University of Tlemcen
Faculty of Sciences
Department of Mathematics

A.Y: 2024/2025
1st year LMD-M and MI

Correction of SW Nº 03
Conductors and capacitors

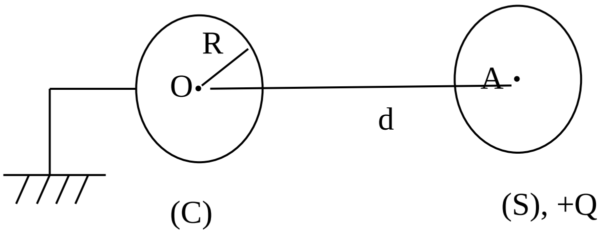
Exercise 1

We have $E = \frac{\sigma}{\varepsilon_0}$ with $Q = \sigma . S = \sigma . 4\pi R^2$

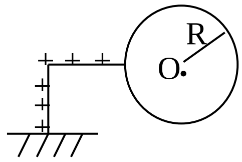
so $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 R^2}$

NA. $E = \frac{10^{-8}}{4\pi 10^{-2}.(8.85.10^{-12})} = 9.10^3\ V/M$

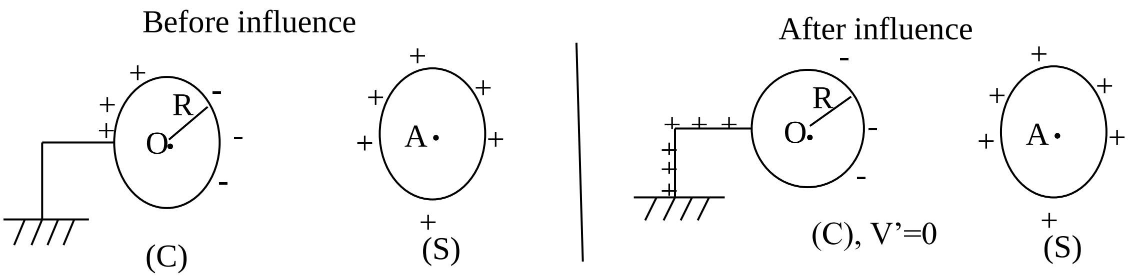
Exercise 2



Before influence,



Neglecting the influence of (C) on (S), let's find the charge q of (C).
Remember that when the conductor is connected to the ground, positive charges flow towards the ground and the potential V=0 .



After the influence, positive charges flow to ground and the conductor (C) will have a negative charge and zero potential. $\boldsymbol{V_C{}'=0}$

$V'_C = V_C + V_{S/C} = 0$

with V_C : is the potential of (C) before influence and VS/C: is the potential of (C) after influence of (S) on (C).

$\boldsymbol{V_C = \frac{kq_C}{R}}$

$\boldsymbol{V_{S/C} = \frac{kQ_S}{d}}$ **with (Q_S=+Q)**

So $\boldsymbol{V'_C = \frac{kq_C}{R} + \frac{kQ}{d} = 0}$

$\Rightarrow q_C = -\frac{QR}{d}$ with d= OA

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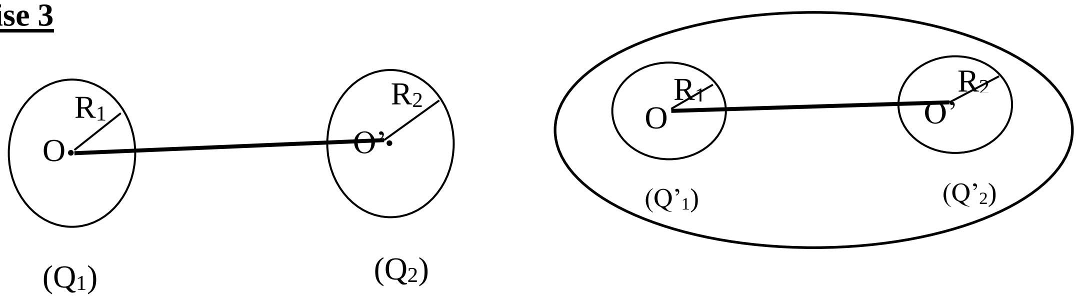
<u>Exercise 3</u>



Let's find Q1'and Q2' after influence:

By connecting the two conductors with a wire ; a single conductor is created with Q'1 and Q'2 are the charges of the two conductors.

After influence when this conductor is in electrostatic equilibrium:

- The potential is constant: $V_1' = V_2' \Rightarrow \frac{kQ_1'}{R_1} = \frac{kQ_2'}{R_2}$ $\quad So\ \mathbf{\frac{Q_1'}{R_1} = \frac{Q_2'}{R_2}}$

- The total charge in the formed conductor is the sum of the charges of the two conductors, since the charge carried by the wire is neglected.

