

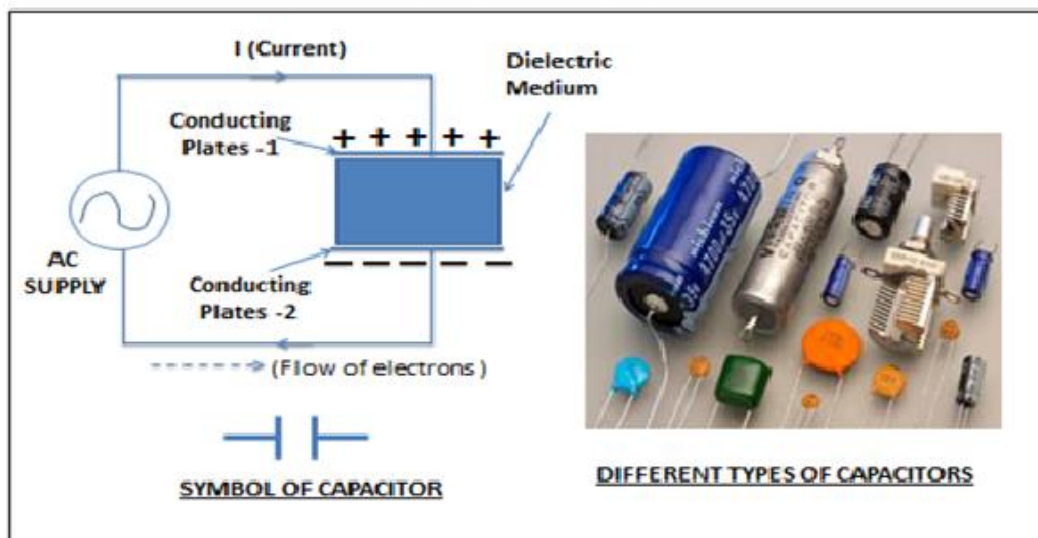
1ST YEAR LMD-M AND MI

ELECTRICITY COURSE

Chapter III: The Conductors and Capacitors

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CAPACITOR



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1st part : The conductors النواقل

1. Definition

An electrical conductor is a medium in which positive and negative charges can move under the action of an electric field. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. These materials are called conductors.

Example: water, metals.....

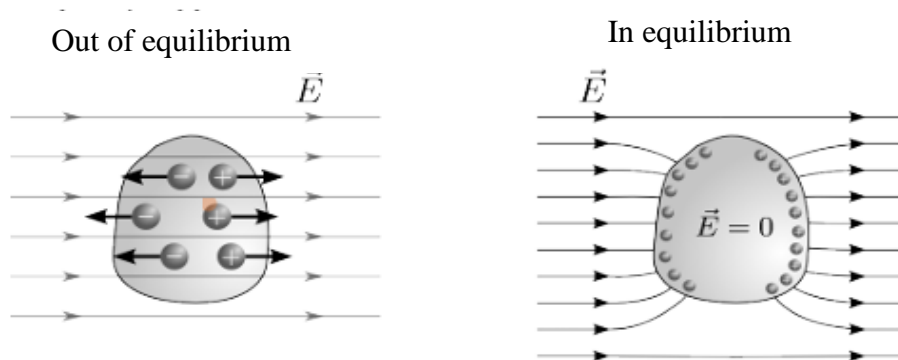
Another class of materials is called isolator (مادة عازلة) in which all the electrons are tightly bound to their respective atoms or molecules. In effect, there are no free electrons.

Example: wood, plastic.....

2. Conductor in electrostatic equilibrium

A conductor is said to be in electrostatic equilibrium if all charges are immobile (no charge displacement in this medium).

Charges in a neutral conductor placed in a uniform electrostatic field move to cancel out the field in the conductor. This changes the external field.



2.1. Properties of a conductor in isolated electrostatic equilibrium

a- The electric field

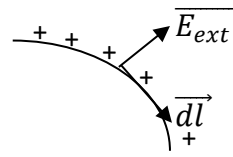
Electrostatic equilibrium means that the charges inside a conductor are immobile, and therefore not subject to any force. This condition translates into a zero electrostatic force for each charge. $\vec{F} = q \vec{E} = \vec{0}$

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Therefore the electric field $\vec{E}_{ins} = \vec{0}$

Note: the electric field is normal to the surface of a conductor in equilibrium.

b- Electric potentiel



We have $\vec{E} = -\overrightarrow{grad}v \Rightarrow E = -\frac{dv}{dl}$ and $\vec{E}_{int} = \vec{0}$

