Correction of SW N° 01 Electricity

Part 1: ELECTROSTATICS "point charges »

Exercise 1 :

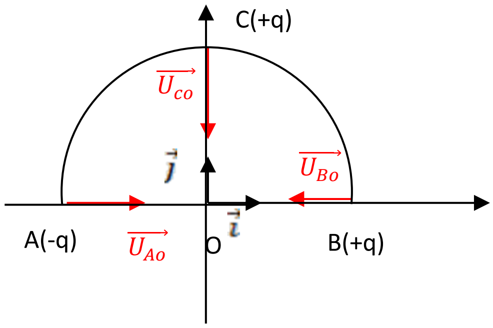
- The potentiel on point O :

$V_O = V_A + V_B + V_C = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC}$

OA=OB=OC=R

$V_O = k\frac{(-q)}{R} + k\frac{(+q)}{R} + k\frac{(+q)}{R} \Rightarrow V_O = k\frac{q}{R}$

- Electric Field on point O :



$\overrightarrow{E_O} = \overrightarrow{E_A} + \overrightarrow{E_B} + \overrightarrow{E_C}$

with

$\overrightarrow{E_A} = k\frac{q_A}{(OA)^2}\overrightarrow{u_{AO}}$, $\overrightarrow{E_B} = k\frac{q_B}{(OB)^2}\overrightarrow{u_{BO}}$

$\overrightarrow{E_C} = k\frac{q_C}{(OC)^2}\overrightarrow{u_{CO}}$

$\overrightarrow{u_{AO}} = \vec{\imath}$, $\overrightarrow{u_{BO}} = -\vec{\imath}$, $\overrightarrow{u_{CO}} = -\vec{\jmath}$

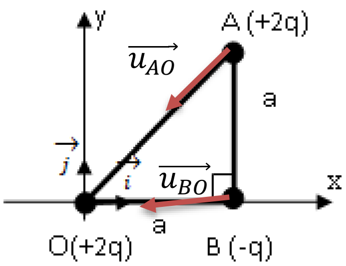
so $\overrightarrow{E_O} = k\frac{(-q)}{R^2}\vec{\imath} + k\frac{q}{R^2}(\overrightarrow{-\imath}) + k\frac{q}{R^2}(\overrightarrow{-\jmath}) \Rightarrow \overrightarrow{E_0} = -k\frac{q}{R^2}(2\vec{\imath} + \vec{\jmath})$

- Electrostatic Force on point O

$$\overrightarrow{F_0} = q'\overrightarrow{E_O} = q\overrightarrow{E_O} = -k\frac{q^2}{R^2}(2\vec{\imath} + \vec{\jmath})$$

Exercise 2

- **Electrostatic force exerted on the charge q_A :**



$$\overrightarrow{E_o} = \overrightarrow{E_{Bo}} + \overrightarrow{E_{Ao}}$$

$\overrightarrow{E_{Bo}} = k\frac{q_B}{(BO)^2}\overrightarrow{u_{Bo}}$, $\overrightarrow{E_{AO}} = k\frac{q_A}{(OA)^2}\overrightarrow{u_{oA}}$

With **q_A=+2q, q_B=-q, q_O=+2q** and **AB= OB= a**

According to Pitagort's right triangle rule (ABO) ; $OA^2=BO^2+BA^2$

$$OA^2 = a^2 + a^2 = 2a^2$$

$\overrightarrow{u_{BO}} = -\vec{\imath}, \quad \overrightarrow{u_{AO}} = -\cos\frac{\pi}{4}\,\vec{\imath} - \sin\frac{\pi}{4}\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}$

$\overrightarrow{E_o} = k\frac{-q}{a^2}\vec{\imath} - k\frac{(2q)}{2a^2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) \Rightarrow \overrightarrow{F_A} = -\frac{kq}{a^2}\left[\left(1+\frac{\sqrt{2}}{2}\right)\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right]$

- **Electric force on point O:**

$$\overrightarrow{F_O} = q_O\overrightarrow{E_O}$$

$$\Rightarrow \overrightarrow{F_O} = -\frac{k2q^2}{a^2}\left[\left(1+\frac{\sqrt{2}}{2}\right)\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right]$$

