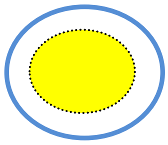
University of Tlemcen
Faculty of Sciences
Department of Mathematics

Correction of SW N°02 of Electricity
Gauss's theorem

Exercise 1 :



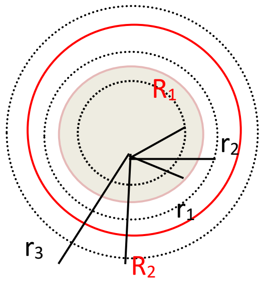
The Gaussian surface is a sphere with center O and radius r.
For reasons of symmetry, the field is radial and constant at any point on the Gaussian surface.

$$\emptyset = \oiint \vec{E}.\overrightarrow{ds} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$\vec{E} \parallel \overrightarrow{ds}$ Donc : $\oiint \vec{E}.\overrightarrow{ds} = \iint E.ds = E \iint ds = E.S = E4\pi r^2 \Rightarrow \boldsymbol{E4\pi r^2} = \frac{\boldsymbol{\sum Q_{int}}}{\boldsymbol{\varepsilon_0}}$

$$\Rightarrow \boldsymbol{E} = \frac{\boldsymbol{Q_{int}}}{\boldsymbol{4\pi r^2 \varepsilon_0}} \quad (*)$$

1- The electrostatic field E(r) at any point in space.



we have 3 cases :

1st case r<R₁ ($r \in [0, R_1[$

$dq = \rho dv = \rho 4\pi r^2 dr \Rightarrow Q_{int} = \rho \iiint dv = \rho 4\pi \int_0^r r^2 dr$

$\Rightarrow Q_{int} = \rho \ \frac{4}{3}\pi r_1^3$

so $(*) \Rightarrow E_1 = \frac{\rho \ \frac{4}{3}\pi r_1^3}{4\pi r^2 \varepsilon_0}$ then $\boldsymbol{E_1} = \frac{\boldsymbol{\rho}}{\boldsymbol{3\varepsilon_0}}\boldsymbol{r_1}$

2nd case $R_1 \le r < R_2$ ($r \in [R_1, R_2[$

$dq = \rho dv = \rho 4\pi r_2^2 dr \Rightarrow Q_{int} = \rho \iiint dv = \rho 4\pi \int_0^{R_1} r^2 dr$

so $Q_{int} = \rho \ \frac{4}{3}\pi R_1^3$

$$(*) \Rightarrow E_2 = \frac{\rho \ \frac{4}{3}\pi R_1^3}{4\pi r^2 \varepsilon_0} \quad so \ \boldsymbol{E_2} = \frac{\boldsymbol{\rho \ R_1^3}}{\boldsymbol{3\varepsilon_0 r^2}}$$

3rd case $r \ge R_2$ ($r \in [R_2, +\infty[$)

$Q_{int} = Q_1 + Q_2$ with $Q_1 = \rho \ \frac{4}{3}\pi R_1^3$ and $dq_2 = \sigma ds \Rightarrow Q_2 = \sigma 4\pi R_2^2$

So $Q_{int} = \rho \ \frac{4}{3}\pi R_1^3 + \sigma 4\pi R_2^2$

Then $(*) \Rightarrow E_3 = \frac{\rho \ \frac{4}{3}\pi R_1^3 + \sigma 4\pi R_2^2}{4\pi r^2 \varepsilon_0}$ hence $\boldsymbol{E_3} = \frac{\boldsymbol{\rho \ R_1^3}}{\boldsymbol{3\varepsilon_0 r^2}} + \frac{\boldsymbol{\sigma R_2^2}}{\boldsymbol{\varepsilon_0 r^2}}$

1- The electric potential v(r) at any point in space.

