University of Tlemcen
Faculty of Sciences
Department of Mathematics
Sunday: 14/01/2024
Duration: 01 h 30mn

Academic Year 2023/2024
1st Year LMD-M and MI

Final Exam of Mechanics

Course questions: (5pts)

1- Why use dimensional analysis?

2- What can it say about the total mechanical energy of a system in the presence of frictional forces?

3- What is the difference between a conservative force (قوة منحفضة) and a non-conservative force? Give an example for each one.

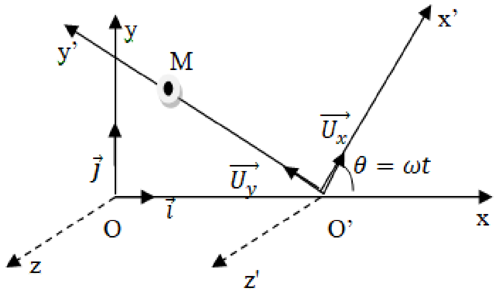
4- Calculate the work of a force F=1.5 10^4N supplied to move a body a height (AB) of 3 meters (vertically).

5- Calculate the work of the spring return force with stiffness constant k ($\overrightarrow{dl} = dx.\vec{\imath}$).

Exercise 1: (7pts)

Consider the fixed reference frame R(Oxyz) where point O' moves **along axis (Ox)** with **constant velocity v_0**. Linked to O' is the moving reference frame (O'x'y'z') which rotates **around (Oz)** with **constant** angular velocity ω. A moving point M moves **along the (O'y')** axis with constant **acceleration γ**.

At time t=0, the axes (Ox) and (O'x') are coincident and M is at O.



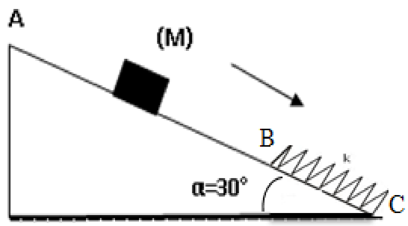
Calculate in the moving frame:

1- The relative velocity $\overrightarrow{v_r}$ and the entrainment velocity $\overrightarrow{v_e}$, deduce the absolute velocity $\overrightarrow{v_a}$.

2- The relative acceleration $\overrightarrow{a_r}$, the entrainment acceleration $\overrightarrow{a_e}$ and Coriolis acceleration $\overrightarrow{a_c}$, deduce the absolute acceleration $\overrightarrow{a_a}$.

Exercise 2: (8pts)

Consider a small block of mass m =2kg dropped without initial velocity at point A of an inclined plane at an angle α=30° to the horizontal. Point A is at a height h_A=5m from the horizontal.



1- Knowing that the coefficient of dynamic friction on **plane AB is μ_d=0.2**, applying the fundamental principle of dynamics, what is the acceleration of the block on plane AB=8m?

2- Calculate the speed of the block when it reaches point B.

3- Using the kinetic energy theorem, find the speed of the block at point B.

4- At point B, the block hits a spring with stiffness constant k=100N/m at speed V_B. Calculate the maximum compression (x) of the spring (given g =10 m/s^2).

Good Luck

<u>Answers to the final Mechanics exam</u>

<u>Course question</u>: (5pts)

1- The advantages of using dimensional analysis are :

Find the dimension of a physical quantity, the units and nature of the quantities, check the homogeneity of a physical law and find the physical law of a physical quantity. **(01 pts)**

2- Total mechanical energy is non-conservative because the system is subject to non-conservative forces such as friction. **(0.5pts)**

3- The difference between a conservative force and a non-conservative force is:

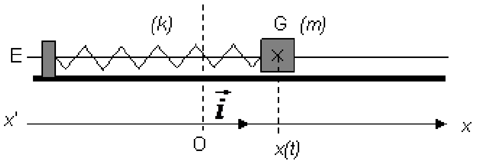
- A force is said to be conservative if its work does not depend on the path followed, and it is said to derive from a potential **(0.25 pts)** Examples: Force of gravity **(0.25 pts)**
- A force is said to be non-conservative if its work depends on the path followed **(0.25 pts)**, like the force of friction. **(0.25 pts)**

4- Calculate the work done by this force to move the car by a height (AB) of 3 meters.

Translated with www.DeepL.com/Translator (free version)

$\boldsymbol{W_{AB}}(\vec{\mathbf{F}}) = |\vec{\boldsymbol{F}}|.|\overrightarrow{\boldsymbol{AB}}|.\boldsymbol{cos\alpha}$=**F.d.cos 0** =1.5 10^4 . 3=4.5 10^4 J **(01 pts)**

5- Calculate the work of the restoring force of a spring with stiffness constant k.