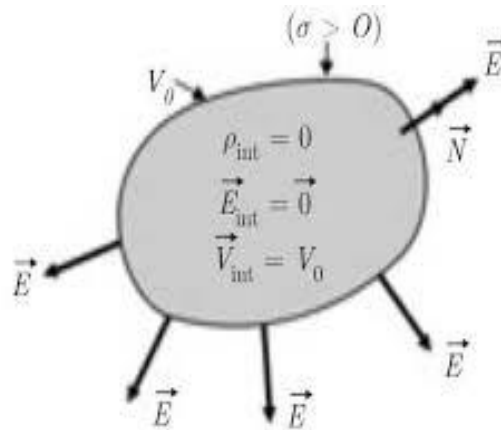
so $V_A - V_B = dV = 0$ and $V = \text{Constant}$

We can conclude that the conductor in equilibrium is an equipotential volume. In other words, all points inside the conductor have the same potential, so the surface of the conductor is equipotential.

c- Distribution of charges

We have $\oint \vec{E} \cdot \vec{ds} = \frac{\Sigma Q_{int}}{\epsilon_0}$ (GT) and $\vec{E}_{int} = \vec{0}$ so $Q_{int} = 0$

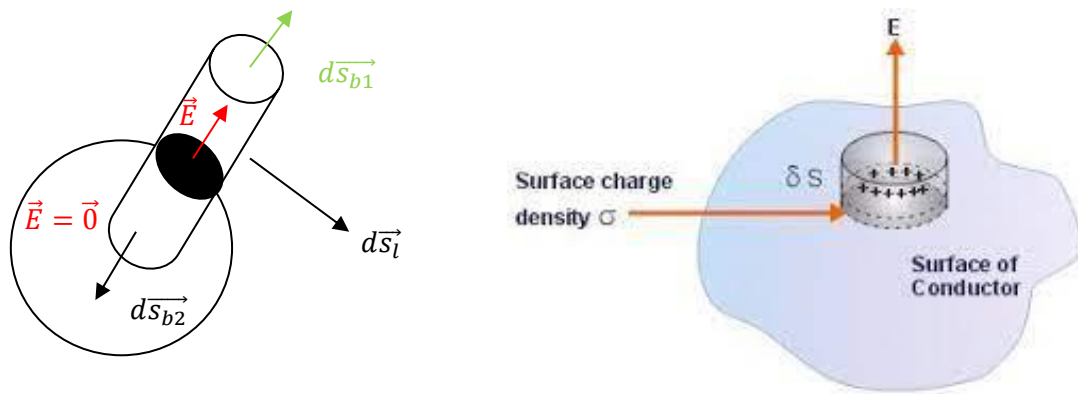
The total electric charge inside the conductor is then said to be zero, and the charge is localized on the surface of the conductor (surface distribution).



2.2. Electric field in the immediate vicinity of a conductor in electrostatic equilibrium (Coulomb's theorem)

Given a conductor of any shape, we calculate the field in the vicinity of the conductor's outer surface. Apply Gauss's Theorem

Choose a cylinder intersecting the surface of the conductor as the Gauss surface.



We have: $\Phi = \iint \vec{E} \cdot \vec{ds} = \frac{\sum Q_{ins}}{\epsilon_0}$

$$\Phi = \iint \vec{E} \cdot \vec{ds} = \iint \vec{E} \cdot \vec{ds}_{base\ 1} + \iint \vec{E} \cdot \vec{ds}_{lat} + \iint \vec{E} \cdot \vec{ds}_{base\ 2}$$

$$\vec{E} \cdot \vec{ds}_{base\ 1} = 0 \text{ because } \vec{E}_{ins} = \vec{0}$$

$$\vec{E} \perp \vec{ds}_{lat} \Rightarrow \iint \vec{E} \cdot \vec{ds}_{lat} = 0$$

$$\vec{E} \parallel \vec{ds}_{base\ 2} \text{ so : } \Phi = \iint \vec{E} \cdot \vec{ds}_{base} = \iint E \cdot ds_{base} = E \cdot \int ds_{base} = E \cdot S_{base}$$

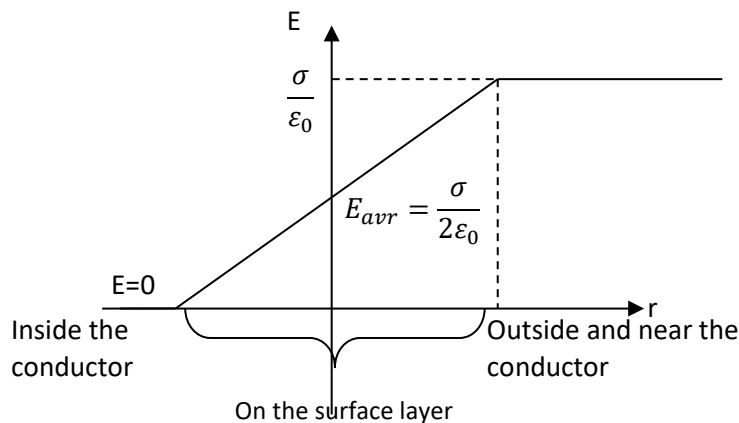
$$\Phi = E \cdot S_{base} = \frac{\sum Q_{ins}}{\epsilon_0}$$

$$dq = \sigma ds \Rightarrow Q_{ins} = \sigma \iint ds = \sigma S_{base} \Rightarrow E \cdot S_{base} = \frac{\sigma S_{base}}{\epsilon_0} \text{ so } \mathbf{E} = \frac{\sigma}{\epsilon_0}$$

Statement of Coulomb's Theorem

The electric field created in the immediate vicinity of the surface of a conductor in electrostatic equilibrium carrying a surface-distributed charge distribution is given by: $\vec{E} =$

$\frac{\sigma}{\epsilon_0} \vec{n}$ with \vec{n} : outward vector normal to the surface.