$Q'_1+Q'_2=Q_1+Q_2$

$$\begin{cases} \frac{Q_1'}{R_1} = \frac{Q_2'}{R_2} \\ Q_1' + Q_2' = Q_1 + Q_2 \end{cases} \Rightarrow \begin{cases} \frac{Q_1'}{2} = \frac{Q_2'}{3} \\ Q_1' + Q_2' = 25 \end{cases}$$

$$\Rightarrow \begin{cases} Q_1' = 2\frac{Q_2'}{3} \\ 2\frac{Q_2'}{3} + Q_2' = 25 \end{cases} \quad \text{so } Q_2' = 15\mu C = Q_2 \text{ et } Q_1' = 10\mu C = Q_1$$

The charges on the two conductors have not changed, so there has been no charge displacement, as the two conductors are far apart and the charge on the wire is negligible.

<u>Exercise 4</u>

1- C_1 and C_2 in series: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$C_{eq1} = \frac{C_1 C_2}{C_1+C_2} \Rightarrow C_{eq1}=2\ \mu F$

C_3 et C_4 in series: $\frac{1}{C_{eq2}} = \frac{1}{C_3} + \frac{1}{C_4}$

$\Rightarrow C_{eq2} = \frac{C_3 C_4}{C_3+C_4} \Rightarrow C_{eq2}=1{,}33\ \mu F$

$C_! \therefore C_2 \Rightarrow C_{eq} = C_{ea1} + C_{ea2} = 3{,}33\mu F \Rightarrow$ $\mathbf{C_{eq}=3,33\mu F}$

2- The voltage across the armatures of each capacitor:

$Q_1 = Q_2$ (C_1 and C_2 in séries)

$$C_1\ V_1 = C_2\ V_2 \Rightarrow V_1 = \frac{C_2\ V_2}{C_1} = 2V_2$$

$V_1 + V_2 = 90$ volt

so

$$2V_2 + V_2 = 90 \Rightarrow 3V_2 = 90 \Rightarrow \mathbf{V_2 = 30\ volt} \Rightarrow \mathbf{V_1 = 60\ volt}$$

$Q_3 = Q_4$ (C_3 and C_4 in séries)

$C_3 V_3 = C_4 V_4 \Rightarrow V_3 = \frac{C_4 V_4}{C_3} = 2V_4 \Rightarrow V_3 + V_4 = 90$ volt

So

$2V_4 + V_4 = 90 \Rightarrow 3V_4 = 90 \Rightarrow \mathbf{V_4 = 30\ volt} \Rightarrow \mathbf{V_3 = 60\ volt}$

3- the electrical charge carried by each capacitor.

$$\mathbf{Q_1 = Q_2 = C_1 V_1 = C_2 V_2 = 180 \mu C}$$
$$\mathbf{Q_3 = Q_4 = C_3 V_3 = C_4 V_4 = 120 \mu C}$$

<u>Exercise 4</u>

1) The capacity of the two capacitors in parallel, $C_{23} = C_2 + C_3 = 6$ mF
and the capacity of the entire circuit:

$$\frac{1}{C_{ab}} = \frac{1}{C_1} + \frac{1}{C_{23}}$$

$$C_{eq} = 2{,}0\ m\ F$$

2) The charge on the first capacitor is the same as the charge on the whole combination, because it's the only thing to which the left wire is connected. This charge can be found from the capacity.

$$Q_{eq} = Q_1 = C_{eq}.V_{eq} = 12\ m\ C$$

There is a charge Q 1 on the opposite side of the first capacitor, which must also come from the next capacitors as they are equal to each other. Therefore,

$$Q_2 = Q_3 = (1/2)\ Q_1.$$
$$Q_1 = 12\ m\ C,\ Q_2 = Q_3 = 6{,}0\ m\ C$$

2) We have $V_i = Q_i / C_i$. hence $V_1 = (Q_1/C_1) = 4{,}0\ V$

And $V_2 = V_3 = (Q_2/C_2) = 2{,}0\ V$

4) the capacity is:

$$C = \frac{Q}{V} = \frac{Q}{Ed}$$ so $C = 2.0\ 10^{-2}\ F$

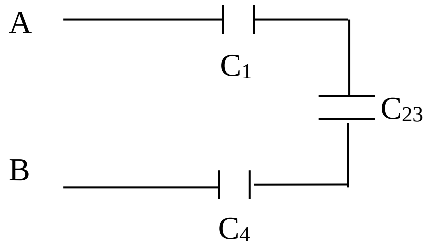
5) Energie is :

$E_p = \frac{1}{2} C U^2 = \frac{1}{2}.QU$ so $E_p = 18\ 10^{-9}\ J$

<u>Supplementary exercise 1:</u>

A 1- Equivalent capacity

$C_{23} = C_2 + C_3 = 10 + 4 = 14\ \mu F$



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{2} + \frac{1}{14} + \frac{1}{7} = \frac{10}{14} \Rightarrow C_{eq} = 1{,}4\mu F$$