$\vec{E} = -\overrightarrow{grad}v \Rightarrow E = -\frac{dv}{dr}$

so $v = -\int E dr$

<u>1st case : r<R1 ($r \in [0, R_1[$)</u>

$$E_1 = \frac{\rho}{3\varepsilon_0} r \Rightarrow v_1 = -\frac{\rho}{3\varepsilon_0}\int r dr \text{ so } \quad \boldsymbol{v_1 = -\frac{\rho}{6\varepsilon_0} r^2 + C_1}$$

<u>2st case R1 ≤ r<R2 ($r \in [R_1, R_2[$</u>

$$E_2 = \frac{\rho R_1^3}{3\varepsilon_0 r^2} \Rightarrow v_2 = -\frac{\rho R_1^3}{3\varepsilon_0}\int \frac{1}{r^2} dr \text{ so } \quad \boldsymbol{v_2 = \frac{\rho R_1^3}{3\varepsilon_0}\frac{1}{r} + C_2}$$

<u>3rd case r≥R2 ($r \in [R_2, +\infty[$)</u>

$$E_3 = \frac{\rho R_1^3}{3\varepsilon_0 r^2} + \frac{\sigma R_2^2}{\varepsilon_0 r^2} \Rightarrow v_3 = -\left(\frac{\rho R_1^3}{3\varepsilon_0} + \frac{\sigma R_2^2}{\varepsilon_0}\right)\int \frac{1}{r^2} dr$$

So $\boldsymbol{v_3 = \left(\frac{\rho R_1^3}{3\varepsilon_0} + \frac{\sigma R_2^2}{\varepsilon_0}\right)\frac{1}{r} + C_3}$

Infinite potential ($r \to \infty$) v=0 so C_3=0 and $\boldsymbol{v_3 = \left(\frac{\rho R_1^3}{3\varepsilon_0} + \frac{\sigma R_2^2}{\varepsilon_0}\right)\frac{1}{r}}$

- Potential is a continuous function:
- **at R2 so $\boldsymbol{v_3(R_2) = v_2(R_2)}$**

$$\frac{\rho R_1^3}{3\varepsilon_0}\frac{1}{R_2} + \frac{\sigma R_2^2}{\varepsilon_0}\frac{1}{R_2} = \frac{\rho R_1^3}{3\varepsilon_0}\frac{1}{R_2} + C_2 \Rightarrow C_2 = \frac{\sigma R_2}{\varepsilon_0}$$

donc $\boldsymbol{v_2 = \frac{\rho R_1^3}{3\varepsilon_0}\frac{1}{r} + \frac{\sigma R_2}{\varepsilon_0}}$

- **at R1 so $\boldsymbol{v_2(R_1) = v_1(R_1)}$**

$$-\frac{\rho}{6\varepsilon_0}R_1^2 + C_1 = \frac{\rho R_1^3}{3\varepsilon_0}\frac{1}{R_1} + \frac{\sigma R_2}{\varepsilon_0} \Rightarrow C_1 = \frac{\rho R_1^2}{3\varepsilon_0} + \frac{\rho R_1^2}{6\varepsilon_0} + \frac{\sigma R_2}{\varepsilon_0} = \frac{3\rho R_1^2}{6\varepsilon_0} + \frac{\sigma R_2}{\varepsilon_0}$$

$$C_1 = \frac{\rho R_1^2}{2\varepsilon_0} + \frac{\sigma R_2}{\varepsilon_0}$$

so $\boldsymbol{v_1 = -\frac{\rho}{6\varepsilon_0}r^2 + \frac{\rho R_1^2}{2\varepsilon_0} + \frac{\sigma R_2}{\varepsilon_0}}$

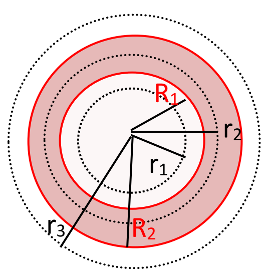
<u>Exercise 2</u>

The Gaussian surface is a sphere with center O and radius r. For reasons of symmetry, the field is radial and constant at any point on the Gaussian surface.

