Université de Tlemcen
Faculté des Sciences
Département de Mathématiques

Année Universitaire 2019/2020
Durée : 01 h 30min
1ère Année LMD-MI

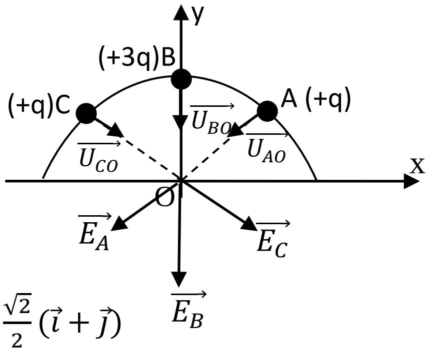
Corrigé du Rattrapage d'Électricité

Exercice 1: (8pts)

1- **Le champ électrique au point « O » : (5,25pts)**

$$\vec{E_O} = \vec{E_B} + \vec{E_A} + \vec{E_C} \text{(0.5pts)}$$

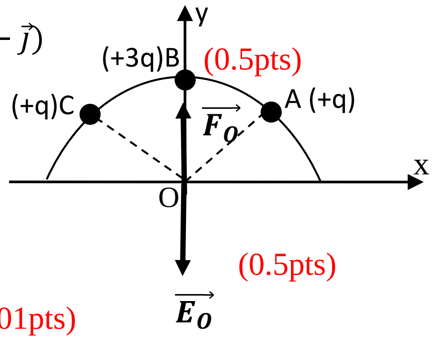
$$\begin{cases} \vec{E_A} = kq\frac{1}{AO^2}\overrightarrow{U_{AO}} \\ \vec{E_B} = k3q\frac{1}{BO^2}\overrightarrow{U_{BO}} \text{(0.75pts)} \\ \vec{E_C} = kq\frac{1}{CO^2}\overrightarrow{U_{CO}} \end{cases}$$



avec $\begin{cases} AO = BO = CO = R \\ \overrightarrow{U_{AO}} = -cos\alpha\vec{\imath} - sin\alpha\vec{\jmath} \\ \overrightarrow{U_{BO}} = -\vec{\jmath} \\ \overrightarrow{U_{CO}} = cos\alpha\vec{\imath} - sin\alpha\vec{\jmath} \end{cases}$ (01,25 pts)

on à $cos\alpha = sin\alpha = \frac{\sqrt{2}}{2}$

D'où $\begin{cases} \overrightarrow{U_{AO}} = -\frac{\sqrt{2}}{2}(\vec{\imath} + \vec{\jmath}) \text{(0.5pts)} \\ \overrightarrow{U_{BO}} = -\vec{\jmath} \\ \overrightarrow{U_{CO}} = \frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath}) \text{(0.5pts)} \end{cases}$ donc $\begin{cases} \vec{E_A} = -kq\frac{1}{R^2}\frac{\sqrt{2}}{2}(\vec{\imath} + \vec{\jmath}) \\ \vec{E_B} = -k3q\frac{1}{R^2}\vec{\jmath} \quad \text{(0.75pts)} \\ \vec{E_C} = kq\frac{1}{R^2}\frac{\sqrt{2}}{2}(\vec{\imath} - \vec{\jmath}) \end{cases}$



$$\boldsymbol{\vec{E_O} = \vec{E_B} + \vec{E_A} + \vec{E_C} = -kq\frac{(\sqrt{2}+3)}{R^2}\vec{\jmath}} \text{(0.5pts)}$$

2- **Le force électrostatique au point « O » (01,5 pts):**

avec $q_o = q' = -q$

$$\boldsymbol{\vec{F_o} = q'\vec{E_O} = -qE_o = kq^2\frac{(\sqrt{2}+3)}{R^2}\vec{\jmath}} \text{(01pts)}$$

3- **Le potentiel au point « O » : (01,25 pts)**

$V_O = V_A + V_B + V_C$ (0.5pts)

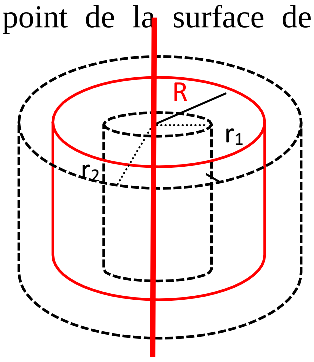
$\Rightarrow V_O = kq\frac{(1+1+3)}{R}$ (0.25pts)