Summary :

Isolated, neutral body: electric charge is conserved $\Delta Q=0$

Conductive body in electrostatic equilibrium: $Q_{\text{int}}=0$, $E_{\text{int}}=0$, $E_{\text{ext}} = \frac{\sigma}{\epsilon_0}$, $V_{\text{int}}=V_0$, $Q_{\text{ext}}=\sigma \cdot dS$

3. Capacity of a conductor in electrostatic equilibrium

For an electrostatically equilibrium conductor, there's a link between the potential at which the conductor is located and the charge distributed over its surface. Experience shows that the ratio Q/V is a constant. This is called the conductor's capacity; denoted $C=Q/V$

The unit of capacity is the **Farad**, symbol **F**.

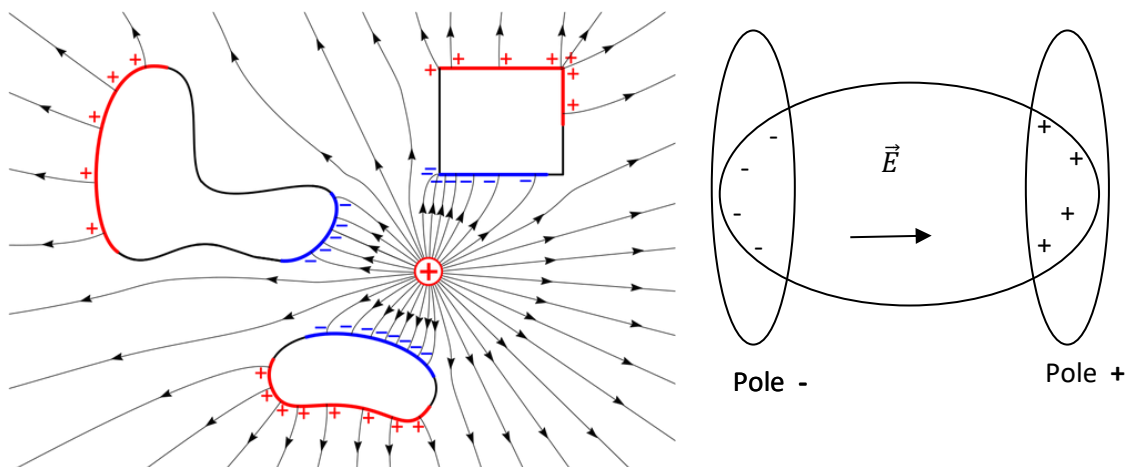
4. Potential energy of a conductor in electrostatic equilibrium

A conductor in electrostatic equilibrium carrying charge Q , let V be its potential and C its capacity, its potential energy is written as:

$$E_p = \frac{1}{2} Q \cdot V = \frac{1}{2} \cdot C \cdot V^2 = \frac{1}{2} \cdot \frac{Q^2}{C} \text{ (Joule)}$$

5. Electrostatic influence between conductors

If we place a conductor in an electric field, the positive charges go in the same direction as the field and the negative charges go in the opposite direction, creating two poles, one positive and one negative.



There are two types of electrostatic influence in the presence of two charged conductors.

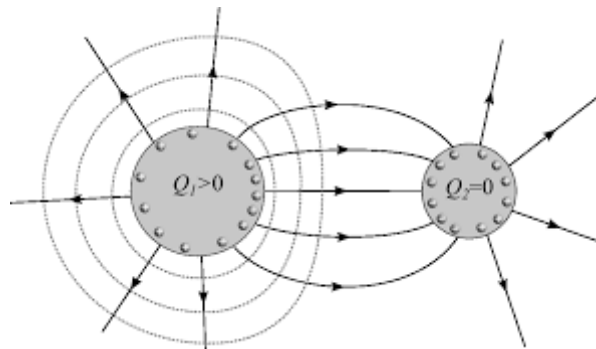
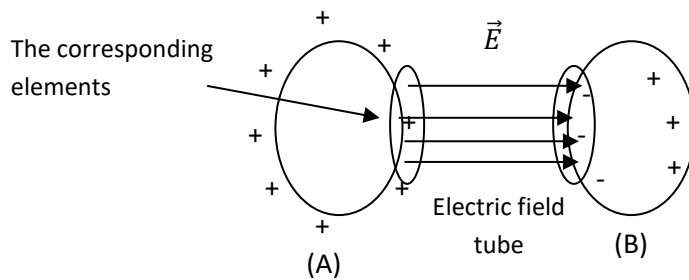
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5.1. Partial influence

The charge on conductor B remains the same; there's just a change in charge distribution.