2- Charges carried by capacitors

In a series connection: $Q_{AB} = Q_{C1} = Q_{C23} = Q_{C4}$ avec $Q_{AB} =$ Ceq U_{AB} $U_{AB}=U_{C1}+U_{C23}+U_{C4}$

$Q_{eq} = C_{eq}U \Rightarrow Q_{eq} = 1{,}4x12 = 16{,}8\mu C$ U=U_{AB}=12 volt

$$Q_{eq} = Q_{C_1} = Q_{C_4} = Q_{C_{23}} = 16{,}8\mu C$$

And $\boldsymbol{U_{23}} = \boldsymbol{U_2} = \boldsymbol{U_3} \Rightarrow \frac{Q_{C_{23}}}{C_{23}} = \frac{Q_{C_2}}{C_2} = \frac{Q_{C_3}}{C_3}$

$\Rightarrow Q_{C_2} = \frac{Q_{C_{23}} x\, C_2}{C_{23}} = \frac{16{,}8x10}{14} = 12\mu C$ and $Q_{C_3} = \frac{Q_{C_{23}} x\, C_3}{C_{23}} = \frac{16{,}8x4}{14} = 4{,}8\mu C$

1- Capacitors voltage

$U_1 = \frac{Q_{C_1}}{C_1} = \frac{16{,}8}{2} = 8{,}4Volt$ and $U_4 = \frac{Q_{C_4}}{C_4} = \frac{16{,}8}{7} = 2{,}4Volt$
and $U_3 = U_2 = 12 - 8{,}4 - 2{,}4 = 1{,}2Volt$

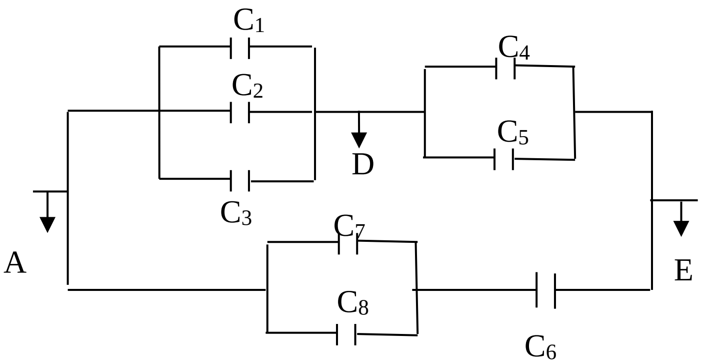
B. The capacity of capacitor
$V = \int Edl = E\int_A^B dl = Ed$

$$C = \frac{Q}{V} = \frac{Q}{Ed} \quad \text{so } C = \frac{30x10^{-3}}{100x0{,}015} = 20.\ 10^{-3}\ F$$

C. The energie is :

$$E_p = \frac{1}{2}CU^2 = \frac{1}{2}.QU \text{ so } E_p = 18\ 10^{-9}\ J$$

<u>Supplimentary exercise 2:</u>



1. $Q_{C1} = C_1U_{AD} \Rightarrow U_{AD} = \frac{Q_{C1}}{C_1} = \frac{10}{4}$
$\Rightarrow U_{AD} = 2.5\ Volt$

2. $Q_{C2} = C_2U_{AD} = 3.5x2.5 = 8.75\mu c$
$Q_{C3} = C_3U_{AD} = 2.5x2.5 = 6.25\mu c$

3. $U_{BD} = 2\ Volt$
$Q_{C4} = C_4U_{BD} = 5x2 = 10\mu C$

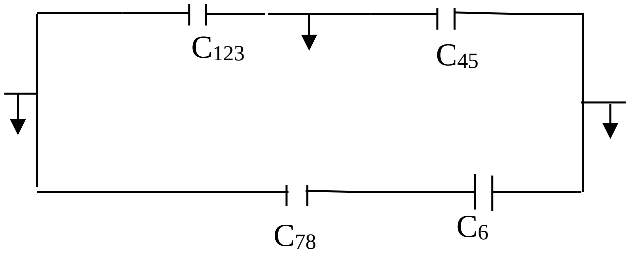
and $Q_{C5} = C_5 U_{BD} = 5x2 = 10\mu C$

4. Let's calculate C_{eq} :

$C_{123}=C_1+C_2+C_3=4+3,5+2,5=10\mu F$

$C_{45}=C_4+C_5=5+5=10\mu F$

$C_{78}=C_7+C_8 =5+5=10\mu F$



$\frac{1}{C_{eq1}} = \frac{1}{C_{123}} + \frac{1}{C_{45}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Rightarrow C_{eq1} = 5\mu F$

$$\frac{1}{C_{eq2}} = \frac{1}{C_{78}} + \frac{1}{C_6} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} \Rightarrow C_{eq2} = 5\mu F$$

$C_{eq}=C_{eq1}+C_{eq2}=5+5=10\ \mu F$

5. Energy stored in the capacitor C_1 :
   $E_{C1} = \frac{1}{2} C_1 U_{AD}^2 = \frac{1}{2} 4\ (2,5)^2 = 12,5\mu j$