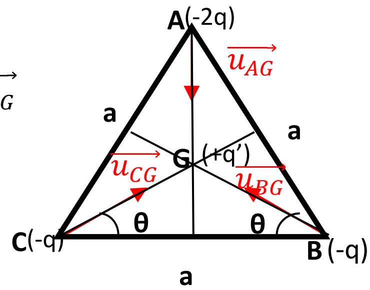
- **Potentiel on point O**

$$V_O = V_A + V_B = k\frac{q_A}{AO} + k\frac{q_B}{BO}$$

$$\Rightarrow V_M = k\frac{-q}{a} + k\frac{2q}{a\sqrt{2}}$$

Exercise 3 :

- **Electrostatic Force on point G :**



$$\overrightarrow{F_G} = \overrightarrow{F_{BG}} + \overrightarrow{F_{CG}} + \overrightarrow{F_{AG}}$$

$\overrightarrow{F_{BG}} = k\frac{q_G q_B}{(GB)^2}\overrightarrow{u_{BG}}, \quad \overrightarrow{F_{CG}} = k\frac{q_G q_C}{(CG)^2}\overrightarrow{u_{CG}}, \quad \overrightarrow{F_{AG}} = k\frac{q_G q_A}{(AG)^2}\overrightarrow{u_{AG}}$

with q_A= -2q, q_B= -q, q_C= - q et q_M= +q'
$AG^2 = BG^2 = CG^2 = a^2/3$

$\overrightarrow{u_{AG}} = -\vec{\jmath}, \quad \overrightarrow{u_{BG}} = -\cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath}$ and $\overrightarrow{u_{CG}} = \cos\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath}$

$$\theta = \frac{\pi}{6}(30°)\,so\ \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\ and\ \sin\frac{\pi}{6} = \frac{1}{2}$$

$\overrightarrow{u_{BG}} = -\frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}$ and with $\overrightarrow{u_{CG}} = \frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}$

$\overrightarrow{F_{BG}} = k\frac{(-qq')}{a^2/3}\left(-\frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}\right), \quad \overrightarrow{F_{CG}} = k\frac{(-qq')}{a^2/3}\left(\frac{\sqrt{3}}{2}\,\vec{\imath} + \frac{1}{2}\,\vec{\jmath}\right), \quad \overrightarrow{F_{AG}} = k\frac{(-2qq')}{a^2/3}(-\vec{\jmath})$

$$\overrightarrow{F_G} = -3k\frac{qq'}{a^2}\left(\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)\vec{\imath} + \left(\frac{1}{2} + \frac{1}{2} - 2\right)\vec{\jmath}\right)$$

$$\overrightarrow{F_G} = -3k\frac{qq'}{a^2}(-\vec{\jmath})$$

$$\overrightarrow{F_G} = 3k\frac{qq'}{a^2}\vec{\jmath}$$

- **Electric field on point G**

$$\overrightarrow{F_G} = q_G\overrightarrow{E_G} \Rightarrow \overrightarrow{E_G} = \frac{\overrightarrow{F_G}}{q_G}$$
$$q_G = q'$$

$$\Rightarrow \overrightarrow{E_G} = \frac{\overrightarrow{F_G}}{q'} = 3k\frac{q}{a^2}\vec{j}$$

- **Potentiel on point M**

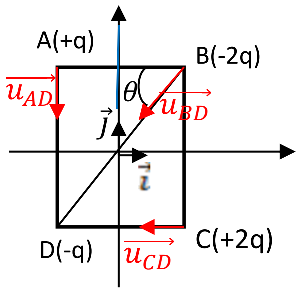
$$V_G = V_A + V_B + V_C = k\frac{q_A}{AG} + k\frac{q_B}{BG} + k\frac{q_C}{CG}$$
$$\Rightarrow V_G = k\frac{(-2q)}{a/\sqrt{3}} + k\frac{(-q)}{a/\sqrt{3}} + k\frac{(-q)}{a/\sqrt{3}}$$

$$\Rightarrow V_G = -4\sqrt{3}\,\frac{kq}{a}$$

Exercise 4 :