Consider two conductors: a positively charged conductor A and an isolated, neutral conductor B. As the two conductors approach each other, the negative charges in conductor B move closer to conductor A, and the positive charges move to the other side.

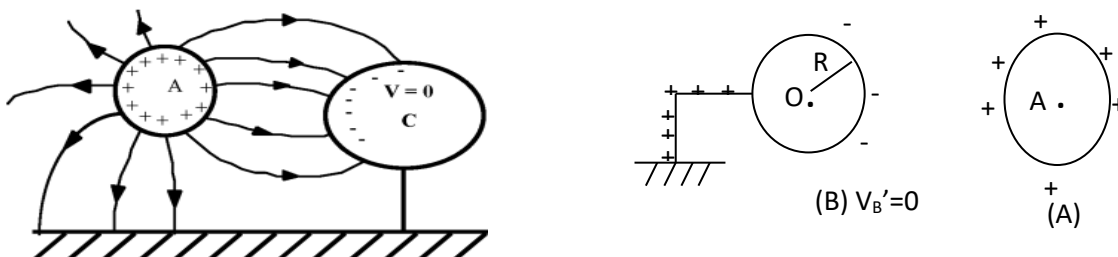
The charges carried by the two opposing conductors are equal and of opposite signs.



Corresponding elements theorem: Faraday's theorem

The charges carried by two corresponding surface elements facing each other are equal and of opposite sign.

Note: If conductor B is connected to earth, positive charges flow to earth, and the charge in conductor B will be Q'_B is negative due to the partial influence of the earth.

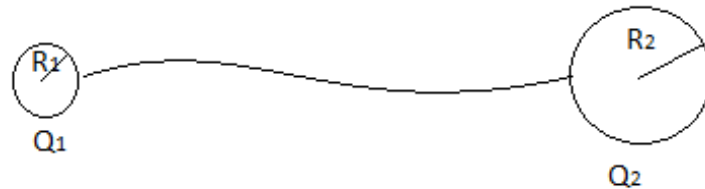


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Special case: Conductor with irregular geometry

When two charged conductors are brought into contact or connected by a conducting wire, charge is transferred between the conductors. If these two conductors have charges Q_1 and Q_2 and are at potential V_1 and V_2 before contact, their charges and potentials after contact will become Q'_1 , Q'_2 , V_1 and V_2 they will be at the same potential so $V'_1 = V'_2$.

And the charge must be maintained: $Q_1 + Q_2 = Q'_1 + Q'_2$

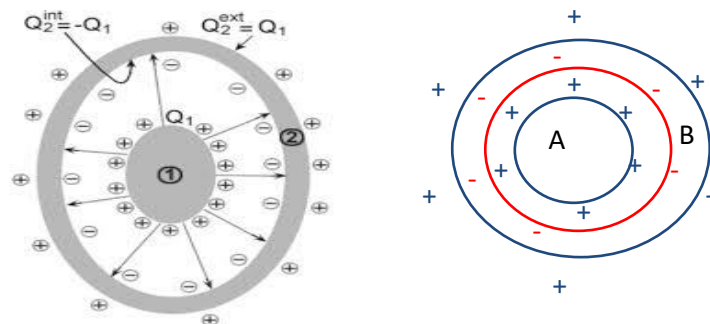


5.2. Total Influence

The conductor influencing A is said to influence conductor B, resulting in the appearance of negative charges on the inner layer of B and positive charges on the outer layer.

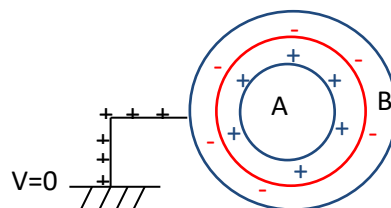
This is the case when conductor A is completely surrounded by conductor B. We say that conductor A is the influencing conductor and conductor B is the influenced conductor.

$$Q_{B \text{ ins}} = -Q_A \quad \text{and} \quad Q_{B \text{ out}} = Q_A$$



Note 1: For the outer layer of B we have 3 possible cases:

- 1- B isolated and neutral $Q_{B \text{ out}} = +Q_A$
- 2- B isolated and has an initial charge Q' Then $Q_{B \text{ out}} = Q_A + Q'$
- 3- B connected to the earth $V=0$ and $Q_{B \text{ out}} = 0$ because positive charges flow to ground.



Note 2: a system of two conductors in electrostatic equilibrium under total influence forms what's known as a capacitor (مكثفة).

