University of Tlemcen
Faculty of Sciences
Department of Mathematics

U.Y: 2023/2024
1st year LMD-M and MI

Correction of SW N° 01 Electricity

Part 2: ELECTROSTATICS

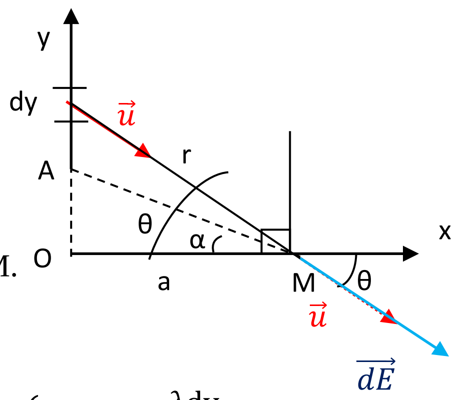
Charges distribution

Exercise 5 :

- The electric field components dEx and dEy resulting from the charge in the elementary element of length dy defined by the angle θ.

The elementary electric field $d\vec{E}$, at point M, created by the linear charge element dq present in the element of length dl.

The charge is on the axis (Oy) so dl=dy and dq=λdy



$d\vec{E} = k\frac{dq}{r^2}\,\vec{u}$

(with r is the distance between the elementary charge dq and the point M) and $\vec{u}$ is directed from dq to the point M.

$\vec{u} = \cos\theta\ \vec{\imath} - \sin\theta\,\vec{\jmath}$

$$d\vec{E} = k\frac{dq}{r^2}\,\vec{u} = k\frac{\lambda dy}{r^2}(\cos\theta\ \vec{\imath} - \sin\theta\,\vec{\jmath}) \Rightarrow \begin{cases} dE_X = k\dfrac{\lambda dy}{r^2}cos\theta \\ dE_y = -k\dfrac{\lambda dy}{r^2}sin\theta \end{cases}$$

We have three variables: y, r and θ; we need to choose one variable and write the other two as a function of this variable

In this case the variable chosen is θ which varies from α to π/2.

Write r and y as functions of θ π/2.

$\cos\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\cos\theta}$ with « a » is the distance OM and it does not depend on θ.

$$\text{tg}\,\theta = \frac{y}{a} \Rightarrow y = a\ \text{tg}\theta$$

$$\Rightarrow dy = a\,d(tg\,\theta) = \frac{a}{cos^2\theta}d\theta$$

$$\begin{cases} dE_X = k\dfrac{\lambda\dfrac{a}{cos^2\theta}d\theta}{\dfrac{a^2}{cos^2\theta}}cos\theta \\ dE_y = -k\dfrac{\lambda\dfrac{a}{cos^2\theta}d\theta}{\dfrac{a^2}{cos^2\theta}}sin\theta \end{cases} \Rightarrow \begin{cases} dE_X = k\dfrac{\lambda}{a}cos\theta d\theta \\ dE_y = -k\dfrac{\lambda}{a}sin\theta d\theta \end{cases}$$

- **The Ex and Ey components of the electric field created by the wire (Ay) and its modulus:**

The studied load changes or is located from point A, corresponding to angle α, to infinity, corresponding to angle π/2.

$$\begin{cases} E_X = \int_{\alpha}^{\pi/2} dE_X = k\dfrac{\lambda}{a}\int_{\alpha}^{\pi/2} cos\theta d\theta \\ E_y = \int_{\alpha}^{\pi/2} dE_y = -k\dfrac{\lambda}{a}\int_{\alpha}^{\pi/2} sin\theta d\theta \end{cases} \Rightarrow \begin{cases} E_X = k\dfrac{\lambda}{a}(\sin(\pi/2) - sin\alpha) \\ E_y = k\dfrac{\lambda}{a}(-\cos(\pi/2) + \cos\alpha) \end{cases}$$

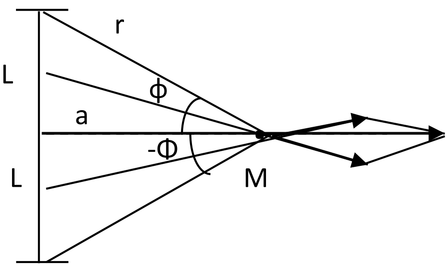
$$\Rightarrow \begin{cases} E_X = k\dfrac{\lambda}{a}(1 - sin\alpha) \\ E_y = k\dfrac{\lambda}{a}cos\alpha \end{cases}$$

$$\Rightarrow \vec{E} = k\frac{\lambda}{a}(1 - sin\alpha)\,\vec{\imath} + k\frac{\lambda}{a}cos\alpha\,\vec{\jmath}$$

