University of Tlemcen
Faculty of Sciences
Department of Mathematics

U.Y: 2023/2024
1st year LMD-M and MI

Correction of SW N° 01 Electricity

Part 1: ELECTROSTATICS "point charges »

Exercise 1 :

- The potentiel on point O :

$$V_O = V_A + V_B + V_C = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC}$$

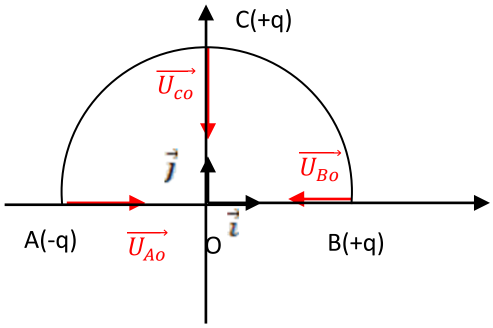
OA=OB=OC=R

$$V_O = k\frac{(-q)}{R} + k\frac{(+q)}{R} + k\frac{(+q)}{R} \Rightarrow V_O = k\frac{q}{R}$$

- Electric Field on point O :

$$\overrightarrow{E_O} = \overrightarrow{E_A} + \overrightarrow{E_B} + \overrightarrow{E_C}$$

with



$$\overrightarrow{E_A} = k\frac{q_A}{(OA)^2}\overrightarrow{u_{AO}}\ ,\ \overrightarrow{E_B} = k\frac{q_B}{(OB)^2}\overrightarrow{u_{BO}}$$

$$\overrightarrow{E_C} = k\frac{q_C}{(OC)^2}\overrightarrow{u_{CO}}$$

$$\overrightarrow{u_{AO}} = \vec{\imath},\ \ \overrightarrow{u_{BO}} = -\vec{\imath},\qquad \overrightarrow{u_{CO}} = -\vec{\jmath}$$

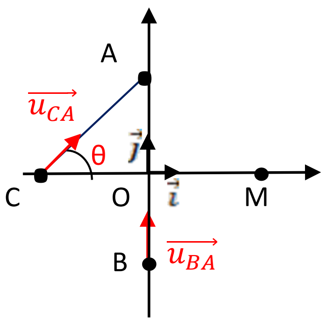
so $\overrightarrow{E_O} = k\frac{(-q)}{R^2}\vec{\imath} + k\frac{q}{R^2}(\overline{-\imath}) + k\frac{q}{R^2}(\overline{-\jmath}) \Rightarrow \overrightarrow{E_0} = -k\frac{q}{R^2}(2\vec{\imath} + \vec{\jmath})$

- Electrostatic Force on point O

$$\overrightarrow{F_0} = q'\,\overrightarrow{E_O} = q\overrightarrow{E_O} = -k\frac{q^2}{R^2}(2\vec{\imath} + \vec{\jmath})$$

Exercise 2

- **Electrostatic force exerted on the charge q_A :**



$$\overrightarrow{F_A} = \overrightarrow{F_{BA}} + \overrightarrow{F_{CA}}$$

$$\overrightarrow{F_{BA}} = k\frac{q_A q_B}{(BA)^2}\overrightarrow{u_{BA}},\ \ \overrightarrow{F_{CA}} = k\frac{q_A q_C}{(CA)^2}\overrightarrow{u_{CA}}$$

With q_A=+q, q_B=+q, q_C= - 2q and BO=AO=a and CO=b

BA=a+a=2a and CA= ?

According to Pitagort's right triangle rule (ACO) ; $CA^2=CO^2+OA^2$

$$CA^2 = a^2 + b^2$$

$\overrightarrow{u_{BA}} = \vec{j},\quad \overrightarrow{u_{CA}} = \cos\theta\,\vec{\imath} + sin\theta\,\vec{j}$ with $\cos\theta = \frac{CO}{CA} = \frac{b}{\sqrt{a^2+b^2}}$ and $\sin\theta = \frac{OA}{CA} = \frac{a}{\sqrt{a^2+b^2}}$