$$\emptyset = \oiint \vec{E}.\overrightarrow{ds} = \frac{\sum Q_{int}}{\varepsilon_0}$$

$\vec{E} \parallel \overrightarrow{ds}$ so : $\oiint \vec{E}.\overrightarrow{ds} = \iint E.ds = E\iint ds = E.S = E4\pi r^2 \Rightarrow \boldsymbol{E4\pi r^2} = \frac{\boldsymbol{\sum Q_{int}}}{\boldsymbol{\varepsilon_0}}$

$$\Rightarrow \boldsymbol{E = \frac{Q_{int}}{4\pi r^2 \varepsilon_0}} \quad (*)$$



The electrostatic field E(r) at any point in space.

<u>We have 3 cases:</u>

1st case r<R1

$Q_{int}=0$ so $\boldsymbol{E_1=0}$

2nd case R1 ≤ r<R2

$dq=\rho dv=\rho 4\pi r^2 dr \Rightarrow Q_{int}=\rho\iiint dv=\rho 4\pi\int_{R_1}^{r} r^2 dr$

So $Q_{int}=\rho\ \frac{4}{3}\pi(r^3-R_1^3)$

$$(*)\Rightarrow E_2=\frac{\rho\ \frac{4}{3}\pi(r^3-R_1^3)}{4\pi r^2\varepsilon_0}\quad so\ \boldsymbol{E_2}=\frac{\boldsymbol{\rho}\ (r^3-R_1^3)}{\boldsymbol{3\varepsilon_0 r^2}}=\frac{\boldsymbol{\rho}}{\boldsymbol{3\varepsilon_0}}\left(r-\frac{R_1^3}{\boldsymbol{r^2}}\right)$$

3rd case r≥R2

$dq=\rho dv=\rho 4\pi r^2 dr \Rightarrow Q_{int}=\rho\iiint dv=\rho 4\pi\int_{R_1}^{R_2} r^2 dr$

So $Q_{int}=\rho\ \frac{4}{3}\pi(R_2^3-R_1^3)$

$$(*)\Rightarrow E_3=\frac{\rho\ \frac{4}{3}\pi(R_2^3-R_1^3)}{4\pi r^2\varepsilon_0}\ donc\ \boldsymbol{E_3}=\frac{\boldsymbol{\rho\ (R_2^3-R_1^3)}}{\boldsymbol{3\varepsilon_0 r^2}}$$

1- The electric potential v(r) at any point in space.

$\vec{E}=-\overrightarrow{grad}v\Rightarrow E=-\frac{dv}{dr}$ so $v=-\int E dr$

1st case : r<R1

$E_1=0\Rightarrow \quad \boldsymbol{v_1=C_1}$

2nd case R1 ≤ r<R2

$E_2=\frac{\boldsymbol{\rho}}{\boldsymbol{3\varepsilon_0}}\left(r-\frac{R_1^3}{\boldsymbol{r^2}}\right)\Rightarrow v_2=-\frac{\rho}{3\varepsilon_0}\left(\int r dr-R_1^3\int\frac{1}{\boldsymbol{r^2}}dr\right)$

so $\boldsymbol{v_2}=-\frac{\boldsymbol{\rho}}{\boldsymbol{3\varepsilon_0}}\left(\frac{\boldsymbol{r^2}}{\boldsymbol{2}}-R_1^3\left(\frac{\boldsymbol{-1}}{\boldsymbol{r}}\right)\right)+\boldsymbol{C_2}$

$$\boldsymbol{v_2}=-\frac{\boldsymbol{\rho}}{\boldsymbol{3\varepsilon_0}}\left(\frac{\boldsymbol{r^2}}{\boldsymbol{2}}+\frac{R_1^3}{\boldsymbol{r}}\right)+\boldsymbol{C_2}$$