The modulus of the electric field: $|\vec{E}| = \sqrt{\left(k\frac{\lambda}{a}(1 - sin\alpha)\right)^2 + \left(k\frac{\lambda}{a}cos\alpha\right)^2}$

- The expression of the electric field at the point M equidistant from the ends of the wire of length 2L:

In this case, the angle θ varies from (-Φ) to Φ.



$$\begin{cases} E_X = \int_{-\Phi}^{\Phi} dE_X = k\frac{\lambda}{a}\int_{-\Phi}^{\Phi} cos\theta d\theta \\ E_y = \int_{-\Phi}^{\Phi} dE_y = -k\frac{\lambda}{a}\int_{-\Phi}^{\Phi} sin\theta d\theta \end{cases}$$

$$\Rightarrow \begin{cases} E_X = k\frac{\lambda}{a}(\sin\Phi - sin(-\Phi)) \\ E_y = -k\frac{\lambda}{a}(-\cos\Phi - (-\cos(-\Phi))) \end{cases}$$

$$\Rightarrow \begin{cases} E_X = k\dfrac{\lambda}{a}(sin\Phi + sin\Phi) \\ E_y = k\dfrac{\lambda}{a}(cos\,\alpha - \cos\alpha) = 0 \end{cases}$$

$$\Rightarrow \vec{E} = 2k\frac{\lambda}{a}sin\Phi\,\vec{\imath}$$

When symmetrical about the (Ox) axis, the electric field will have a single component along the x-axis, the other component being zero.

$sin\Phi = \frac{L}{r} = \frac{L}{\sqrt{L^2+a^2}}$ so $\vec{E} = 2k\frac{\lambda L}{a\sqrt{L^2+a^2}}\vec{\imath}$, $r = \sqrt{L^2+a^2}$

- The electric field for an infinite wire

In this case θ varies from ($-\pi/2$) to $\pi/2$

$$\begin{cases} E_X = \int_{-\pi/2}^{\pi/2} dE_X = k\frac{\lambda}{a}\int_{-\pi/2}^{\pi/2} cos\theta d\theta \\ E_y = \int_{-\pi/2}^{\pi/2} dE_y = -k\frac{\lambda}{a}\int_{-\pi/2}^{\pi/2} sin\theta d\theta \end{cases}$$

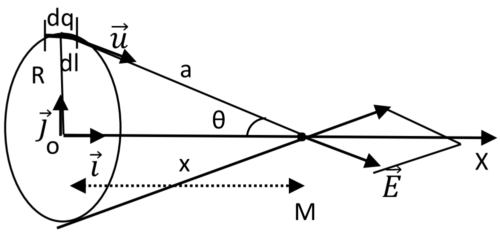
$$\Rightarrow \begin{cases} E_X = k\frac{\lambda}{a}(\sin\pi/2 - sin(-\pi/2)) \\ E_y = -k\frac{\lambda}{a}(-\cos\pi/2 + \cos(-\pi/2)) \end{cases}$$

$$\Rightarrow \begin{cases} E_X = k\frac{\lambda}{a}(sin\,\pi/2 + \sin\pi/2) \\ E_y = k\frac{\lambda}{a}(cos\,\pi/2 - \cos\pi/2) = 0 \end{cases}$$

$$\Rightarrow \vec{E} = 2k\frac{\lambda}{a}\,\vec{\imath}$$

<u>Exercise 6 :</u>

1- **Electrostatic field We're looking for the elementary field ($\overrightarrow{dE}$) created by the Charge element dq present in the element of length dl.**



We are looking for the elementary field $\overrightarrow{dE}$ created by the charge element dq present in the element of length dl. (dq=λdl)

$$\overrightarrow{dE} = \frac{kdq}{a^2}\vec{u} \Rightarrow \overrightarrow{dE} = \frac{k\lambda dl}{R^2+x^2}(\,cos\theta\vec{\imath} - sin\theta\vec{\jmath})$$