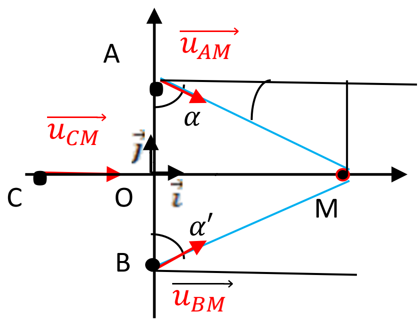
so $\overrightarrow{u_{CA}} = \frac{b}{\sqrt{a^2+b^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+b^2}}\,\vec{j}$

$$\overrightarrow{F_A} = k\frac{q^2}{4a^2}\vec{j} + k\frac{(-2q^2)}{a^2+b^2}\left(\frac{b}{\sqrt{a^2+b^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+b^2}}\,\vec{j}\right) \Rightarrow \overrightarrow{F_A} = kq^2\left[\frac{-2b}{(a^2+b^2)^{\frac{3}{2}}}\,\vec{\imath} + \left(\frac{1}{4a^2} - \frac{2a}{(a^2+b^2)^{\frac{3}{2}}}\right)\vec{j}\right]$$

- **Electrostatic force exerted on the charge q_M:**

$$\overrightarrow{F_M} = \overrightarrow{F_{BM}} + \overrightarrow{F_{CM}} + \overrightarrow{F_{AM}}$$

$$\overrightarrow{F_{BM}} = k\frac{q_M q_B}{(MA)^2}\overrightarrow{u_{BM}},\quad \overrightarrow{F_{CM}} = k\frac{q_M q_C}{(CM)^2}\overrightarrow{u_{CM}},\ \overrightarrow{F_{AM}} = k\frac{q_M q_A}{(AM)^2}\overrightarrow{u_{AM}}$$



with q_A=+q, q_B=+q, q_C= - 2q and q_M=+q
BO=AO=a, CO=b and OM=x

CM= b + x$\Rightarrow CM^2 = (b + x)^2$ and AM= ?, BM= ?

According to Pitagort's right triangle rule (AMO) ;

$AM^2=MO^2+OA^2 \Rightarrow AM^2 = a^2 + x^2$

According to Pitagort's right triangle rule (BMO) ;

$BM^2=MO^2+OB^2 \Rightarrow AM^2 = a^2 + x^2$

$\overrightarrow{u_{CM}} = \vec{\imath},\quad \overrightarrow{u_{AM}} = sin\ \alpha\,\vec{\imath} - \cos\alpha\,\vec{j}$

with $\cos\alpha = \frac{OA}{AM} = \frac{a}{\sqrt{a^2+x^2}}$ and $\sin\alpha = \frac{OM}{AM} = \frac{x}{\sqrt{a^2+x^2}}$

So $\overrightarrow{u_{AM}} = \frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} - \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}$ and $\overrightarrow{F_{AM}} = k\frac{(q^2)}{a^2+x^2}\left(\frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} - \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}\right)$

$\overrightarrow{u_{BM}} = sin\ \alpha'\vec{\imath} + \cos\alpha'\,\vec{j}$ with $\cos\alpha' = \frac{OB}{BM} = \frac{a}{\sqrt{a^2+x^2}}$ and $\sin\alpha' = \frac{OM}{BM} = \frac{x}{\sqrt{a^2+x^2}}$

so $\overrightarrow{u_{BM}} = \frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}$ and $\overrightarrow{F_{BM}} = k\frac{(q^2)}{a^2+x^2}\left(\frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}\right)$

$$\overrightarrow{F_M} = k\frac{-2q^2}{(b+x)^2}\vec{\imath} + k\frac{(q^2)}{a^2+b^2}\left(\frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} - \frac{a}{\sqrt{a^2+x^2}}\,\vec{j} + \frac{x}{\sqrt{a^2+x^2}}\,\vec{\imath} + \frac{a}{\sqrt{a^2+x^2}}\,\vec{j}\right)$$

$$\Rightarrow \overrightarrow{F_M} = kq^2\left[\left(\frac{2x}{(a^2+x^2)^{\frac{3}{2}}} - \frac{2}{(b+x)^2}\right)\vec{\imath}\right]$$

- **Electric field on point M**

$$\overrightarrow{F_M} = q_M \overrightarrow{E_M} \Rightarrow \overrightarrow{E_M} = \frac{\overrightarrow{F_M}}{q_M}$$

$$\Rightarrow \overrightarrow{E_M} = kq\left[\left(\frac{2x}{(a^2+x^2)^{\frac{3}{2}}} - \frac{2}{(b+x)^2}\right)\vec{\imath}\right]$$