$\Rightarrow \boldsymbol{V_O = 5kq\frac{1}{R}}$ (0.5pts)

Exercice 2 (8pts)

On considère comme surface de gauss un cylindre de rayon **r** et de hauteur **h**. (0.5pts)

A cause e la symétrie, le champ est radial et constant en tout point de la surface de Gauss(0.5pts)



D'après le Théorème de Gauss : $\emptyset = \iint \vec{E}.\vec{ds} = \frac{\sum Q_{int}}{\varepsilon_0}$ (0.5pts)

$\emptyset = \iint \vec{E}.\vec{ds} = 2\iint \vec{E}.\overrightarrow{ds}_{base} + \iint \vec{E}.\overrightarrow{ds_{lat}}$

$\vec{E} \perp \overrightarrow{ds}_{base} \Rightarrow \iint \vec{E}.\overrightarrow{ds_{lat}} = 0$

$\vec{E} \parallel \overrightarrow{ds_{lat}}$ Donc : $\emptyset = \iint \vec{E}.\overrightarrow{ds_{lat}} = \iint E.ds_{lat} = E.\int ds_{lat} = E.S_{lat}$

$\Rightarrow \emptyset = \boldsymbol{E2\pi rh} = \frac{\sum Q_{int}}{\varepsilon_0}$ donc $\boldsymbol{E} = \frac{\sum \boldsymbol{Q_{int}}}{\boldsymbol{2\pi rh\varepsilon_0}}$ (0.5pts)

1- **Le champ électrique 3,5pts**

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1er cas $r < R$ $dq = \lambda dl$ (0.5pts) $\Rightarrow Q_{int} = \lambda l$ (0.5pts) donc $E_1 2\pi rh = edc\ \frac{\lambda h}{\varepsilon_0} \Rightarrow \boldsymbol{E_1} = \frac{\lambda}{2\pi r\varepsilon_0}$ (0.5pts)

2eme cas r≥ R $Q_{int} = Q_1 + Q_2$ (0.5pts)

$dq_2 = \sigma ds$ (0.25pts) $\Rightarrow Q_{int} = \sigma S = \sigma 2\pi Rh$ (0.5pts) donc $Q_{int} = \lambda h + \sigma 2\pi Rh$ (0.25pts)

Donc $E_2 2\pi rh = \frac{\lambda h + \sigma 2\pi Rh}{\varepsilon_0} \Rightarrow \boldsymbol{E_2} = \frac{\lambda}{2\pi r\varepsilon_0} + \frac{\sigma R}{\varepsilon_0 r}$ (0.5pts)

2- Le potentiel électrique v(r) en tout point de l'espace 2,5pts

$\vec{E} = -\overrightarrow{grad}v$ (0.5pts) $\Rightarrow E = -\frac{dv}{dr}$ donc $v = -\int Edr$ (0.5pts)

1er cas $r < R$

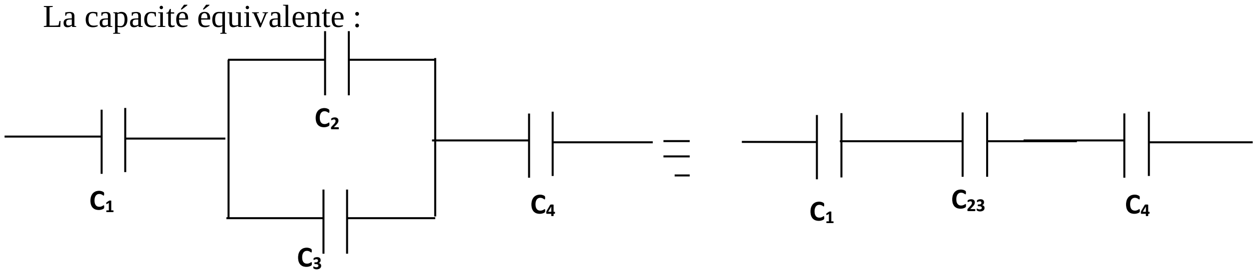
$E_1 = \frac{\lambda}{2\pi r_{\varepsilon_0}} \Rightarrow v_1 = \frac{\lambda}{2\pi\varepsilon_0}\int\frac{dr}{r}$ (0.25pts)donc $v_1 = \frac{\lambda}{2\pi\varepsilon_0}\ln r + C_1$(0.5pts)