$\vec{F} = -kx\vec{\imath}$, $\overrightarrow{dl} = dx.\vec{\imath}$ et $dW = \vec{F}.\overrightarrow{dl}$ (0.5 pts)

$$dW = -dE_p = -kx.dx \Rightarrow dE_p = kxdx$$ (0.5 pts)

$$\Rightarrow \int dEp = k\int_{x_i}^{x_f} xdx \quad \Rightarrow Ep = \frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}kx^2$$ (0.5 pts)

<u>Exercise 1</u> : (7pts)

1- Speeds: 3.5pts

M moves along the Oy' axis with constant acceleration, so: $\overrightarrow{O'M} = Y\,\overrightarrow{u_y}$ with $\gamma = \frac{dv}{dt}$ and at t=0 the point M is at O' :

$\gamma = \frac{dv}{dt} \Rightarrow \int_0^v dv = \gamma\int_0^t dt$ so $v = \gamma t$ (at t=0, v_0 (M)=0)

$v = \gamma t = \frac{dY}{dt} \Rightarrow \int_0^Y dY = \gamma\int_0^t tdt$ so $Y = \frac{1}{2}\gamma t^2$ (at t=0, Y_0 (M)=0)

$\overrightarrow{O'M} = \frac{1}{2}\gamma t^2\,\overrightarrow{u_y}$ (0.5pts)

O' moves on Ox with a constant speed v_0 so $\overrightarrow{OO'} = x\vec{\imath}$ and $v_0 = \frac{dx}{dt}$ and à t=0, axis (O'x') is confused with (Ox).

$v_0 = \frac{dx}{dt} \Rightarrow \int_0^x dx = v_0\int_0^t dt$ so $x = v_0 t$ (at t=0, x_0 (O')=0) then $\overrightarrow{OO'} = v_0 t\vec{\imath}$ (0.5pts)

University of Tlemcen
Faculty of Sciences
Department of Mathematics
Sunday: 14/01/2024
Duration: 01 h 30mn

Academic Year 2023/2024
1st Year LMD-M and MI

$\vec{v_r} = \frac{d\overrightarrow{O'M}}{dt} = \gamma\, t\, \vec{\mathbf{u_y}}$ (0.5pts)

$\vec{v_e} = \frac{d\overrightarrow{OO'}}{dt} + \vec{\omega} \wedge \overrightarrow{O'M}$ (0.25pts) with $\vec{\omega} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$ (0.25pts)

$\vec{u_x} = \cos\theta\vec{\imath} + \sin\theta\vec{\jmath}$ and $\vec{u_y} = -\sin\theta\vec{\imath} + \cos\theta\vec{\jmath}$

Using the passage table :

So $\vec{\imath} = \cos\theta\, \vec{u_x} - \sin\theta\, \vec{u_y}$

	$\vec{u_x}$	$\vec{u_y}$
$\vec{\imath}$	Cosθ	-sin θ
$\vec{\jmath}$	Sinθ	cosθ

$\frac{d\overrightarrow{OO'}}{dt} = v_0\, \vec{\imath} = v_0(\cos\omega t\, \vec{u_x} - \sin\omega t\, \vec{u_y})$ (0. 5pts)

$\vec{\omega} \wedge \overrightarrow{O'M} = \begin{vmatrix} \vec{u_x} & \vec{u_y} & \vec{u_z} \\ 0 & 0 & \omega \\ 0 & \frac{1}{2}\gamma t^2 & 0 \end{vmatrix} = -\frac{1}{2}\gamma t^2\omega\, \vec{u_x}$ (0.25pts)

$\vec{\mathbf{v_e}} = \left(-\frac{1}{2}\gamma t^2\omega + v_0\cos\omega t\right) \vec{\mathbf{u_x}} + (-v_0\sin\omega t)\, \vec{\mathbf{u_y}}$ (0.25pts)

$\vec{\mathbf{v_a}} = \vec{\mathbf{v_r}} + \vec{\mathbf{v_e}} = \left(-\frac{1}{2}\gamma t^2\omega + v_0\cos\omega t\right) \vec{\mathbf{u_x}} + (\gamma t - v_0\sin\omega t)\, \vec{\mathbf{u_y}}$ (0.5pts)

1- **The accelerations :** 3.5pts

$\vec{\mathbf{a_r}} = \frac{\mathbf{d}\vec{\mathbf{v_r}}}{\mathbf{dt}} = \gamma\vec{\mathbf{u_y}}$ (1pts)

$\vec{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2} + \frac{d\vec{\omega}}{dt} \wedge \overrightarrow{O'M} + \vec{\omega} \wedge \vec{\omega} \wedge \overrightarrow{O'M}$ (0. 5pts) with $\frac{d^2\overrightarrow{OO'}}{dt^2} = \vec{0}$ (0.25pts)