- **Potentiel on point M**

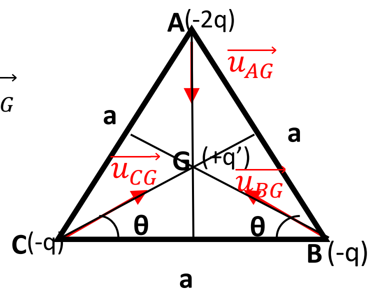
$$V_M = V_A + V_B + V_C = k\frac{q_A}{AM} + k\frac{q_B}{BM} + k\frac{q_C}{CM}$$

$$\Rightarrow V_M = k\frac{q}{\sqrt{a^2+x^2}} + k\frac{q}{\sqrt{a^2+x^2}} - 2k\frac{q}{b+x}$$

$$\Rightarrow V_M = 2kq\left(\frac{1}{\sqrt{a^2+x^2}} - \frac{1}{b+x}\right)$$

Exercise 3 :

- **Electrostatic Force on point G :**



$$\overrightarrow{F_G} = \overrightarrow{F_{BG}} + \overrightarrow{F_{CG}} + \overrightarrow{F_{AG}}$$

$\overrightarrow{F_{BG}} = k\frac{q_G q_B}{(GB)^2}\overrightarrow{u_{BG}}$, $\overrightarrow{F_{CG}} = k\frac{q_G q_C}{(CG)^2}\overrightarrow{u_{CG}}$, $\overrightarrow{F_{AG}} = k\frac{q_G q_A}{(AG)^2}\overrightarrow{u_{AG}}$

with q_A= -2q, q_B= -q, q_C= - q et q_M= +q'
AG^2= BG^2= CG^2=$a^2/3$

$\overrightarrow{u_{AG}} = -\vec{\jmath}$, $\overrightarrow{u_{BG}} = -\cos\ \theta\,\vec{\imath} + \sin\theta\,\vec{\jmath}$ and $\overrightarrow{u_{CG}} = \cos\ \theta\,\vec{\imath} + \sin\theta\,\vec{\jmath}$

$$\theta = \frac{\pi}{6}(30°)so\ \ cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\ and\ sin\frac{\pi}{6} = \frac{1}{2}$$

$\overrightarrow{u_{BG}} = -\frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}$ and with $\overrightarrow{u_{CG}} = \frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}$

$\overrightarrow{F_{BG}} = k\frac{(-qq')}{a^2/3}\left(-\frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}\right)$, $\overrightarrow{F_{CG}} = k\frac{(-qq')}{a^2/3}\left(\frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}\right)$, $\overrightarrow{F_{AG}} = k\frac{(-2qq')}{a^2/3}(-\vec{\jmath})$

$$\overrightarrow{F_G} = -3k\frac{qq'}{a^2}\left(\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\vec{\imath} + \left(\frac{1}{2} + \frac{1}{2} - 2\right)\vec{\jmath}\right)$$

$$\overrightarrow{F_G} = -3k\frac{qq'}{a^2}(-\vec{\jmath})$$

$$\overrightarrow{F_G} = 3k\frac{qq'}{a^2}\vec{\jmath}$$

- **Electric field on point G**

$$\overrightarrow{F_G} = q_G \overrightarrow{E_G} \Rightarrow \overrightarrow{E_G} = \frac{\overrightarrow{F_G}}{q_G}$$

$$q_G = q'$$

$$\Rightarrow \overrightarrow{E_G} = \frac{\overrightarrow{F_G}}{q'} = 3k\frac{q}{a^2}\vec{j}$$

- **Potentiel on point M**

$$V_G = V_A + V_B + V_C = k\frac{q_A}{AG} + k\frac{q_B}{BG} + k\frac{q_C}{CG}$$

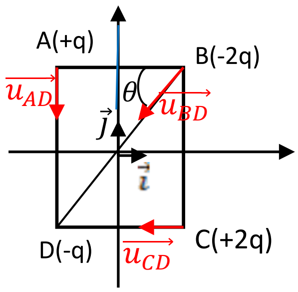
$$\Rightarrow V_G = k\frac{(-2q)}{a/\sqrt{3}} + k\frac{(-q)}{a/\sqrt{3}} + k\frac{(-q)}{a/\sqrt{3}}$$

$$\Rightarrow V_G = -4\sqrt{3}\,\frac{kq}{a}$$

Exercise 4 :