- **Electrostatic Force on point D :**

$$\overrightarrow{F_D} = \overrightarrow{F_{BD}} + \overrightarrow{F_{CD}} + \overrightarrow{F_{AD}}$$



$$\overrightarrow{F_{BD}} = k\frac{q_Dq_B}{(DB)^2}\overrightarrow{u_{BD}},\quad \overrightarrow{F_{CD}} = k\frac{q_Dq_C}{(CD)^2}\overrightarrow{u_{CD}},\ \overrightarrow{F_{AD}} = k\frac{q_Dq_A}{(AD)^2}\overrightarrow{u_{AD}}$$

with q_A= +q, q_B= - 2q, q_C= +2q et q_D= -q

AD=BD=a

$BD^2 = CD^2 + BC^2 = a^2 + a^2 = 2a^2$

BD=$a\sqrt{2}$

Cosθ=$\frac{AB}{BD} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, sinθ=$\frac{AD}{BD} = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2}$ so θ=$\frac{\pi}{4}$

$\cos(\frac{\pi}{2}-\varphi) = \sin\ \varphi$

$\sin(\frac{\pi}{2}-\varphi) = \cos\ \varphi$

$$\overrightarrow{u_{AD}} = -\vec{j},\quad \overrightarrow{u_{CD}} = -\ \vec{i}\ \text{ and }\ \overrightarrow{u_{BD}} = -\cos\ \theta\,\vec{i} - \sin\theta\,\vec{j}$$

$$\theta = \frac{\pi}{4}\quad So\ \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\overrightarrow{u_{BD}} = -\frac{\sqrt{2}}{2}\,\vec{i} - \frac{\sqrt{2}}{2}\,\vec{j}$$

$$\overrightarrow{F_{BD}} = k\frac{(-2q)(-q)}{2a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) = k\frac{q^2}{a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),$$

$$\overrightarrow{F_{CD}} = k\frac{(+2q)(-q)}{a^2}(-\vec{\imath}\,) = 2k\frac{q^2}{a^2}\vec{\imath},$$

$$\overrightarrow{F_{AD}} = k\frac{(+q)(-q)}{a^2}(-\vec{\jmath}\,) = k\frac{q^2}{a^2}\vec{\jmath}$$

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right) + 2k\frac{q^2}{a^2}\vec{\imath} + k\frac{q^2}{a^2}\vec{\jmath}$$

$$\overrightarrow{F_D} = k\frac{q^2}{a^2}\left(\left(-\frac{\sqrt{2}}{2} + 2\right)\vec{\imath} + \left(-\frac{\sqrt{2}}{2} + 1\right)\vec{\jmath}\right)$$

- **Electric Field on point O :**

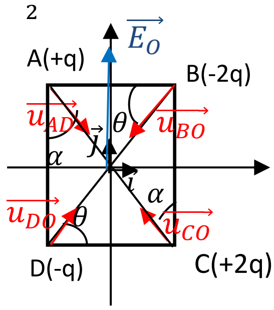
$$\overrightarrow{E_O} = \overrightarrow{E_{BO}} + \overrightarrow{E_{CO}} + \overrightarrow{E_{AO}} + \overrightarrow{E_{DO}}$$

$$\overrightarrow{E_{BO}} = k\frac{q_B}{(oB)^2}\overrightarrow{u_{BO}},\quad \overrightarrow{E_{CO}} = k\frac{q_C}{(CO)^2}\overrightarrow{u_{CO}},\ \overrightarrow{E_{AO}} = k\frac{q_A}{(AO)^2}\overrightarrow{u_{AO}},\ \overrightarrow{E_{DO}} = k\frac{q_D}{(DO)^2}\overrightarrow{u_{DO}}$$

and $q_A = +q$, $q_B = -2q$, $q_C = +2q$ et $q_D = -q$

$AO^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2 = \frac{a^2}{2}$ So $AO^2 = BO^2 = CO^2 = DO^2 = \frac{a^2}{2}$

($BO = a\sqrt{2}/2$ So $BO^2 = a^2/2$)



$$\overrightarrow{u_{BO}} = -\cos\,\theta\,\vec{\imath} - \sin\theta\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}$$

$$\overrightarrow{u_{AO}} = \sin\,\alpha\,\vec{\imath} - \cos\alpha\,\vec{\jmath} = \frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath},$$

$$\overrightarrow{u_{CO}} = -\sin\,\alpha\,\vec{\imath} + \cos\alpha\,\vec{\jmath} = -\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}$$

$$\overrightarrow{u_{DO}} = \cos\,\theta\,\vec{\imath} + \sin\theta\,\vec{\jmath} = \frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\quad \left(\text{with } \theta = \alpha = \frac{\pi}{4}\ So\ \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}\right)$$

$$\overrightarrow{E_{BO}} = k\frac{(-2q)}{a^2/2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),\quad \overrightarrow{E_{CO}} = k\frac{2q}{a^2/2}\left(-\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$$

$$\overrightarrow{E_{AO}} = k\frac{q}{a^2/2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} - \frac{\sqrt{2}}{2}\,\vec{\jmath}\right),\quad \overrightarrow{E_{DO}} = k\frac{(-q)}{a^2/2}\left(\frac{\sqrt{2}}{2}\,\vec{\imath} + \frac{\sqrt{2}}{2}\,\vec{\jmath}\right)$$

