University of Tlemcen
Faculty of Sciences
Department of Mathematics

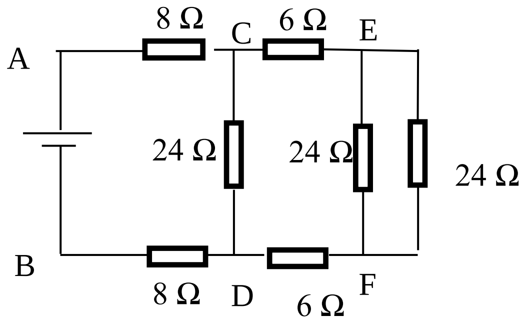
A.Y: 2024/2025
1st year LMD-M and MI

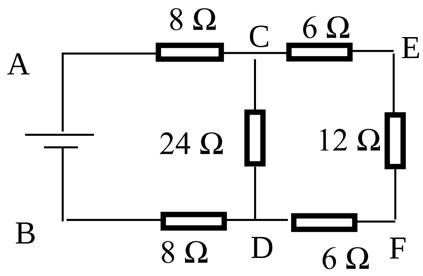
Correction of SW Series N° 04

ELECTRO-KINETICS

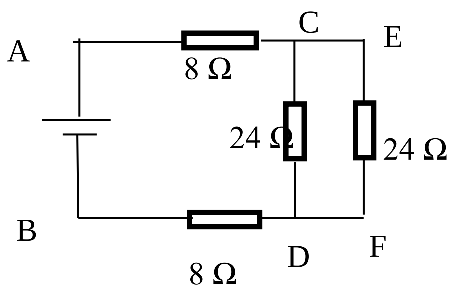
Exercise 1

1- To calculate the equivalent resistance, simplify the circuit:
$R_{eq1} = 16 + 4 + 4 = 24\ \Omega$

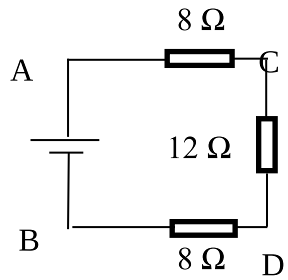




Then $\frac{1}{R_{eq2}} = \frac{1}{24} + \frac{1}{24} = \frac{2}{24} = \frac{1}{12}$
So $R_{eq2}=12\ \Omega$



$R_{eq3}=12+6+6=24\ \Omega$



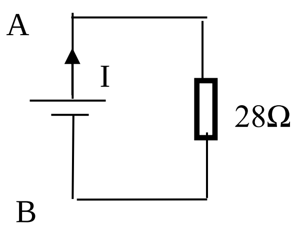
$\frac{1}{R_{eq4}} = \frac{1}{24} + \frac{1}{24} = \frac{2}{24} = \frac{1}{12}$ so $R_{eq4}=12\ \Omega$

So
$R_{eq}=8+8+12=28\ \Omega$

2- The intensity I of the current delivered by the generator, specifying the direction of flow:

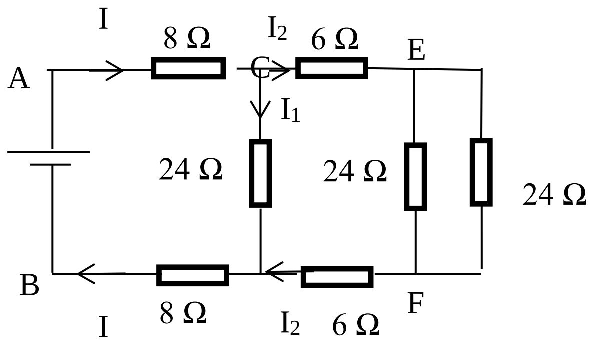
$E = \text{Req}\, I \Rightarrow I = \frac{E}{\text{Req}}$

So $I = \frac{56}{28} = 2A$



3- the potentiel $V_{AC}=R_{AC}\, I=8x2=16V$
The current flowing in the CD branch
In the ACDBA Loop (in this direction)
$E-V_{AC}-V_{CD}-V_{DB}=0$

$\Rightarrow E-8I-24I_1-8I=0$
$I_1 = \frac{E-16I}{24} = \frac{56-32}{24} = 1A$



4- V_{EF} voltage between points E and F
In the CCLD Loop (in this direction).