- **Electrostatic Force on point D :**

$$\overrightarrow{F_D} = \overrightarrow{F_{BD}} + \overrightarrow{F_{CD}} + \overrightarrow{F_{AD}}$$



$$\overrightarrow{F_{BD}} = k\frac{q_D q_B}{(DB)^2}\overrightarrow{u_{BD}},\quad \overrightarrow{F_{CD}} = k\frac{q_D q_C}{(CD)^2}\overrightarrow{u_{CD}},\ \overrightarrow{F_{AD}} = k\frac{q_D q_A}{(AD)^2}\overrightarrow{u_{AD}}$$

with q_A= +q, q_B= - 2q, q_C= +2q et q_D= -q

AD=BD=a

$BD^2 = CD^2 + BC^2 = a^2 + a^2 = 2a^2$

BD=$a\sqrt{2}$

$\text{Cos}\theta = \frac{AB}{BD} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\sin\theta = \frac{AD}{BD} = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$ so $\theta = \frac{\pi}{4}$

$\text{Cos}(\frac{\pi}{2} - \varphi) = \sin\ \varphi$

$\sin(\frac{\pi}{2} - \varphi) = \cos\varphi$

$\overrightarrow{u_{AD}} = -\vec{j}$, $\overrightarrow{u_{CD}} = -\vec{i}$ and $\overrightarrow{u_{BD}} = -\cos\theta\,\vec{i} - \sin\theta\,\vec{j}$

$$\theta = \frac{\pi}{4}\quad So\ \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\overrightarrow{u_{BD}} = -\frac{\sqrt{2}}{2}\vec{i} - \frac{\sqrt{2}}{2}\vec{j}$$

$$\overrightarrow{F_{BD}} = k\frac{(-2q)(-q)}{2a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) = k\frac{q^2}{a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),$$

$$\overrightarrow{F_{CD}} = k\frac{(+2q)(-q)}{a^2}(-\vec{\imath}\,) = 2k\frac{q^2}{a^2}\vec{\imath},$$

$$\overrightarrow{F_{AD}} = k\frac{(+q)(-q)}{a^2}(-\vec{\jmath}\,) = k\frac{q^2}{a^2}\vec{\jmath}$$

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) + 2k\frac{q^2}{a^2}\vec{\imath} + k\frac{q^2}{a^2}\vec{\jmath}$$

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\left(\left(-\frac{\sqrt{2}}{2}+2\right)\vec{\imath} + \left(-\frac{\sqrt{2}}{2}+1\right)\vec{\jmath}\right)$$

- **Electric Field on point O :**

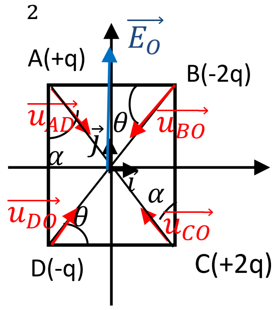
$$\overrightarrow{E_O} = \overrightarrow{E_{BO}} + \overrightarrow{E_{CO}} + \overrightarrow{E_{AO}} + \overrightarrow{E_{DO}}$$

$\overrightarrow{E_{BO}} = k\frac{q_B}{(oB)^2}\overrightarrow{u_{BO}}$, $\overrightarrow{E_{CO}} = k\frac{q_C}{(CO)^2}\overrightarrow{u_{CO}}$, $\overrightarrow{E_{AO}} = k\frac{q_A}{(AO)^2}\overrightarrow{u_{AO}}$, $\overrightarrow{E_{DO}} = k\frac{q_D}{(DO)^2}\overrightarrow{u_{DO}}$

and $q_A= +q$, $q_B= -2q$, $q_C= +2q$ et $q_D= -q$

$AO^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{2}$ So $AO^2 = BO^2 = CO^2 = DO^2 = \frac{a^2}{2}$

($BO= a\sqrt{2}/2$ So $BO^2=a^2/2$)



$\overrightarrow{u_{BO}} = -\cos\,\theta\,\vec{\imath} - \sin\theta\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}$

$\overrightarrow{u_{AO}} = \sin\,\alpha\,\vec{\imath} - \cos\alpha\,\vec{\jmath} = \frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath},$

$$\overrightarrow{u_{CO}} = -\sin\,\alpha\,\vec{\imath} + \cos\alpha\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}$$