According to the relationship of Pitagorth : $a^2=R^2+x^2$ and $\vec{u}= cos\theta\,\vec{\imath} - sin\theta\,\vec{\jmath}$

We have symmetry with respect to the axis (Ox), so the electric field has a single component E_x, (E_y=0) and $cos\theta = \frac{x}{a} = \frac{x}{\sqrt{R^2+x^2}}$

So $dE_x = \frac{k\lambda}{R^2+x^2}\frac{x}{\sqrt{R^2+x^2}}dl \Rightarrow dE_x = k\lambda\frac{x}{(R^2+x^2)^{\frac{3}{2}}}dl$, (There is a single variable (l) and x and R are constant with respect to l).

$E_x = k\lambda\frac{x}{(R^2+x^2)^{\frac{3}{2}}}\int_0^{2\pi R} dl = k\lambda\frac{x}{(R^2+x^2)^{\frac{3}{2}}}\,2\pi R$ with $k=1/(4\pi\varepsilon_0)$

So $E = \frac{\lambda}{2\varepsilon_0}\ \frac{x\,R}{(R^2+x^2)^{\frac{3}{2}}}$

2- Electrostatic Potentiel :

$\vec{E} = -\overrightarrow{grad}\,V = -\frac{dV}{dx}\vec{\imath} \Rightarrow E = -\frac{dV}{dx}$

$$V = -\int Edx = -\frac{\lambda R}{2\varepsilon_0}\frac{1}{2}\int \frac{2x}{(R^2+x^2)^{\frac{3}{2}}}dx$$

$$\int U'U^n = \frac{U^{n+1}}{n+1}\ pour \int \frac{U'}{U} = lnU$$

$U=R^2+x^2$; n=-3/2, U'=2x so $\frac{(R^2+x^2)^{\frac{-3}{2}+1}}{\frac{-3}{2}+1} = \frac{(R^2+x^2)^{\frac{-1}{2}}}{\frac{-1}{2}} = \frac{-2}{\sqrt{R^2+x^2}}$

$$V = -\int Edx = -\frac{\lambda R}{2\varepsilon_0}\frac{1}{2}\left(\frac{-2}{\sqrt{R^2+x^2}}\right)$$

So $V = \frac{\lambda R}{2\varepsilon_0}\frac{1}{\sqrt{R^2+x^2}}$

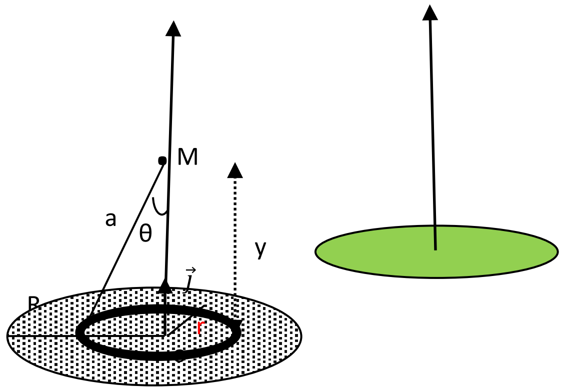
<u>**Exercise 7 :**</u>

1- Electrostatic Potentiel :

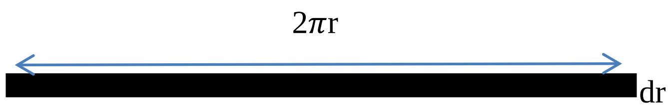
We are looking for the elemental potential dV created by the Charge element dq present in the elemental surface. ds. (dq=σds)

$dv = k\frac{dq}{r}$

dq=σds



The elementary surface in this case is a ring of radius r (with 0<r<R) and thickness dr and ds=2πrdr.



