Chapter 2

Fundamental of statics – Part 2-

Forces

Forces (e.g. the weight of a body), whatever their nature and however they are applied (at a distance or in contact with two bodies) are in materials resistance as in traditional physics vector quantities.

So, each time we consider a force (figure1.1), we have to look for :

- the line of action (direction),

- the sense,

- point of application,

- intensity.

intensity.

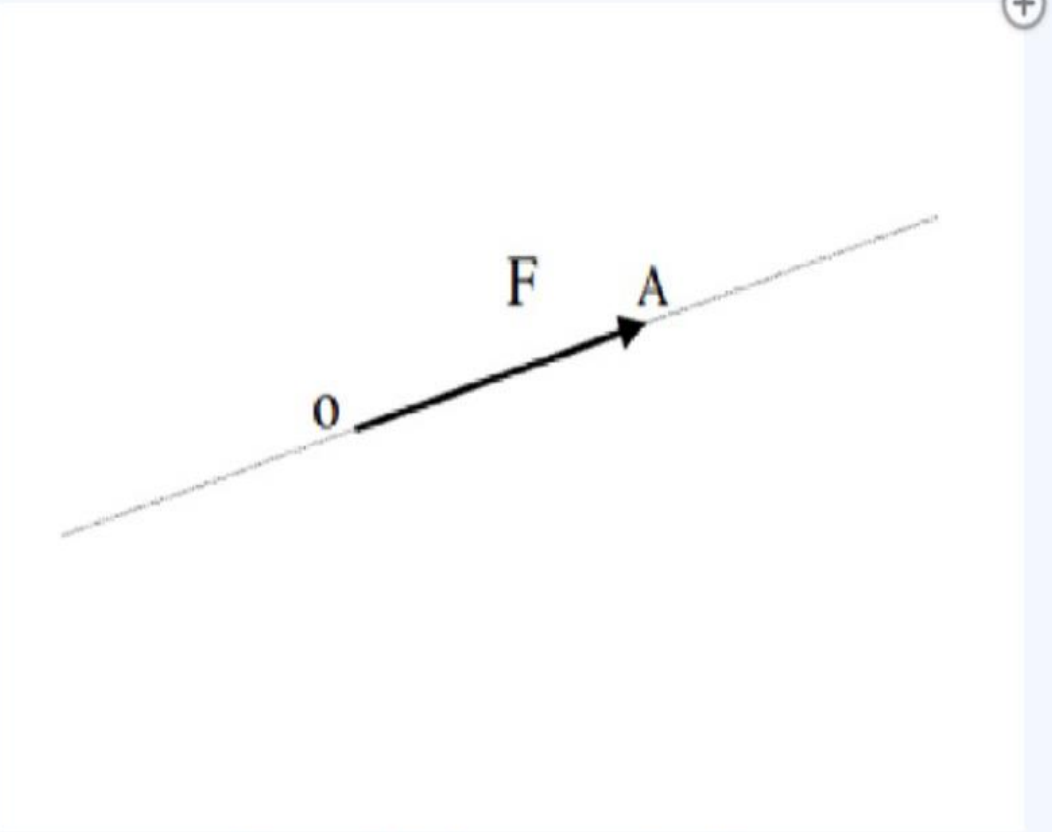


figure 1.1 Force projection

Notion of forces

The line of action

If a force is exerted, for example, by a tensile wire, the line of action of the force is that materialized by the wire. Similarly, if a force is transmitted by a rigid rod, this rod materializes the line of action of the force.

Notion of forces

Sense

The sense of a force is that of the movement it tends to produce; if force and movement are in the same direction, the force is said to be driving; otherwise, the force is said to be resisting. For example, friction forces are resistive forces.

Notion of forces

Point of application

If a solid is pulled by a wire or pushed by a rigid rod, the point of application is the point of attachment of the wire or the point of contact of the rod. In the case of the weight of a body, the point of application is the body's center of gravity.

Notion of forces

Intensity

Intensity measures the magnitude of the force. It is expressed in **Newton (N).**

Equilibrium of a solid subject to concurrent forces

We consider opposing forces (supported by the same axis), and concurrent forces (whose lines of action pass through the same point).

Equilibrium of a solid subject to concurrent forces

Two equal but opposing forces are balanced.

This is because the vectors representing them are opposite sliding vectors, whose summation is null. (figure 1.2)

The equilibrium of the supports or fixations leads us to consider the existence of linking (or reaction) forces opposed to the soliciting forces. For example, in the case of attachment point B in figure 1.2, subjected to the pull of the wire, system equilibrium is only possible if there is, at point B, a reaction R equal but opposite to the soliciting force F.

Equilibrium of a solid subject to concurrent forces

Two equal but opposing forces are balanced.