$\overrightarrow{u_{DO}} = \cos\,\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath} = \frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}$ (with $\theta = \alpha = \frac{\pi}{4}$ So $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$)

$$\overrightarrow{E_{BO}} = k\frac{(-2q)}{a^2/2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),\quad \overrightarrow{E_{CO}} = k\frac{2q}{a^2/2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$$

$\overrightarrow{E_{AO}} = k\frac{q}{a^2/2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$, $\overrightarrow{E_{DO}} = k\frac{(-q)}{a^2/2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$

So $\overrightarrow{E_O} = 2k\frac{q}{a^2}\left[\left(-2\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\vec{\imath} + \left(-2\left(-\frac{\sqrt{2}}{2}\right) + 2\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\vec{\jmath}\right]$

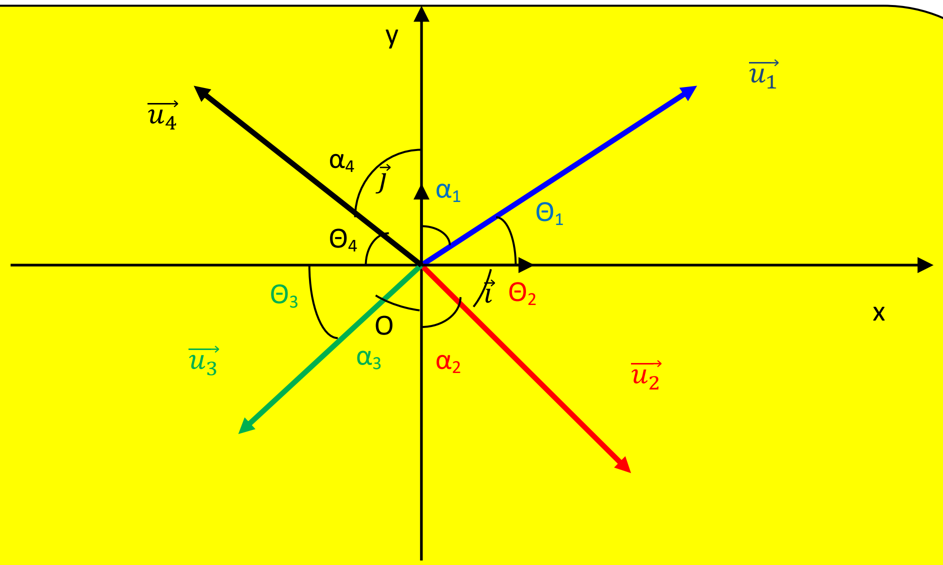
$$\Rightarrow \overrightarrow{E_O} = 2\sqrt{2}k\frac{q}{a^2}\vec{j}$$

- **Potentiel on point O**

$$V_O = V_A + V_B + V_C + V_D = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC} + k\frac{q_D}{OD}$$

$$\Rightarrow V_O = k\frac{(+q)}{a/\sqrt{2}} + k\frac{(-2q)}{a/\sqrt{2}} + k\frac{(2q)}{a/\sqrt{2}} + k\frac{(-q)}{a/\sqrt{2}}$$

$$\Rightarrow V_O = 0$$



$$\overrightarrow{u_1} = \cos\theta_1\vec{i} + \sin\theta_1\vec{j} \quad \text{ou} \quad \overrightarrow{u_1} = \sin\alpha_1\vec{i} + \cos\alpha_1\vec{j}$$

$$\overrightarrow{u_2} = \cos\theta_2\vec{i} - \sin\theta_2\vec{j} \quad \text{ou} \quad \overrightarrow{u_2} = \sin\alpha_2\vec{i} - \cos\alpha_2\vec{j}$$

$$\overrightarrow{u_3} = -\cos\theta_3\vec{i} - \sin\theta_3\vec{j} \quad \text{ou} \quad \overrightarrow{u_3} = -\sin\alpha_3\vec{i} - \cos\alpha_3\vec{j}$$

$$\overrightarrow{u_4} = -\cos\theta_4\vec{i} + \sin\theta_4\vec{j} \quad \text{ou} \quad \overrightarrow{u_4} = -\sin\alpha_4\vec{i} + \cos\alpha_4\vec{j}$$