Part 2 : Capacitors المكثفات

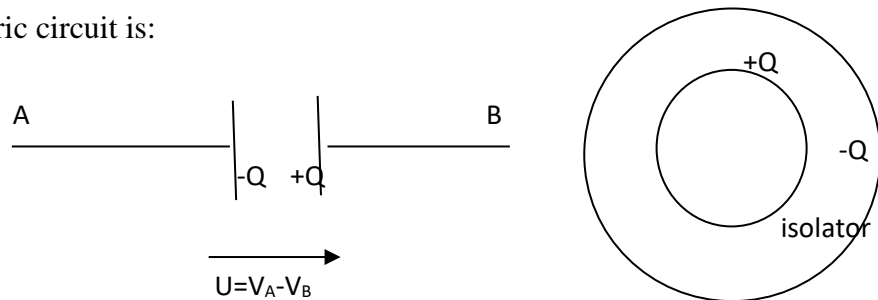
1. Definitions

A capacitor is an assembly of two conductors under total influence. The two conductors are called the capacitor's armatures, and the capacitor's charge is that of its internal armature Q (Coulomb).

V_A is the potential of the internal armature and V_B is the potential of the external armature,

$U = V_A - V_B$ is the potential difference of a capacitor (its unit is **the volt**).

Its symbol in an electric circuit is:



The charge of a capacitor is written as: $Q = C.U$

Where C is the capacity of the capacitor, its unit is **Farad**, and depends on the geometry of the capacitor and the insulation between the two plates.

Note :

- The isolator (material placed between the armatures) increases the capacity of a capacitor.
- The capacity of a capacitor depends on the geometry of the armatures.
- Capacity C is always positive.

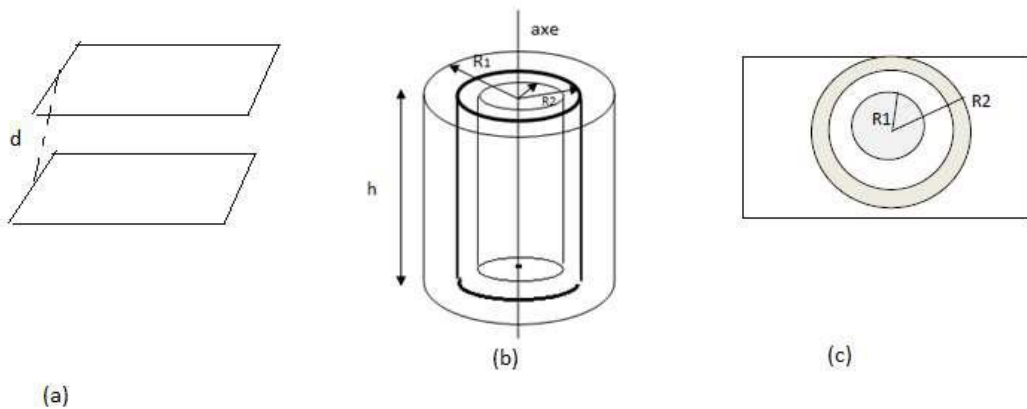
Capacitor types:

There are three types of capacitor: spherical, cylindrical and planar:

- **Planar capacitors** consist of two parallel plates, each with a surface area of S , separated by a small distance d .
- **Cylindrical capacitors** are made up of a conductive **cylinder of radius R_1** and length h , surrounded by a conductive hollow **cylinder of radius R_2** and **length h** . This assembly is called a coaxial cable.

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- **Spherical capacitors** consist of a conductive hollow sphere with **radius R_2** and a conductive sphere of **radius R_1** at the center of the hollow sphere.



The different types of capacitors: (a) planar, (b) cylindrical and (c) spherical

2. Method for calculating capacitor capacity

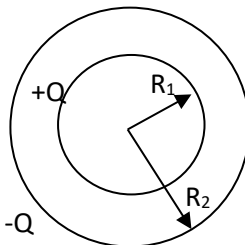
2.1. Capacity of a spherical capacitor

Consider two concentric conducting spheres under total influence, one with charge $+Q$ and the other with charge $-Q$.

To determine the capacitance of this capacitor, we first calculate the electric field using Gauss's Theorem.

- Step 1: Calculate the E field

Gauss's theorem: in this case, we choose a sphere with center O and radius r as the Gauss surface. For reasons of symmetry, the field is radial and constant at any point on the Gaussian surface.



$$\oint \vec{E} \cdot d\vec{s} = \frac{\sum Q_{int}}{\epsilon_0}$$

$$\vec{E} // d\vec{s} \quad \text{so: } \oint \vec{E} \cdot d\vec{s} = \int E \cdot ds = E \int ds = E \cdot S = E 4\pi r^2 \Rightarrow E 4\pi r^2 = \frac{\sum Q_{int}}{\epsilon_0}$$

We're interested in the field between the armatures, so:

$$\text{For } R_1 < r < R_2 : \quad Q_{ins} = Q \Rightarrow E = \frac{Q}{4\pi r^2 \epsilon_0}$$

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- Step 2: Calculating the potential V