3rd case r≥R2

$E_3=\frac{\rho\ (R_2^3-R_1^3)}{3\varepsilon_0 r^2}\Rightarrow v_3=-\frac{\rho\ (R_2^3-R_1^3)}{3\varepsilon_0}\int\frac{1}{r^2}dr$ so $\boldsymbol{v_3}=\frac{\boldsymbol{\rho\ (R_2^3-R_1^3)}}{\boldsymbol{3\varepsilon_0}}\frac{\boldsymbol{1}}{\boldsymbol{r}}+\boldsymbol{C_3}$

Infinite potentiel at (r→ ∞) v=0 so $C_3=0$ and $\boldsymbol{v_3}=\frac{\boldsymbol{\rho\ (R_2^3-R_1^3)}}{\boldsymbol{3\varepsilon_0}}\frac{\boldsymbol{1}}{\boldsymbol{r}}$

- The potential is a continuous function:
  - **At R2** *so* $\boldsymbol{v_3(R_2)=v_2(R_2)}$

$$\frac{\boldsymbol{\rho\ (R_2^3-R_1^3)}}{\boldsymbol{3\varepsilon_0}}\frac{1}{R_2}=-\frac{\boldsymbol{\rho}}{\boldsymbol{3\varepsilon_0}}\left(\frac{\boldsymbol{R_2^2}}{\boldsymbol{2}}+\frac{R_1^3}{\boldsymbol{R_2}}\right)+C_2$$

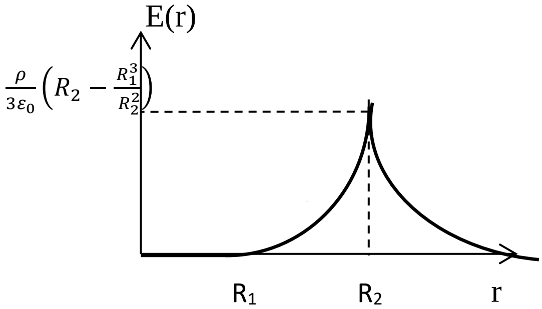
$$\Rightarrow C_2 = \frac{\rho R_2^2}{3\varepsilon_0} + \frac{\rho R_2^2}{6\varepsilon_0} - \frac{\rho\ (R_1^3)}{3R_2\varepsilon_0} + \frac{\rho\ (R_1^3)}{3R_2\varepsilon_0} = \frac{3\rho R_2^2}{6\varepsilon_0} = \frac{\rho R_2^2}{2\varepsilon_0}$$

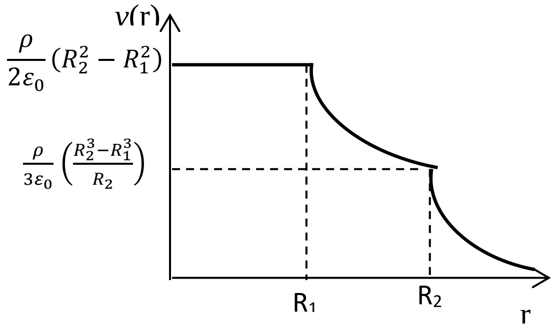
so $\boldsymbol{v_2} = -\frac{\rho}{3\varepsilon_0}\left(\frac{r^2}{2} + \frac{R_1^3}{r}\right) + \frac{\rho R_2^2}{2\varepsilon_0}$

- **at R₁ so** $-\frac{\rho}{3\varepsilon_0}\left(\frac{R_1^2}{2} + \frac{R_1^3}{R_1}\right) + \frac{\rho R_2^2}{2\varepsilon_0} \Rightarrow C_1 = -\frac{\rho}{3\varepsilon_0}\left(\frac{R_1^2}{2} + R_1^2\right) + \frac{\rho R_2^2}{2\varepsilon_0}$

$$\Rightarrow C_1 = -\frac{\rho}{3\varepsilon_0}\left(\frac{3R_1^2}{2}\right) + \frac{\rho R_2^2}{2\varepsilon_0}$$

then $v_1 = -\frac{\rho\ R_1^2}{2\varepsilon_0} + \frac{\rho R_2^2}{2\varepsilon_0}$





