)^2}\overrightarrow{u_{CO}}$, $\overrightarrow{E_{AO}} = k\frac{q_A}{(AO)^2}\overrightarrow{u_{AO}}$, $\overrightarrow{E_{DO}} = k\frac{q_D}{(DO)^2}\overrightarrow{u_{DO}}$

Avec $q_A = +q$, $q_B = -2q$, $q_C = +2q$ et $q_D = -q$

$AO^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{2}$ donc $AO^2 = BO^2 = CO^2 = DO^2 = \frac{a^2}{2}$

(BO= $a\sqrt{2}/2$ donc $BO^2=a^2/2$)

$$\overrightarrow{u_{BO}} = -\cos\,\theta\,\vec{\imath} - \sin\theta\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}$$

$$\overrightarrow{u_{AO}} = \sin\,\alpha\,\vec{\imath} - \cos\alpha\,\vec{\jmath} = \frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath},$$

$$\overrightarrow{u_{CO}} = -\sin\,\alpha\,\vec{\imath} + \cos\alpha\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}$$

$\overrightarrow{u_{DO}} = \cos\,\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath} = \frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}$ (avec $\theta = \alpha = \frac{\pi}{4}$ $donc\ \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$)

$$\overrightarrow{E_{BO}} = k\frac{(-2q)}{a^2/2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),\quad \overrightarrow{E_{CO}} = k\frac{2q}{a^2/2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$$

$$\overrightarrow{E_{AO}} = k\frac{q}{a^2/2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),\quad \overrightarrow{E_{DO}} = k\frac{(-q)}{a^2/2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$$

donc $\overrightarrow{E_O} = 2k\frac{q}{a^2}\left[\left(-2\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\vec{\imath} + \left(-2\left(-\frac{\sqrt{2}}{2}\right) + 2\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\vec{\jmath}\right]$

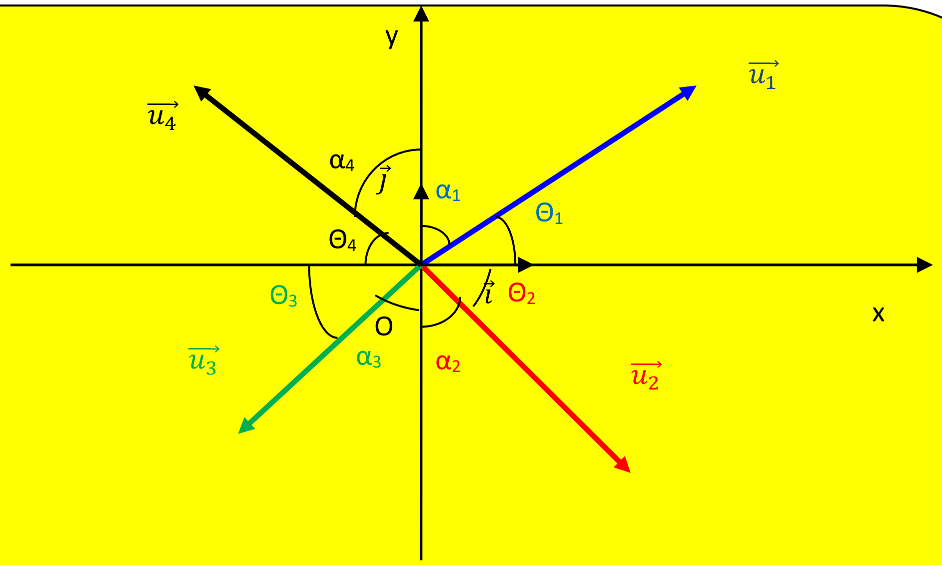
$$\Rightarrow \overrightarrow{E_O} = 2\sqrt{2}k\frac{q}{a^2}\vec{\jmath}$$

- **Le potentiel au point O**

$$V_O = V_A + V_B + V_C + V_D = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC} + k\frac{q_D}{OD}$$

$$\Rightarrow V_O = k\frac{(+q)}{a/\sqrt{2}} + k\frac{(-2q)}{a/\sqrt{2}} + k\frac{(2q)}{a/\sqrt{2}} + k\frac{(-q)}{a/\sqrt{2}}$$

$$\Rightarrow V_O = 0$$



$\overrightarrow{u_1} = \cos\theta_1\vec{\imath} + \sin\theta_1\vec{\jmath}$ ou $\overrightarrow{u_1} = \sin\alpha_1\vec{\imath} + \cos\alpha_1\vec{\jmath}$

$\overrightarrow{u_2} = \cos\theta_2\vec{\imath} - \sin\theta_2\vec{\jmath}$ ou $\overrightarrow{u_2} = \sin\alpha_2\vec{\imath} - \cos\alpha_2\vec{\jmath}$

$\overrightarrow{u_3} = -\cos\theta_3\vec{\imath} - \sin\theta_3\vec{\jmath}$ ou $\overrightarrow{u_3} = -\sin\alpha_3\vec{\imath} - \cos\alpha_3\vec{\jmath}$

$\overrightarrow{u_4} = -\cos\theta_4\vec{\imath} + \sin\theta_4\vec{\jmath}$ ou $\overrightarrow{u_4} = -\sin\alpha_4\vec{\imath} + \cos\alpha_4\vec{\jmath}$