We have : $\vec{E} = -\overrightarrow{grad}v \Rightarrow E = -\frac{dv}{dr}$ so $v = -\int E dr$

$$\text{For } R_1 < r < R_2 : \int_{V_1}^{V_2} dV = -\frac{Q}{4\pi\epsilon_0} \int_{R_1}^{R_2} \frac{dr}{r^2} \Rightarrow V_2 - V_1 = \frac{Q}{4\pi\epsilon_0} \left(\frac{1}{R_2} - \frac{1}{R_1} \right) = \frac{Q}{4\pi\epsilon_0} \left(\frac{R_1 - R_2}{R_1 R_2} \right)$$

$$\Rightarrow V_2 - V_1 = -\frac{Q}{4\pi\epsilon_0} \left(\frac{R_2 - R_1}{R_1 R_2} \right)$$

$$U = V_1 - V_2 \Rightarrow U = \frac{Q}{4\pi\epsilon_0} \left(\frac{R_2 - R_1}{R_1 R_2} \right)$$

- Step 3: Capacity calculation

The charge $Q = C U \Rightarrow C = \frac{Q}{U}$ then $C = 4\pi\epsilon_0 \frac{R_1 R_2}{R_2 - R_1}$

In the case of a spherical capacitor where the armatures are very close, we have:

$$R_2 \approx R_1 \text{ and } e = R_2 - R_1$$

$$C = \frac{4\pi\epsilon_0 R^2}{e} = \frac{4\pi R^2}{e} \epsilon_0 \quad \text{and} \quad s = 4\pi R^2 \quad \text{so} \quad C = \frac{\epsilon_0 s}{e}$$

2.2. Capacity of a cylindrical capacitor

Consider two coaxial conductive cylinders

under total influence, one with charge +Q

and the other with charge -Q.

- Step 1: Calculating the E field

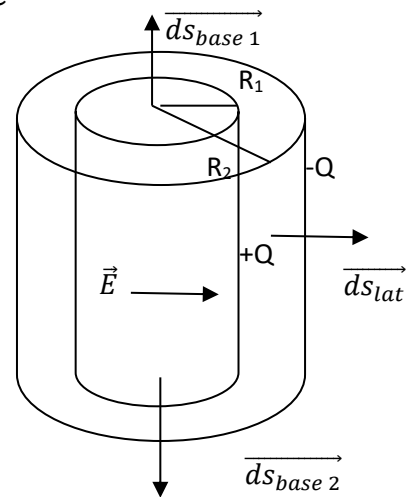
The Gaussian surface is a cylinder of radius r and height h.

Because of symmetry, the radial field is constant in the Gaussian surface.

$$\text{According to Gauss's Theorem : } \phi = \iint \vec{E} \cdot \overrightarrow{ds} = \frac{\sum Q_{ins}}{\epsilon_0}$$

$$\phi = \iint \vec{E} \cdot \overrightarrow{ds} = \iint \vec{E} \cdot \overrightarrow{ds}_{base 1} + \iint \vec{E} \cdot \overrightarrow{ds}_{lat} + \iint \vec{E} \cdot \overrightarrow{ds}_{base 2}$$

$$\vec{E} \perp \overrightarrow{ds}_{base} \Rightarrow \iint \vec{E} \cdot \overrightarrow{ds}_{lat} = 0$$



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$$\vec{E} \parallel \overrightarrow{ds_{lat}} \text{ so : } \phi = \iint \vec{E} \cdot \overrightarrow{ds_{lat}} = \iint E \cdot ds_{lat} = E \cdot \int ds_{lat} = E \cdot S_{lat}$$

$$\text{Then } \phi = E \cdot 2\pi r h = \frac{Q_i}{\epsilon_0}$$

$$\text{For } R_1 < r < R_2 : \quad Q_{ins} = Q \Rightarrow E = \frac{Q}{2\pi h r \epsilon_0}$$

- Step 2: Calculating the potential V

$$\text{The potential : } \vec{E} = -\overrightarrow{grad}v \Rightarrow E = -\frac{dv}{dr} \text{ so } v = -\int E dr$$

$$\text{For } R_1 < r < R_2 \quad \int_{V_1}^{V_2} dV = -\frac{Q}{2\pi h \epsilon_0} \int_{R_1}^{R_2} \frac{dr}{r} \Rightarrow V_2 - V_1 = -\frac{Q}{2\pi h \epsilon_0} (\ln R_2 - \ln R_1)$$

$$\Rightarrow V_2 - V_1 = -\frac{Q}{2\pi h \epsilon_0} \left(\ln \frac{R_2}{R_1} \right)$$

$$U = V_1 - V_2 \Rightarrow U = \frac{Q}{2\pi h \epsilon_0} \left(\ln \frac{R_2}{R_1} \right)$$

- Step 3: Capacity calculation

$$\text{The charge is } Q = C U \Rightarrow \text{the capacity will be } C = \frac{Q}{U} \text{ so } C = 2\pi h \epsilon_0 \left(\ln \frac{R_1}{R_2} \right)$$

