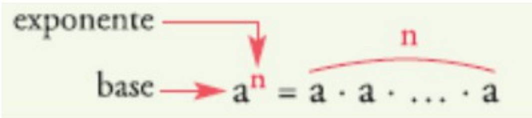
Chapter 2: Powers, Roots, and Logarithm

I. Introduction Exponents, Roots and logatithm are all related notions!
One of the most common areas relating to inverses where novices encounter trouble is in terms of powers, where the inverse operations are roots and logarithms. A root is the inverse operation of raising a quantity to a given power. A logarithm is the inverse operation of an exponential operation.

II. Powers and Exponents When multiplying two or more numbers, each number is called a factor of the product. When the same factor is repeated, one can use an exponent to simplify theyour writing. An exponent tells many times a number is used as a factor. A power is a number that is expressed using exponents ; it is used when we want to multiply a number by itself several times. In this term,



a is called base and n is called index or exponent. The word power sometimes also means the exponent alone rather than the result of an exponential expression.

*How to Say Powers

x^2 : x squared,
or x to the second power.

x^3 : x cubed.
or x to the third power.

x^n : x to the power of n ,
or x to the n -th power,
or x to the n ,
or x to the n -th,
or x upper n ,
or x raised n ,

$(x+y)^2$: x plus y all squared,
or bracket x plus y bracket closed squared,
or x plus y in bracket squared.

Exemples

- ♣ 5^2 :five to the second power or five squared
- ♣ 5^3 :five to the third power or five cubed
- ♣ 4^5 :four (raised) to the fifth power
- ♣ 15^{21} :fifteen to the twenty-first
- ♣ 33^{22} :thirty-three to the power of twenty-two

II. Properties of Powers Wh

A. for equal exponents: A product (fraction) raised by an exponent is equal to product (fraction) of factors raised by the same exponent:

$$(a\,b)^n=a^n b^n \quad \text{and} \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$$

A. for equal bases: A product (fraction) raised by an exponent is equal to product (fraction) of factors raised by the same exponent:

$$(a)^{n+m}=a^n a^m \quad \text{and} \quad a^{\frac{n}{m}}=\frac{a^n}{a^m}$$

III. Roots and Radicals

Root is inversion of exponentiation

$$\sqrt[n]{a}=b \quad \text{means that} \quad a=b^n$$

$\sqrt[n]{a}$ a is called radical expression (or radical form) because it contains a root.
The radical expression has several parts:

- the radical sign √❑

• the radicand: the entire quantity under the radical sign
• the index: the number that indicates the root that is being taken
Example: $\sqrt[3]{5}$: 5 is the radicand, 3 is the index.

The radical expression can be written in exponential form (powers with fractional exponents)

Example: $\sqrt[n]{a}=a^{\frac{1}{n}}$

So the law of powers can be used in calculating root

Example: $\sqrt[n]{ab}=(ab)^{\frac{1}{n}}=a^{\frac{1}{n}}b^{\frac{1}{n}}=\sqrt[n]{a}\sqrt[n]{b}$

*A number is said **perfect square** if its roots are integers.
example: 9, 16, 36, and 100 are perfect squares, but 12 and 20 are not.

***How to Say Radicals**

$\sqrt{x}$: (square) root of x

$\sqrt[3]{y}$: cube root of y

$\sqrt[n]{z}$; n-th root of z

$\sqrt[5]{x^2 y^3}$: fifth root of (pause) x squared times y cubed
fifth root of x squared times y cubed in bracket

***Square Root** The square root is in simplest form if:
a. the radicand does not contain perfect squares other than 1.
b. no fraction is contained in radicand.
c. no radicals appear in the denominator of a fraction.
Example $\sqrt{24}$ is not a simplest form because we can write it as $\sqrt{4\times 6}$ where 4 is a perfect square. We can simplify the radical into $2\sqrt{6}$

A radical and a number is called **a binomial**. Its **conjugate** is another binomial with the same number and radical, but the sign of second term is changed.
Example $5+\sqrt{7}$ is a binomial and its conjugate is $5-\sqrt{7}$

III. Logarithm

If $b = a^c$ <=> $c = log_a b$
a, b, c are *real numbers* and $b > 0,\ a > 0,\ a \neq 1$
a is called "base" of the logarithm.
$log_a b$ is pronounced as "the logarithm of b to base a ", "the base-a logarithm of b ", or most commonly "the log, base a, of b ")
Remark

1) In litterature you can find the notation ${}^a\log b$
2) There are standard notation of logarithms if the base is ***10*** or ***e***.
3) $\log_{10} b$ is denoted by $lg\, b$
   $\log_e b$ is denoted by $\log b$ or $\ln b$

Examples
${}^n\log x$: log x to the base of n or « log base n of x »
$\ln 2$: natural log of two or “L N” of two
${}^5\log^2 25$: log squared of twenty-five to the base of five or log base five of twenty-five all squared

***Laws for Logarithm**
First Law for logarithm: The logarithm of a product is equal to the sum of the logarithm of the factors

$${}^a\log(xy)={}^a\log(x)+{}^a\log(y)$$

Second Law for logarithm: The logarithm of a quotient is equal to the difference of the logarithms of the dividend and divisor

$$^{a}\log(x/y) = {}^{a}\log(x) - {}^{a}\log(y)$$

Third Law for logarithm: The logarithm of a power is equal to the exponent times the logarithm of the basis

$$^{a}\log(x^{y}) = y\,{}^{a}\log(x)$$