Exercise 3 :

Consider a cylinder of radius r and height h as a Gaussian surface.
Due to symmetry, the field is radial and constant at any point on the Gauss surface.
According to Gauss's Theorem:

$$\emptyset = \iint \vec{E}.\overrightarrow{ds} = \frac{\sum Q_{int}}{\varepsilon_0}$$

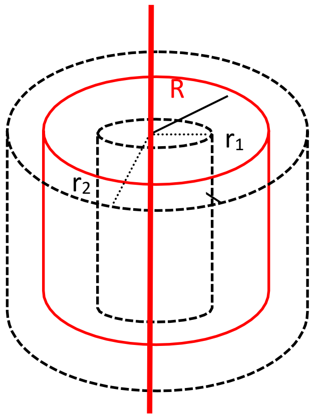
$$\emptyset = \iint \vec{E}.\overrightarrow{ds} = 2\iint \vec{E}.\overrightarrow{ds}_{base} + \iint \vec{E}.\overrightarrow{ds_{lat}}$$

$$\vec{E} \perp \overrightarrow{ds}_{base} \Longrightarrow \iint \vec{E}.\overrightarrow{ds_{lat}} = 0$$

$\vec{E} \parallel \overrightarrow{ds_{lat}}$ so: $\emptyset = \iint \vec{E}.\overrightarrow{ds_{lat}} = \iint E.ds_{lat} = E.\int ds_{lat} = E.S_{lat}$

$\Rightarrow \emptyset = \boldsymbol{E2\pi rh} = \frac{\sum Q_{int}}{\varepsilon_0}$ then $\boldsymbol{E} = \frac{\sum \boldsymbol{Q_{int}}}{\boldsymbol{2\pi rh\varepsilon_0}}$

1- Electric field



1st case $r < R$ $\quad dq = \lambda dl \Rightarrow Q_{int} = \lambda h$

$$E_1 2\pi rh = \frac{\lambda h}{\varepsilon_0} \Rightarrow \boldsymbol{E_1} = \frac{\boldsymbol{\lambda}}{2\pi r_{\varepsilon_0}}$$

2nd case r≥ R $\quad Q_{int} = Q_1 + Q_2$

$$dq_2 = \sigma ds \Rightarrow Q_{int} = \sigma S = \sigma 2\pi Rh$$

So $Q_{int} = \lambda h + \sigma 2\pi Rh$

Then $E_2 2\pi rh = \frac{\lambda h + \sigma 2\pi Rh}{\varepsilon_0} \Rightarrow \boldsymbol{E_2} = \frac{\boldsymbol{\lambda}}{2\pi r\boldsymbol{\varepsilon_0}} + \frac{\boldsymbol{\sigma R}}{\boldsymbol{\varepsilon_0 r}}$

2- Let's find λ for which E_2=0

$$\frac{\lambda}{2\pi r\boldsymbol{\varepsilon_0}} + \frac{\sigma R}{\varepsilon_0 r} = 0 \Rightarrow \frac{\lambda}{2\pi r\boldsymbol{\varepsilon_0}} = -\frac{\sigma R}{\varepsilon_0 r} \; so \;\; \lambda = -2\pi\sigma R$$

Exercise 4:

1- the field : Consider a cylinder of radius r and height h as a Gauss surface.