V_{DC}-V_{CE}-V_{EF}-V_{FD}=0 ⇒V_{EF}= V_{DC}-V_{CE}-V_{FD}

⇒ V_{EF}=24I_1-6I_2-6I_2
I_2=I-I_1=2-1=1A
V_{EF}=24-12=12 V
The current flowing through the EF

V_{EF}=$R_{EF}I_{EF}$ $\Rightarrow I_{EF} = \frac{V_{EF}}{R_{EF}} = \frac{12}{24} = 0.5\ A$

2. Calculate the voltage VGH between points G and H, and deduce the current in the branche GH.

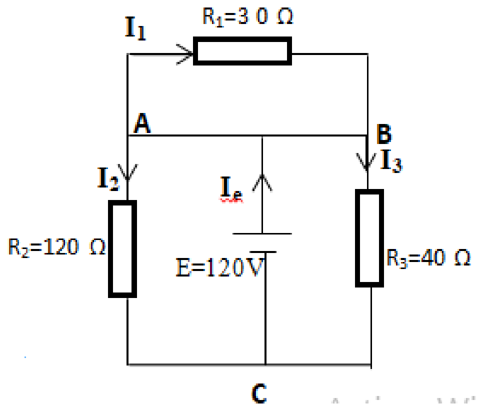
V_{EF}= V_{EG}+V_{GH}+V_{HF}=(4+16+4)I' so I'=12/24=0.5A
Then V_{GH}=0.5 * 16=8Ω

3. Power dissipated by the generator:

P = EI = 56V x 2mA = 112mW

Exercise 2 :

1- Calculation of currents I_2 et I_3 : (figure 3)



Loop 1 : R_1I_1=0 ⇒ I_1=0
Loop 2 : R_2I_2- E=0 ⇒ I_2=1A
Loop 3 : R_3I_3- E=0 ⇒ I_3=3A
Calculation of generator output current:
Node C we have : I_2+ I_3= I_e=4A

2- Calculation of currents I_1, I_2 et I_3 : (figure 4)

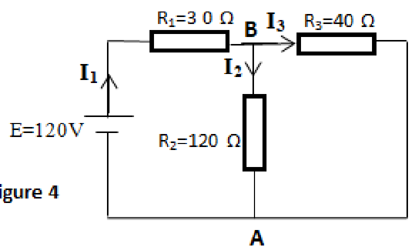


Figure 4

Node A: I_1 =I_2+ I_3
Loop 1: E- R_1I_1 – R_2I_2=0
Loop 2: R_2I_2 –R_3I_3=0

$I_1 = \frac{E - R_2I_2}{R_1} = 4(1 - I_2)$

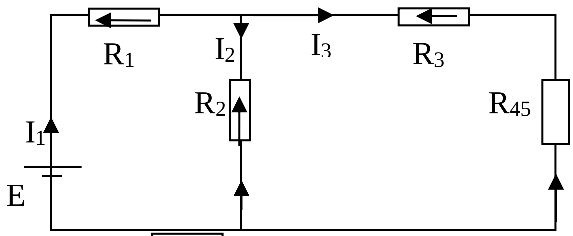
and $I_3 = \frac{R_2I_2}{R_3} = 3I_2$ so $4(1 - I_2) = I_2 + 3I_2$

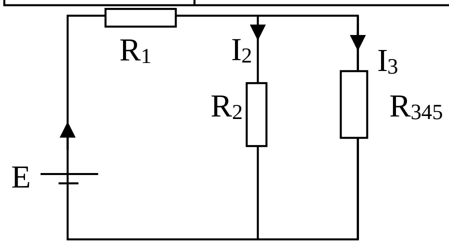
$I_2 = 0.5A$ and $I_1 = 2A$ so $I_3 = 3I_2 = 1.5A$

3- Find I_1 using R_{eq} :

R_{eq}= R_1+(R_2* R_3)/(R_2+ R_3)=60Ω
We have E=R_{eq} I_1 so I_1=E/ R_{eq}=120/60=2A

Exercice 3:





The current intensity I using

Kirchoff's laws :

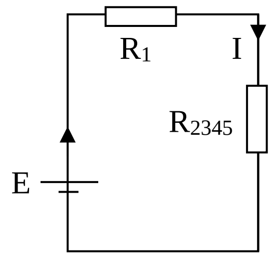
- Law of nodes : I_1=I_2+I_3

Law of loop:

E-R_1I_1-R_2I_2=0

R_2I_2-R_3I_3-$R_{45}I_3$=0

$\frac{1}{R_{45}} = \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12} \Rightarrow R_{45} = 4\Omega$

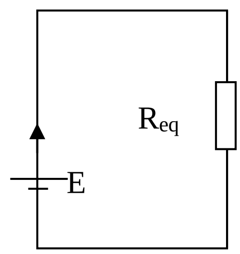


$$\begin{cases} 12 - 2I_1 - 20\, I_2 = 0 \\ 20I_2 - 16I_3 - 4I_3 = 0 \\ I_1 = I_2 + I_3 \end{cases}$$

$$\Rightarrow \begin{cases} 12 - 2(I_2 + I_3) - 20\, I_2 = 0 \\ 20I_2 - 16I_3 - 4I_3 = 0 \end{cases} \Rightarrow \begin{cases} 12 - 2I_3 - 22I_2 = 0 \\ 20I_2 - 20I_3 = 0 \end{cases}$$