2.3. Capacity of a planar capacitor

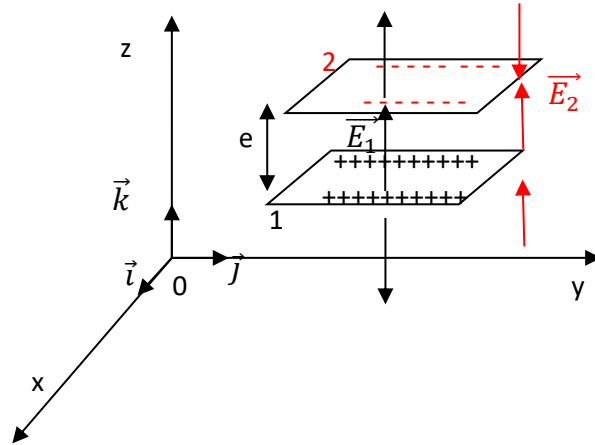
This capacitor is composed of two parallel charged planes spaced by a thickness e . The charge distribution in the two armatures is surface-based.

Step 1: Calculating the E field

Let's assume that C_1 has a +Q charge and C_2 has a -Q charge.

$$\text{- In the first armature: } \vec{E}_1 = \frac{\sigma}{2\epsilon_0} \vec{k}$$

$$\text{- In the second armature: } \vec{E}_2 = -\frac{\sigma}{2\epsilon_0} (-\vec{k}) = \frac{\sigma}{2\epsilon_0} \vec{k}$$



Between the two armatures: $\vec{E} = \vec{E}_1 + \vec{E}_2$

$$\vec{E} = \left(\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \right) \vec{k} = \frac{\sigma}{\epsilon_0} \vec{k}$$

- **Step 2: Calculating potential**

The potentiel : $\vec{E} = -\overrightarrow{grad}v$; E is according (oz) $\Rightarrow E = -\frac{dv}{dz}$ so $v = -\int Edz$

$$\int_{V_1}^{V_2} dV = -\int_{z_1}^{z_2} Edz \Rightarrow V_2 - V_1 = -E(z_2 - z_1) = -E \cdot e$$

$$U = V_1 - V_2 = E \cdot e = \frac{\sigma}{\epsilon_0} \cdot e$$

- **Step 3: Capacity calculation**

There is a surface distribution of charges: $Q = \sigma \cdot dS$ hence $\sigma = Q/S$ then $U = \frac{e \cdot Q}{S \cdot \epsilon_0}$

We have $Q = C \cdot U$ so $C = \frac{S \cdot \epsilon_0}{e}$

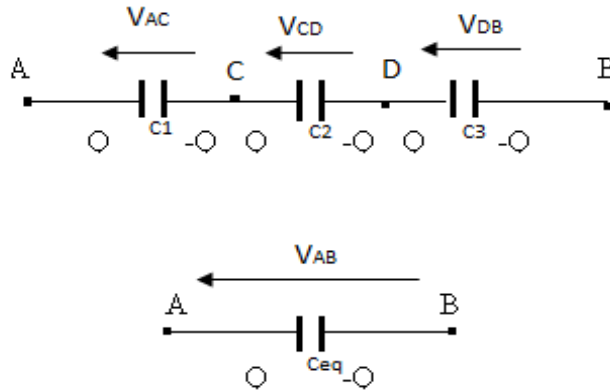
3. Association of capacitors

There are two types of capacitor grouping:

3.1. Grouping in series ربط على التسلسل

In a series connection of n capacitors, all capacitors store **the same charge Q** due to the total influence between the capacitor plates.

$$Q_{eq} = Q_{C1} = Q_{C2} = Q_{C3} = \dots = Q_{Cn}$$



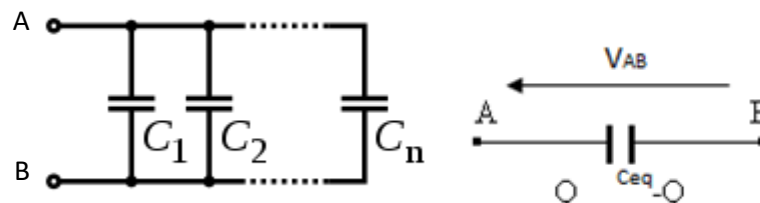
On the other hand, the voltage between all the capacitors **is equal to the sum of the voltages** of the individual capacitors (series connection).

$$U = V_A - V_B = U_1 + U_2 + \dots + U_n = \frac{Q}{C_{eq}}$$

$$= (V_A - V_1) + (V_1 - V_2) + (V_2 - V_3) + \dots + (V_n - V_B) = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots + \frac{Q}{C_n} = \frac{Q}{C_{eq}}$$

