University of Tlemcen
Faculty of Sciences
Department of Mathematics

University Year: 2023/2024
1st Year LMD-M and MI

Supervised work N° 1 of Mecanics

Dimensional analysis and uncertainty calculation

Exercise 1

Find the dimension of the following physical quantities:

Surface, Volume, Density, Frequency, Linear Velocity, Angular Velocity, Linear Acceleration, Angular Acceleration, Force, Work, Energy, Power, and Pressure.

Exercise 2

The characteristic equation of a constant temperature fluid is as follows:

$$\left(p+\frac{a}{V^2}\right)(V-b)=c$$

Or p is the pressure and V is the volume.
Determine the dimensions of quantities a, b and c.

Exercise 3

Check the homogeneity of this formula:

$$p = \rho g h_1 + h_2 F$$

Such as: P pression, ρ density, g an acceleration of gravity, h_1 and h_2 are heights and F a force.

Exercise 4

1. In a fluid, a ray ball (نصف القطر) r animated by a velocity v, is subjected to a friction force given by $F=-6\pi\eta\, rv$, where η is the viscosity of the fluid.
   What is the dimension of η?
2. When the ball is dropped without initial speed at the moment t = 0, its speed is written to t > 0:
   $$v = a\left(1-exp\left(-\frac{t}{b}\right)\right)$$

Where a and b are two quantities that depend on the characteristics of the fluid. What are the dimensions of a and b?

Exercise 5

The sound emitted by the wire of a guitar is characterized by its frequency f. This frequency is a function of the force F of the wire tension, the length L and the density ρ of the wire.
Find the expression of frequency f assuming the form:

$$f=K\,F^a\,L^b\,\rho^c$$

(With K a dimensionless constant and the frequency dimension $[f]=T^{-1}$).

Exercise 6

Let the simple pendulum formed of a ball (sphere) of radius R and mass m. The study of the effect of the air on this pendulum shows that its period T depends on a constant k, the coefficient of the air η, the radius of the ball R and its density ρ .

1- Find the expression of the period assuming the form:
$T = K\eta^x R^y \rho^z$ avec $[\eta] = ML^{-1}T^{-1}$
2- Determine relative uncertainty on T based on $\Delta\eta, \Delta R \; and \; \Delta m$.

Exercise 7

The speed limit reached by a weighted parachute is a function of its weight P and its surface S, is given by: $v = \sqrt{\frac{P}{K.S}}$

1) Give the dimension of the constant k.

2) Calculate the speed limit of a parachute having the following characteristics:

M=90 kg, S=80 m2, g=9,81 m/s2, and k=1,15 MKS.

3) The weight being known to the nearest 2 % and the surface to 3 %, calculate the relative uncertainty Δv/v on the velocity v, thus the absolute uncertainty Δv and deduce the condensed writing of this velocity.

Suplimentary Exercises

Exercise 1

The height H of a liquid of mass M contained in a cylinder of radius R is given by the relation:

$$H = \frac{(2.\sigma.cos\alpha)}{(R.g.\rho)}$$

Where α is the liquid-cylinder contact angle, ρ the density of the liquid and g the gravity acceleration.

1- Using the dimensional equations, find the dimension of σ.
2- Determine relative uncertainty on σ based on absolute uncertainties ΔR, Δg, ΔM and Δα.

Exercise 2

The resonance frequency f of an electric circuit is given by the formula:

$$f = \frac{1}{2\pi\sqrt{L.C}}$$

L and C are known with absolute uncertainties ΔL and ΔC.

Determine as a function of L, C, ΔL and ΔC absolute and relative uncertainties on f with the two differential methods.