$\vec{\omega} \wedge \vec{\omega} \wedge \overrightarrow{O'M} = \begin{vmatrix} \vec{u_x} & \vec{u_y} & \vec{u_z} \\ 0 & 0 & \omega \\ -\frac{1}{2}\gamma t^2\omega & 0 & 0 \end{vmatrix} = -\frac{1}{2}\gamma t^2\omega^2\, \vec{u_y}$ (0.25pts)

$\vec{a_e} = -\frac{1}{2}\gamma t^2\omega^2\, \vec{u_y}$ (0.25pts)

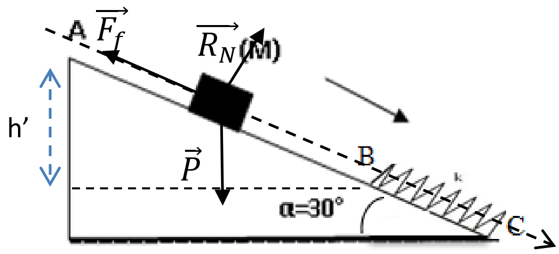
$\vec{\mathbf{a_c}} = \mathbf{2}\vec{\omega} \wedge \vec{\mathbf{v_r}} = \begin{vmatrix} \vec{u_x} & \vec{u_y} & \vec{u_z} \\ 0 & 0 & 2\omega \\ 0 & \gamma t & 0 \end{vmatrix} = -2\gamma t\, \boldsymbol{\omega}\, \vec{\mathbf{u_x}}$ (0.5pts)

$\vec{a_a} = \vec{a_r} + \vec{a_e} + \vec{a_c}$ (0.25pts)

So $\vec{\mathbf{a_a}} = (-2\gamma t\,\omega)\, \vec{u_x} + \left(\gamma - \frac{1}{2}\gamma t^2\omega^2\right) \vec{u_y}$ (0.5pts)

Exercise 2 : (8pts)

(0.5pts)



1- The acceleration of mass m on AB: 03pts

By applying the PFD : $\Sigma\vec{F} = m\vec{a} \Rightarrow \vec{p} + \overrightarrow{R_N} + \vec{f} = m\vec{a}$ (0.5pts)

Following (Ox) – f +p_x= -f + m g sinα=ma….(1)(0.5pts)

Following (Oy) R_N-p_y=0⇒ R_N=m g cosα……(2) (0.5pts)

μ_d=tgφ=F_f/R_N (0.5pts) ⇒F_f=N tg φ so F_f= μ_d m g cosα (0.5pts)

(1): - μ_d m g cosα + m g sinα = m.a ⇒ a= g (sinα- μ_d cosα)=3.27m/s^2 (0.5pts)

2- The velocity at point B : we have v_A=0

$and\ v_B^2 - v_A^2 = 2a(AB) \Rightarrow v_B^2 = 2a(AB)$ (0.5pts)

with (AB)=8m $\Rightarrow v_B = \sqrt{2(3.27)(8)} = 7.23 m/s^{-1}$ (0.5pts)

3- Applying the kinetic energy theorem, find the speed of the block when it reaches point B.

$\Delta Ec = \Sigma W_{f\ ext} \Rightarrow Ec_B - Ec_A = W_p + W_{Ff} + W_{RN}$(0.5pts)

$\frac{1}{2}mv_B^2 = mgsin\alpha\ AB - F_f AB$ (0.5pts)

And

$\frac{1}{2}mv_B^2 = mgsin\alpha\ AB - \mu_d\ m\ g\ \cos\alpha\ AB$ so $v_B = \sqrt{g.2.AB\ sin\alpha - 2\mu_d\ g\ \cos\alpha\ AB}$

$v_B = \sqrt{g.2.AB(sin\alpha - \mu_d\ \cos\alpha\ )}$ (0.5pts)

4- At point B, the block touches a spring with stiffness constant k=100N/m at speed VB.

Calculate the maximum compression (x) of the spring? (we give g =10 m/s^2).

$\Delta E_M = E_{M_C} - E_{M_B} = \Sigma W_{f\ NC} \Rightarrow (E_{C_C} + E_{P_C}) - (E_{C_B} + E_{P_B}) = W_{Ff}$ (01pts)

$\Rightarrow \frac{1}{2}kx^2 - \frac{1}{2}mv_B^2 - mg\ h' = \frac{1}{2}kx^2 - \frac{1}{2}mv_B^2 - mg\ (AB\ sin\alpha) = -\mu_d\ m\ g\ \cos\alpha\ AB$ (0.5pts)

So; x =$\sqrt{\frac{m(v_B^2 + g\ 2(AB\ sin\alpha) - \mu_d\ 2\ g\ \cos\alpha\ AB)}{k}}$ =1.07m (0.5pts)