So $U = V_A - V_B = Q \sum_{i=1}^n \frac{1}{C_i}$ and $\frac{1}{C_{eq}} = \sum_{i=1}^n \frac{1}{C_i}$

3.2. Parallel grouping ربط على التفرع



In a parallel connection of **n** capacitors, all the capacitors have **the same voltage U** and the total charge is **the sum of the charges** of the individual capacitors.

$$U = U_1 = U_2 = \dots = U_n$$

and $Q_{eq} = Q_{C1} + Q_{C2} + Q_{C3} + \dots + Q_{Cn} = C_{eq} U$

$$Q_{eq} = C_1 U + C_2 U + \dots + C_n U = C_{eq} U$$

so $Q_{eq} = (\sum_{i=1}^n C_i) \cdot U = C_{eq} U$ then $C_{eq} = \sum_{i=1}^n C_i$

4. Energy of a capacitor

The potential energy of a capacitor with charge Q and capacitance C subjected to a voltage U is:

$$E_p = \frac{1}{2} Q \cdot U = \frac{1}{2} \cdot C \cdot U^2 = \frac{1}{2} \cdot \frac{Q^2}{C} \text{ (Joule)}$$

Application exercise :

Consider the circuit diagram below.

1- Knowing that capacitor C_1 carries charge $Q_1=10 \mu\text{C}$, what will be the potential difference V_{AD} between points A and D?

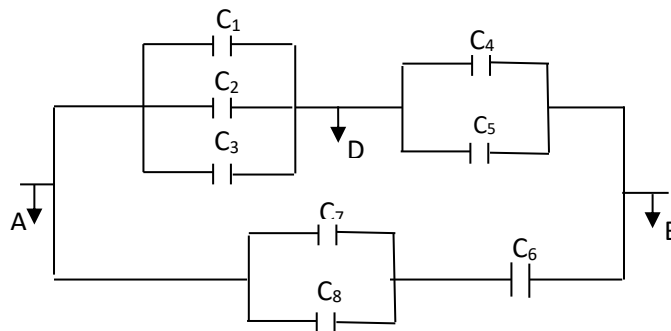
2- Determine the charges Q_2 and Q_3 of capacitors C_2 and C_3 respectively.

3- Since the PD between B and D is equal to 2V, calculate the charges Q_4 and Q_5 of capacitors C_4 and C_5 .

4- What is the equivalent capacity C_{eq} of the entire circuit?

5- Calculate the energy stored by capacitor C_1 .

We give : $C_1=4\mu\text{F}$, $C_2=3.5\mu\text{F}$, $C_3=2.5\mu\text{F}$, $C_4= C_5= C_7= C_8=5\mu\text{F}$, $C_6=10\mu\text{F}$.



Answers:

1. $Q_{C1} = C_1 U_{AD} \Rightarrow U_{AD} = \frac{Q_{C1}}{C_1} = \frac{10}{4} \Rightarrow U_{AD} = 2.5 \text{ Volt}$

2. $Q_{C2} = C_2 U_{AD} = 3.5 \times 2.5 = 8.75 \mu\text{C}$ $Q_{C3} = C_3 U_{AD} = 2.5 \times 2.5 = 6.25 \mu\text{C}$

3. $U_{BD} = 2 \text{ Volt}$

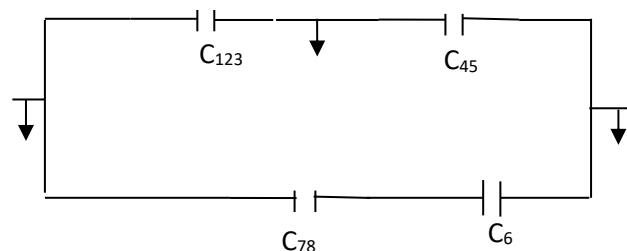
$Q_{C4} = C_4 U_{BD} = 5 \times 2 = 10 \mu\text{C}$

and $Q_{C5} = C_5 U_{BD} = 5 \times 2 = 10 \mu\text{C}$

4. Calculate C_{eq}

$C_{123} = C_1 + C_2 + C_3 = 4 + 3.5 + 2.5 = 10 \mu\text{F}$

$C_{45} = C_4 + C_5 = 5 + 5 = 10 \mu\text{F}$



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$$C_{78} = C_7 + C_8 = 5 + 5 = 10 \mu F$$

$$\frac{1}{C_{eq1}} = \frac{1}{C_{123}} + \frac{1}{C_{45}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Rightarrow C_{eq1} = 5 \mu F$$

$$\frac{1}{C_{eq2}} = \frac{1}{C_{78}} + \frac{1}{C_6} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Rightarrow C_{eq2} = 5 \mu F$$

$$C_{eq} = C_{eq1} + C_{eq2} = 5 + 5 = 10 \mu F$$

5. Energy stored in the capacitor C_1 :

$$E_{C1} = \frac{1}{2} C_1 U_{AD}^2 = \frac{1}{2} 4 (2,5)^2 = 12,5 \mu j$$