$I_2=I_3$ so $12-24I_2=0$ Then $I_2=I_3=0{,}5$ A and $I_1=1$A

2- The current I using the equivalent resistance:



$R_{345}=16+4=20\ \Omega$, $\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_{345}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} \Rightarrow R_{2345} = 10\ \Omega$

$R_{eq}=R_1+R_{2345}=2+10=12\ \Omega$ with $E-R_{eq}I_1=0$ so $I_1 = \frac{E}{R_{eq}} = \frac{12}{12} = 1\ A$

2- The ddp to the R_2 : $U_2=R_2I_2 =20\text{x}\ 0{,}5=10$ V

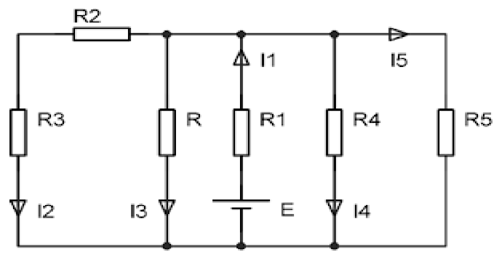
3- The power generated by R_2 : $P_2=R_2(I_2)^2= U_2\text{x}I_2 =10\text{x}\ 0{,}5=5$ W

4- Circulating currents in resistances R_4 and R_5

$U_{45}=R_{45}I_3=4\text{x}0{,}5=2$V with $U_{45} = U_4 = U_5 \Rightarrow U_{45} = R_4 I_3' = R_5 I_3''$

So $I_3' = \frac{U_{45}}{R_4} = \frac{2}{6} = \frac{1}{3}A$ and $I_3'' = \frac{U_{45}}{R_5} = \frac{2}{12} = \frac{1}{6}A$

Exercise 4 :



$U_{R2} =R_2I_2$ so $I_2 = U_{R2} /R_2 = 8V/2k\Omega = 4mA$

$U_{R3} = R_3I_2 = 4k\Omega \times 4mA = 16V$

$U_{AB} = U_{R2} + U_{R3} =8+ 16=24V$

$U_{AB} =RI_3$ so $R = U_{AB} /I_3 = 24V / 2mA$

Then R = 12kΩ

$U_{AB} = U_{R4} =U_{R5} = 24V$ et $R_4 = R_5$ and $I_4=I_5 =U_{AB} / R_4$ with $I_4=I_5 = 24V/3k\Omega = 8mA$

We apply the law of knots to the point A :

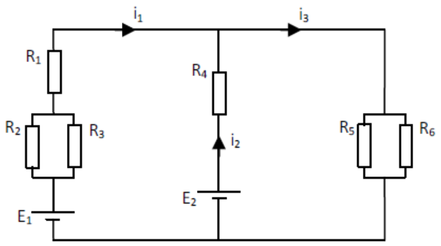
$I_1 = I_2+ I_3 + I_4+I_5 = 2mA + 4mA + 8mA + 8mA = 22mA$

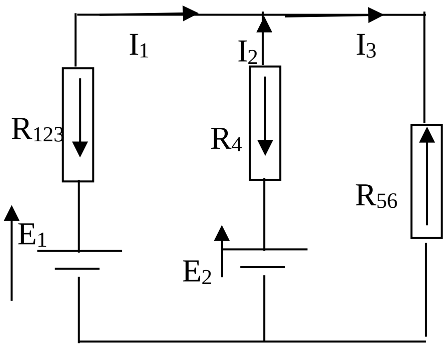
$U_{AB} = E - R_1I_1$ so $E =U_{AB} + R_1I_1$

E = 24V + 1kΩ x22mA then E = 46 V

Exercise 5 :

Circuit simplification:





$\frac{1}{R_{23}} = \frac{1}{20} + \frac{1}{20} = \frac{1}{10} \Rightarrow R_{23} = 10\Omega,$

and R_{123}=10+10=20 Ω ,

$\frac{1}{R_{56}} = \frac{1}{6} + \frac{1}{3} = \frac{3}{6} \Rightarrow R_{23} = 2\Omega$

In this circuit, we have two generators: E_1 and E_2.

E_1 is a generator because it gives the current I_1 , E_2 is also a generator, it gives the current I_2.