So $\overrightarrow{E_O} = 2k\frac{q}{a^2}\left[\left(-2\left(-\frac{\sqrt{2}}{2}\right) + 2\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\vec{\imath} + \left(-2\left(-\frac{\sqrt{2}}{2}\right) + 2\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)\vec{\jmath}\right]$

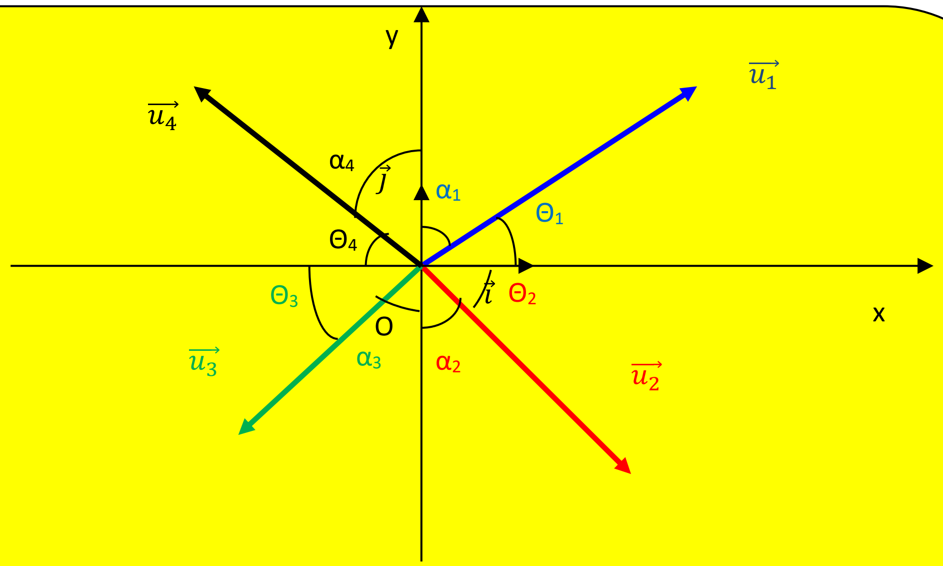
$$\Rightarrow \overrightarrow{E_O} = 2\sqrt{2}k\frac{q}{a^2}\vec{j}$$

- **Potentiel on point O**

$$V_O = V_A + V_B + V_C + V_D = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC} + k\frac{q_D}{OD}$$

$$\Rightarrow V_O = k\frac{(+q)}{a/\sqrt{2}} + k\frac{(-2q)}{a/\sqrt{2}} + k\frac{(2q)}{a/\sqrt{2}} + k\frac{(-q)}{a/\sqrt{2}}$$

$$\Rightarrow V_O = 0$$



$\overrightarrow{u_1} = \cos\theta_1\vec{\imath} + \sin\theta_1\vec{\jmath}$ ou $\overrightarrow{u_1} = \sin\alpha_1\vec{\imath} + \cos\alpha_1\vec{\jmath}$

$\overrightarrow{u_2} = \cos\theta_2\vec{\imath} - \sin\theta_2\vec{\jmath}$ ou $\overrightarrow{u_2} = \sin\alpha_2\vec{\imath} - \cos\alpha_2\vec{\jmath}$

$\overrightarrow{u_3} = -\cos\theta_3\vec{\imath} - \sin\theta_3\vec{\jmath}$ ou $\overrightarrow{u_3} = -\sin\alpha_3\vec{\imath} - \cos\alpha_3\vec{\jmath}$

$\overrightarrow{u_4} = -\cos\theta_4\vec{\imath} + \sin\theta_4\vec{\jmath}$ ou $\overrightarrow{u_4} = -\sin\alpha_4\vec{\imath} + \cos\alpha_4\vec{\jmath}$