2eme cas r≥ R

$E_2 = \frac{\lambda}{2\pi r\varepsilon_0} + \frac{\sigma R}{\varepsilon_0 r} \Rightarrow v_2 = \frac{\lambda}{2\pi\varepsilon_0}\int\frac{dr}{r} + +\frac{\sigma R}{\varepsilon_0}\int\frac{dr}{r}$ (0.25pts) donc $v_2 = \left(\frac{\lambda}{2\pi\varepsilon_0} + \frac{\sigma R}{\varepsilon_0}\right)\ln r + C_2$(0.5pts)

Exercice 3 : 2pts

La capacité équivalente :

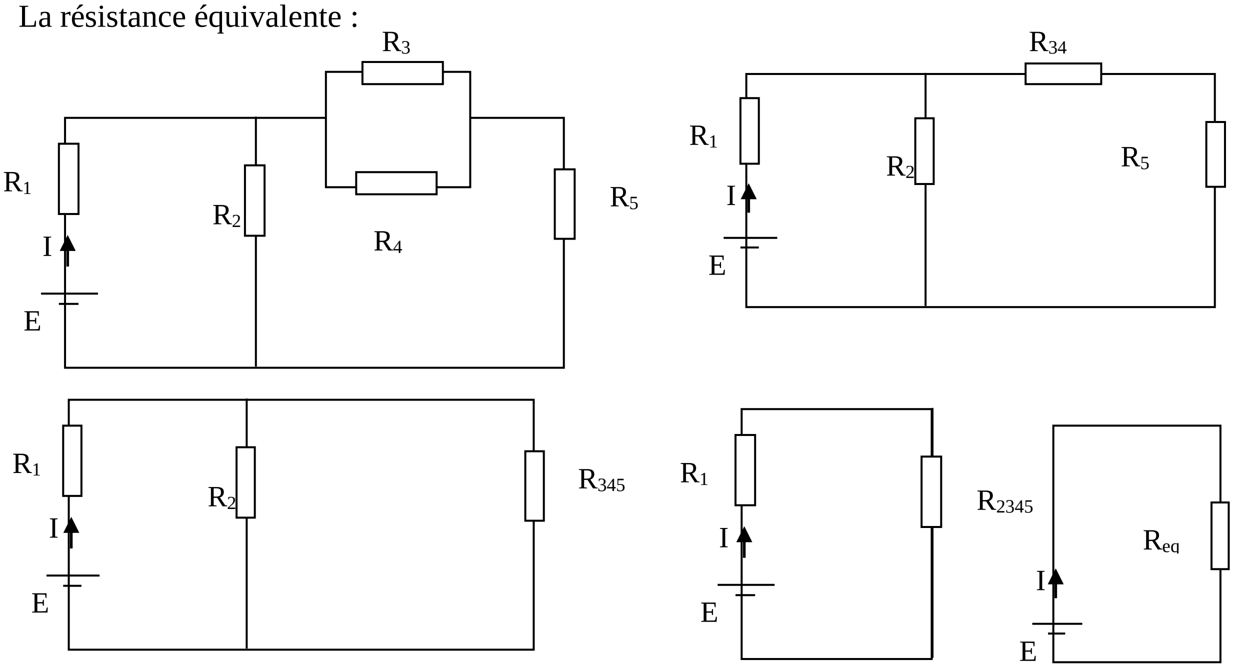


$C_{23} = C_2 + C_3$(0.5pts) $\Rightarrow C_{23} = 10 + 4 = 14\ \mu F$ (0.5pts)

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{5} + \frac{1}{14} + \frac{1}{8} = \frac{222}{560}$ (0.5pts) $\Rightarrow C_{eq} = \frac{280}{111}\mu F$ (0.5pts)

Exercice 3 : 2pts

La résistance équivalente :



$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{6} + \frac{1}{12} = \frac{3}{12} \Rightarrow R_{34} = 4\Omega$ (0.5pts)

R_{345}=16+4=20 Ω (0.5pts), $\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_{345}} = \frac{1}{20} + \frac{1}{20} = \frac{2}{20} \Rightarrow R_{2345} = 10\ \Omega$ (0.5pts)

$R_{eq}=R_1+R_{2345}$=2+10=12 Ω (0.5pts)