Gauss's theorem : $\varphi = \iint \overrightarrow{E}.\overrightarrow{ds} = \frac{\sum Q_{int}}{\varepsilon_0}$

$$\varphi = \iint \overrightarrow{E}.\overrightarrow{ds} = 2\iint \overrightarrow{E}.\overrightarrow{ds}_{base} + \iint \overrightarrow{E}.\overrightarrow{ds}_{lat}$$

$\overrightarrow{E} \;∴∴\; \overrightarrow{ds}_{lat}$ et $\overrightarrow{E} \perp \overrightarrow{ds}_{base} \Rightarrow \iint \overrightarrow{E}.\overrightarrow{ds}_{lat} = 0$

so : $\varphi = \iint \overrightarrow{E}.\overrightarrow{ds}_{lat} = \overrightarrow{E}.\int \overrightarrow{ds}_{lat} = E.S_{lat} = E2\pi rh$

For : $\boldsymbol{r < R_1}$

$\boldsymbol{Q_{int} = 0 \Rightarrow \overrightarrow{E_1} = 0}$

For : $\boldsymbol{R_1 \le r < R_2}$

$Q_{int} = \iint \sigma ds = \sigma \iint ds = \sigma s = \sigma 2\pi R_1 h$

$E_2 2\pi rh = \frac{\sigma 2\pi R_1 h}{\varepsilon_0} \Rightarrow \boldsymbol{E_2 = \frac{\sigma R_1}{\varepsilon_0}\frac{1}{r}}$

For : $\boldsymbol{r \ge R_2}$

$$Q_{int} = Q_1 + Q_2$$

$Q_1 = \sigma 2\pi R_1 h$ and $Q_2 = -\sigma 2\pi R_2 h$ donc $\boldsymbol{Q_{int}} = \sigma 2\pi h(R_1 - R_2)$

$$E_3 2\pi rh = \frac{\sigma 2\pi h(R_1 - R_2)}{\varepsilon_0} \Rightarrow \boldsymbol{E_3 = \frac{\sigma(R_1 - R_2)}{\varepsilon_0}\frac{1}{r}}$$

2- The potentiel

$\overrightarrow{E} = -\overrightarrow{grad}\,V$

$E = E(r) \Rightarrow E = -\frac{dV}{dr} \Rightarrow V = -\int E.dr$

- $V_1 = -\int E_1.dr \Rightarrow \boldsymbol{V_1 = C_1}$
- $V_2 = -\int E_2.dr = -\frac{\boldsymbol{\sigma R_1}}{\boldsymbol{\varepsilon_0}}\int \frac{\boldsymbol{1}}{\boldsymbol{r}}.dr = -\frac{\boldsymbol{\sigma R_1}}{\boldsymbol{\varepsilon_0}}\boldsymbol{lnr + C_2}$
- $V_3 = -\int E_3.dr = -\frac{\boldsymbol{\sigma(R_1 - R_2)}}{\boldsymbol{\varepsilon_0}}\int \frac{\boldsymbol{1}}{\boldsymbol{r}}.dr = -\frac{\boldsymbol{\sigma(R_1 - R_2)}}{\boldsymbol{\varepsilon_0}}\boldsymbol{lnr + C_3}$

<u>Supplementary exercises :</u>

<u>Exercise 1:</u>

Consider a cylinder of radius r and height h as a Gauss surface.

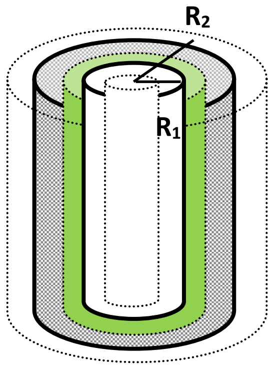
Due to symmetry, the field is radial and constant at all points on the Gauss surface.