2- The curents I_1, I_2 et I_3

Law of nodes: $I_1 + I_2 = I_3$

Law of loops :

$E_1-R_{123}I_1-+R_4I_2-E_2=0 \Rightarrow 20-20\ I_1-+5I_2-10=10-20\ I_1+5I_2=0$ (1)

$E_2-R_4I_2-R_{56}I_3=0 \Rightarrow 10-5I_2-2I_3=0$

$10-5I_2-2(I_1+I_2)=10-2\ I_1-7\ I_2=0$

$10-2I_1-7I_2=0$ (2)

We do (1) – 10*(2) we will ahve $(10-20\ I_1-+5I_2)-(100-20I_1-70I_2)=-90+75\ I_2=0$

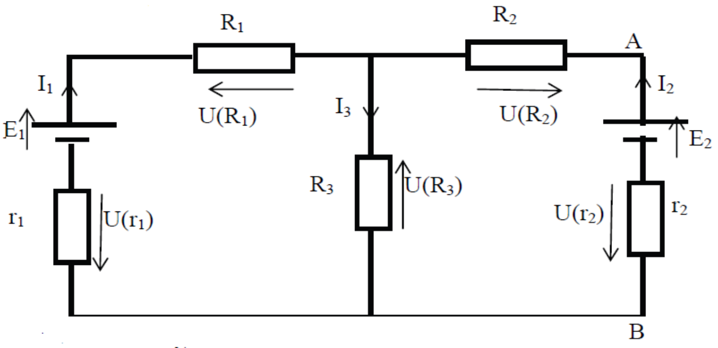
(1)-10x (2)=$90-75I_2=0 \Rightarrow I_2=1{,}2$A, and (2) $\Rightarrow 10-8{,}4-2I_1$ so $I_1=0{,}8$ A

And $I_3=1{,}2+0{,}8=2$ A

Additional exercise

1- Assuming the capacitor is fully charged, calculate the currents I1, I2 and I3.

Since the capacitor is fully charged, the current does not flow, so the circuit will be as follows:



Law of nodes $I_3 = I_1 + I_2$

According to the law of loopes:

$E_1 - R_1 I_1 + R_2 I_2 - E_2 + r_2 I_2 - r_1 I_1 = 0$

$E_1 - R_1 I_1 - R_3 I_3 - r_1 I_1 = 0$ so $E_1 - R_1 I_1 - R_3 I_1 - R_3 I_2 - r_1 I_1 = 0$

By replacing I_3 with $I_1 + I_2$, we will have:

$E_1 - (R_1 + R_3 + r_1) I_1 - R_3 I_2 = 0$

$E_2 - (R_2 + R_3 + r_2) I_2 - R_3 I_1 = 0$

$E_1 = 12V$, $E_2 = 8V$, $r_1 = r_2 = 1\Omega$, $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$, $R_3 = 5\ \Omega$

$12 - 10 I_1 - 5 I_2 = 0$

$(8 - 5 I_1 - 9 I_2 = 0) \times 2 \quad = 16 - 10 I_1 - 18 I_2 = 0$

$-4 + 12 I_2 = 0$ so $I_2 = 0{,}3$ A

$10 I_1 + 5 I_2 - 12 = 0$ (1)

$5 I_1 + 9 I_2 - 8 = 0$ (2)

In doing so : (1)-2x (2)$= 4 - 13 I_2 = 0$

We then find : $I_2 = 0.3$ A , $I_1 = 1.05$A and $I_3 = 1.35$A

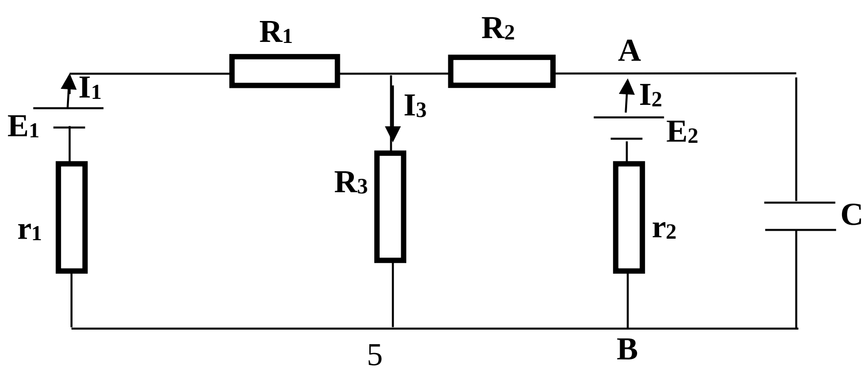
2- The difference in potential between A and B

$U_{AB} = V_A - V_B = E_2 - r_2 I_2 = 7.7$ V

3- Capacitor charge Q_C

Find the potential difference across the capacitor:

$U_C = U_{AB} = 7.7$ V



The charge Q_C

Q_C=C U_C=2x7.7x10^{-6}=1.54 µC

The energy stored in the capacitor is :

E_C=1 /2(C U^2)=1/2(2x 10^{-6}) $(7.7)^2$=59.29x 10^{-6} joule

4- The power generated by R_3

P=U_3 I_3=$R_3 I_3^2$=5x$(1.35)^2$=9.112 Watt