ds=2πrdr

S=$\pi r^2 \Rightarrow ds = 2\pi r dr$

$$dv = k\frac{dq}{a} = k\frac{\sigma ds}{\sqrt{r^2+y^2}} = k\frac{\sigma 2\pi rdr}{\sqrt{r^2+y^2}}$$

$$v = k\sigma\pi\int_0^R \frac{2rdr}{\sqrt{r^2+y^2}}$$

$$\int U'U^n = \frac{U^{n+1}}{n+1}$$

U=r^2+y^2, n=-1/2 ; U'=2r dr

$$v = k\sigma\pi\int_0^R 2rdr(r^2+y^2)^{\frac{-1}{2}}$$

$$v = k\sigma\pi\frac{(r^2+y^2)^{\frac{1}{2}}}{\frac{1}{2}}$$

$$v = 2k\sigma\pi\sqrt{r^2+y^2}$$

$$v = 2k\sigma\pi\left(\sqrt{R^2+y^2}-\sqrt{y^2}\right)$$

$$v = 2k\sigma\pi\left(\sqrt{R^2+y^2}-|y|\right)$$

$$v = k\sigma\pi\int_0^R \frac{2rdr}{\sqrt{r^2+y^2}} = 2k\sigma\pi\sqrt{R^2+y^2} - 2k\sigma\pi\sqrt{0+y^2}$$

$$v = 2k\sigma\pi\left(\sqrt{R^2+y^2}-|y|\right)$$

$$v =: \begin{cases} y>0 & v = 2k\sigma\pi\left(\sqrt{R^2+y^2}-y\right) \\ y<0 & v = 2k\sigma\pi\left(\sqrt{R^2+y^2}+y\right) \end{cases}$$

- **Calculate of electrostatic Feild :**

$\overrightarrow{E} = -\overrightarrow{grad}\, V = -\frac{dV}{dy}\vec{j} \Rightarrow E = -\frac{dV}{dy}$

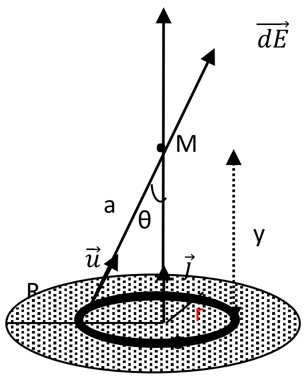
$$\overrightarrow{E} = \begin{cases} y > 0 & E_y = -2k\sigma\pi\left(\dfrac{\mathrm{y}}{\sqrt{\mathrm{R}^2+\mathrm{y}^2}} - 1\right) \\ y < 0 & E_y = -2k\sigma\pi\left(\dfrac{\mathrm{y}}{\sqrt{\mathrm{R}^2+\mathrm{y}^2}} + 1\right) \end{cases}$$

- **Calculating the electric field using the direct method:**

with dq=σds= dq=σ2πrdr, according to the relationship of Pitagorth:

$$a^2=r^2+y^2 \text{ and } \vec{u}= sin\theta\vec{\imath} + cos\theta\vec{\jmath}$$

We have symmetry with respect to the axis (Oy), so the electric field has a single component E_y, (E_x=0) and $cos\theta = \frac{y}{a} = \frac{y}{\sqrt{r^2+y^2}}$.



$\overrightarrow{dE} = \dfrac{kdq}{a^2}\vec{u} \Rightarrow \overrightarrow{dE} = \dfrac{k\sigma ds}{R^2+x^2}(sin\theta\vec{\imath} + cos\theta\vec{\jmath})$

so $dE_y = \frac{k\sigma 2\pi rdr}{r^2+y^2}\frac{y}{\sqrt{r^2+y^2}} \Rightarrow dE_y = k\sigma\pi y\frac{2rdr}{(r^2+y^2)^{\frac{3}{2}}}$

$E_y = k\sigma\pi y\int_0^R \frac{2rdr}{(r^2+y^2)^{\frac{3}{2}}} = -2k\sigma\pi y\left(\frac{1}{\sqrt{R^2+y^2}} - \frac{1}{|y|}\right)$,

with k=1/(4$\pi\varepsilon_0$)

so we have tow cases

$$E = E_y\colon \begin{cases} y > 0 & E_y = -\dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{\mathrm{y}}{\sqrt{R^2+y^2}} - 1\right) \\ y < 0 & E_y = -\dfrac{\sigma}{2\varepsilon_0}\left(\dfrac{\mathrm{y}}{\sqrt{R^2+y^2}} + 1\right) \end{cases}$$

2- The electric field as the disk radius R tends towards infinity :

$\lim_{R\to\infty} E\colon \begin{cases} y > 0 & E_y = \frac{\sigma}{2\varepsilon_0} \\ y < 0 & E_y = -\frac{\sigma}{2\varepsilon_0} \end{cases}$ so $\lim\limits_{R\to\infty}|E| = \frac{\sigma}{2\varepsilon_0}$