According to Gauss's Theorem: $\emptyset = \iint \vec{E}.\overrightarrow{ds} = \frac{\sum Q_{int}}{\varepsilon_0}$

$\emptyset = \iint \vec{E}.\overrightarrow{ds} \ = 2\iint \vec{E}.\overrightarrow{ds}_{base} + \iint \vec{E}.\overrightarrow{ds_{lat}}$

$\overrightarrow{E} \perp \overrightarrow{ds}_{base} \Longrightarrow \iint \vec{E}.\overrightarrow{ds_{lat}} = 0$

$\vec{E} \parallel \overrightarrow{ds_{lat}}$ so : $\emptyset = \iint \vec{E}.\overrightarrow{ds_{lat}} = \iint E.ds_{lat} = E.\int ds_{lat} = E.S_{lat}$



$\Rightarrow \emptyset = \boldsymbol{E2\pi rh} = \frac{\sum Q_{int}}{\varepsilon_0}$

- **Electric field**

<u>1st case r<R₁</u>

$Q_{int} = 0$ so $\boldsymbol{E_1 = 0}$

<u>2nd case $R_1 \leq r < R_2$</u>

$dq = \rho dv = \rho 2\pi rhdr \Rightarrow Q_{int} = \rho \iiint dv = \rho 2\pi h \int_{R_1}^{r} rdr$

so $Q_{int} = \rho 2\ \pi h\left(\frac{r^2}{2} - \frac{R_1^2}{2}\right) = \rho\ \pi h(r^2 - R_1^2)$

or $Q_{int} = \rho(\pi r^2 h - \pi R_1^2 h)$

$$then\ \ E_2 2\pi rh = \frac{\rho\ \pi h(r^2 - R_1^2)}{\varepsilon_0} \quad hence\ \boldsymbol{E_2} = \frac{\boldsymbol{\rho}\ (r^2 - R_1^2)}{\boldsymbol{2\varepsilon_0 r}} = \frac{\boldsymbol{\rho}}{\boldsymbol{2\varepsilon_0}}\left(r - \frac{R_1^2}{\boldsymbol{r}}\right)$$

<u>3rd case $r \geq R_2$</u>

$dq = \rho dv = \rho 4\pi r^2 dr \Rightarrow Q_{int} = \rho \iiint dv = \rho 2\pi h \int_{R_1}^{R_2} r^2 dr$ so $Q_{int} = \rho\ \pi h(R_2^2 - R_1^2)$

$$E_3 2\pi rh \ = \frac{\rho\ \pi h(R_2^2 - R_1^2)}{\varepsilon_0}\ so\ \boldsymbol{E_3} = \frac{\boldsymbol{\rho}\ (\boldsymbol{R_2^2 - R_1^2})}{\boldsymbol{2\varepsilon_0 r}}$$

- The electric potential v(r) at any point in space.

$\vec{E} = -\overrightarrow{grad}v \Rightarrow E = -\frac{dv}{dr}$ so $v = -\int Edr$

<u>1st case : r<R₁</u>

$E_1 = 0 \Rightarrow \quad \boldsymbol{v_1 = C_1}$

<u>2nd case $R_1 \leq r < R_2$</u>

$E_2 = \frac{\rho}{2\varepsilon_0}\left(r - \frac{R_1^2}{r}\right) \Rightarrow v_2 = -\frac{\rho}{2\varepsilon_0}\left(\int rdr - R_1^2 \int \frac{1}{r}dr\right)$ and $\boldsymbol{v_2} = -\frac{\boldsymbol{\rho}}{\boldsymbol{2\varepsilon_0}}\left(\frac{\boldsymbol{r^2}}{\boldsymbol{2}} - R_1^2\ \ln r\right) + \boldsymbol{C_2}$

<u>3rd cas $r \geq R_2$</u>

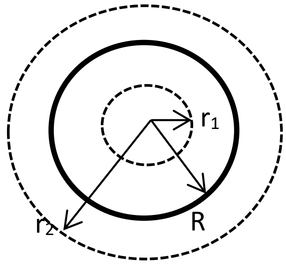
$E_3 = \frac{\rho\ (R_2^2 - R_1^2)}{2\varepsilon_0 r} \Rightarrow v_3 = -\frac{\rho\ (R_2^2 - R_1^2)}{2\varepsilon_0}\int \frac{1}{r}dr$ so $\boldsymbol{v_3} = -\frac{\boldsymbol{\rho}\ (\boldsymbol{R_2^2 - R_1^2})}{\boldsymbol{2\varepsilon_0}} \boldsymbol{\ln r} + \boldsymbol{C_3}$

Exercise 2:

The Gauss surface is a sphere with center O and radius r.
Due to symmetry, the field is radial and constant at any point on the Gauss surface.

