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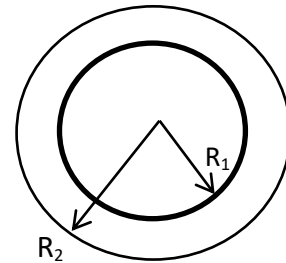
Last name:

## Face-To-Face Test

Let be two concentric spheres with center O and respective radii  $R_1$  and  $R_2$  such that  $R_1 < R_2$ .

The sphere of radius  $R_1$  is surface-loaded with **density**  $\sigma$ . The second of radius  $R_2$  carries a surface distribution of **density**  $\sigma'$ .

- 1- Using GAUSS' theorem find the expression of the electrostatic field  $E(r)$  outside the system.
- 2- Deduce the expression of the electric potential  $V(r)$  outside the system.
- 3- Tracer l'allure de  $E(r)$  et  $V(r)$ .



**Correction of Test:**

The Gaussian surface is a sphere with center O and radius r. For reasons of symmetry, the field is radial and constant at any point on the Gaussian surface.

$$\oint \vec{E} \cdot \vec{ds} = \frac{\sum Q_{int}}{\epsilon_0}$$

$$\vec{E} // \vec{ds} \quad \text{so } \oint \vec{E} \cdot \vec{ds} = \iint E \cdot ds = E \iint ds = E \cdot S = E 4\pi r^2 \Rightarrow E 4\pi r^2 = \frac{\sum Q_{int}}{\epsilon_0}$$

1- The electrostatic field  $E_3(r)$ :

We have 2 cases

**3<sup>rd</sup> case  $r \geq R_2$**

$$Q_{int} = Q_1 + Q_2 \quad \text{with } Q_1 = \sigma 4\pi R_1^2 \text{ and } dq_2 = \sigma' ds \Rightarrow Q_2 = \sigma' 4\pi R_2^2$$

$$\text{so } Q_{int} = \sigma 4\pi R_1^2 + \sigma' 4\pi R_2^2$$

$$\text{and } E_3 = \frac{\sigma 4\pi R_1^2 + \sigma' 4\pi R_2^2}{4\pi r^2 \epsilon_0}$$

$$\text{then } E_3 = \frac{\sigma R_1^2}{\epsilon_0 r^2} + \frac{\sigma' R_2^2}{\epsilon_0 r^2}$$

The electrostatic potential  $V_3(r)$ .

$$\vec{E} = -\overrightarrow{grad} v \Rightarrow E = -\frac{dv}{dr} \quad \text{so } v = -\int E dr$$

**3<sup>rd</sup> case  $r \geq R_2$**

$$E_3 = \frac{\sigma R_1^2 + \sigma' R_2^2}{r^2 \epsilon_0} \Rightarrow v_3 = -\frac{\sigma R_1^2 + \sigma' R_2^2}{\epsilon_0} \int \frac{dr}{r^2} = \frac{\sigma R_1^2 + \sigma' R_2^2}{\epsilon_0 r} + C_3$$

Infinite potential ( $r \rightarrow \infty$ )  $v=0$  so  $\lim_{r \rightarrow \infty} v = 0$

Then  $C_3=0$

$$\text{And } v_3 = \frac{\sigma R_1^2 + \sigma' R_2^2}{\epsilon_0 r}$$