The flux through the Gauss.

$$\emptyset = \iint \vec{E}.\overrightarrow{ds} = \sum \frac{Q_{int}}{\varepsilon_0}$$



1- Electric Field

Charge volume density $\boldsymbol{\rho = \frac{A}{r}}$

$$\begin{cases} \iint \vec{E}.\overrightarrow{ds} = \sum \frac{Q_{int}}{\varepsilon_0} \\ \vec{E} \parallel \overrightarrow{ds} \; and \; E = cst \end{cases} \Rightarrow \iint E.ds = E.4\pi r^2 = \sum \frac{Q_{int}}{\varepsilon_0}$$

We have 2 cases :

1st case : r<R

$$\begin{cases} dq = \rho dv \\ \rho = \frac{A}{r} \\ dv = 4\pi r^2 dr \end{cases} \Rightarrow \int dq = 4\pi \int_0^{r_1} \frac{A}{r} r^2 dr$$

So $Q_{int} = 2\pi A r_1^2$ where $E_1 4\pi r_1^2 = \frac{2\pi A}{\varepsilon_0} r_1^2 \Rightarrow \boldsymbol{E_1 = \frac{A}{2\varepsilon_0}}$

2nd case : r≥R

$\int dq = 4\pi \int_0^R \frac{A}{r} r^2 dr \Rightarrow Q_{int} = 2\pi A R^2$ where $E_2 4\pi r_2^2 = \frac{2\pi A}{\varepsilon_0} R^2 \Rightarrow \boldsymbol{E_2 = \frac{AR^2}{2r_2^2\varepsilon_0}}$

2- The potentiel

$\vec{E} = -\overrightarrow{grad}v \Rightarrow E = -\frac{dv}{dr}$ so $v = -\int E dr$

1st case : r<R

$E_1 = \frac{A}{2\varepsilon_0} \Rightarrow v_1 = -\frac{A}{2\varepsilon_0}\int dr$ so $v_1 = -\frac{A}{2\varepsilon_0} r + c_1$

2nd case : r≥R

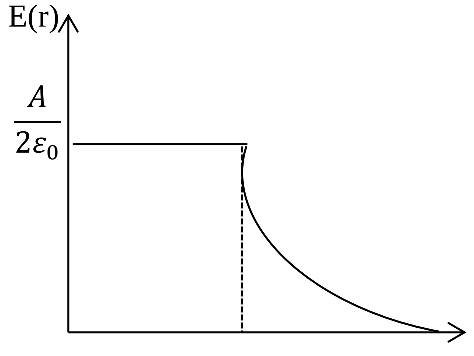
$E_2 = \frac{AR^2}{2r_2^2\varepsilon_0} \Rightarrow v_2 = -\frac{AR^2}{2\varepsilon_0}\int \frac{dr}{r_2^2}$ so $v_2 = \frac{AR^2}{2r\varepsilon_0} + c_2$

At infinity, the potential is zero: $\lim_{r\to\infty} v = 0 \Rightarrow c_2 = 0$ **so** $v_2 = \frac{AR^2}{2r\varepsilon_0}$

The potential is a continuous function, so : $v_1(R) = v_2(R)$

$\frac{AR}{2\varepsilon_0} = -\frac{A}{2\varepsilon_0} R + c_1 \Rightarrow c_1 = \frac{AR}{\varepsilon_0}$ so $v_1 = -\frac{A}{2\varepsilon_0} r + \frac{AR}{\varepsilon_0}$

3- The graph E(r)=f(r)



the graph *v*(r)=f(r)