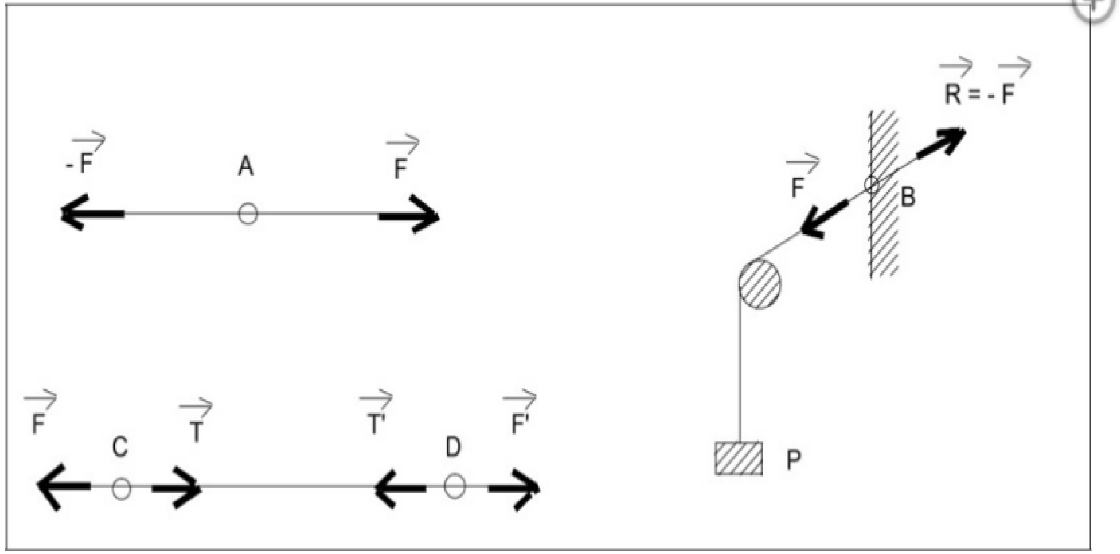
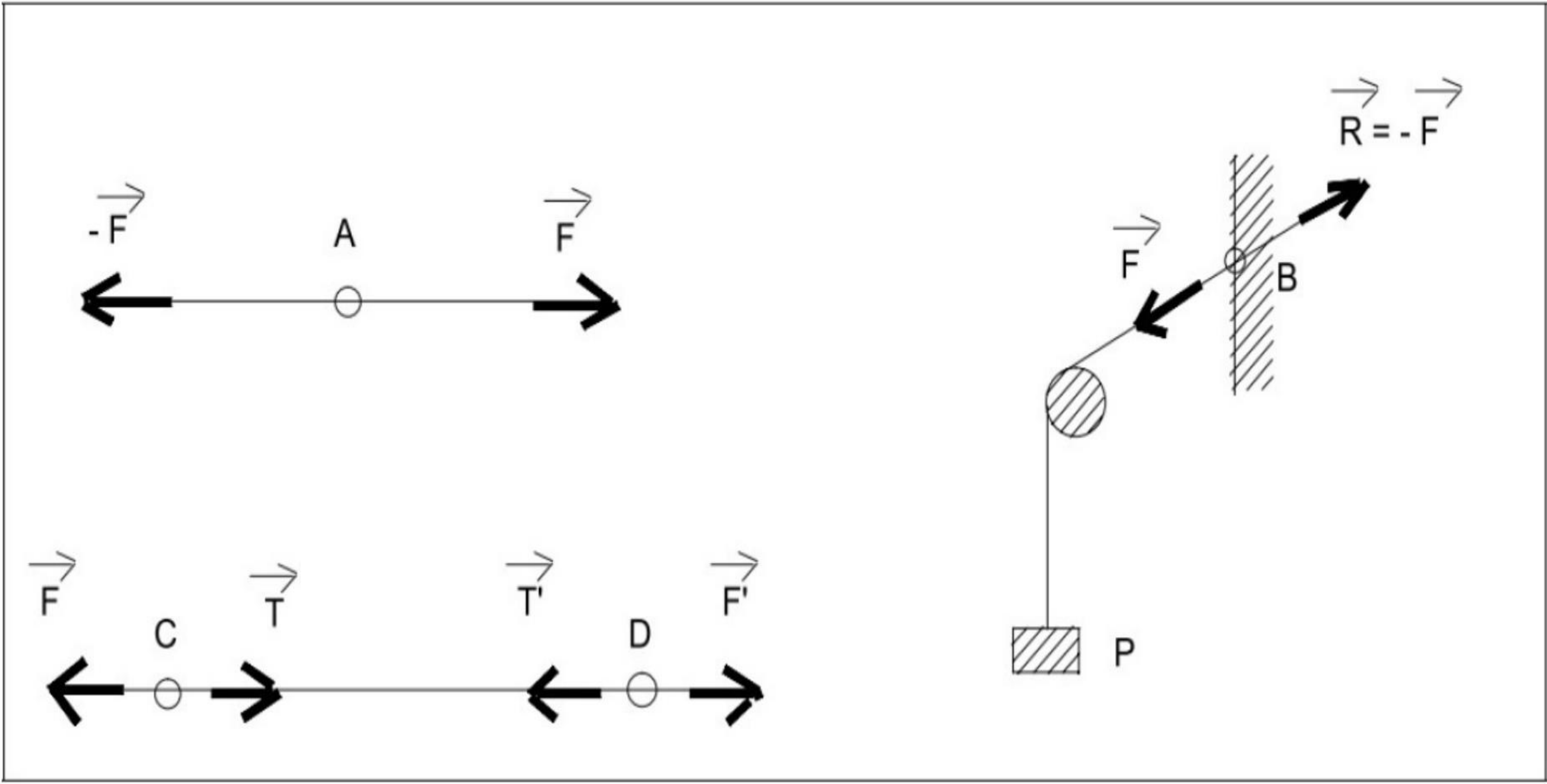


figure 1.2 Opposite forces



$\vec{R} = -\vec{F}$
$-\vec{F}$
A
$\vec{F}$
$\vec{F}$
B
P
$\vec{F}$
C
$\vec{T}$
$\vec{T'}$
D
$\vec{F'}$

Equilibrium of a solid subject to concurrent forces

Concurrent forces

These are forces whose lines of action pass through the same point.

The resultant R of concurrent forces is represented vectorially by the diagonal of the parallelogram built on the vectors representing these forces. The resultant vector's abscissa is equal to the sum of the abscissas of the component vectors. The same is applicable to the ordinates (figure 1.3).

Equilibrium of a solid subject to concurrent forces

Concurrent forces

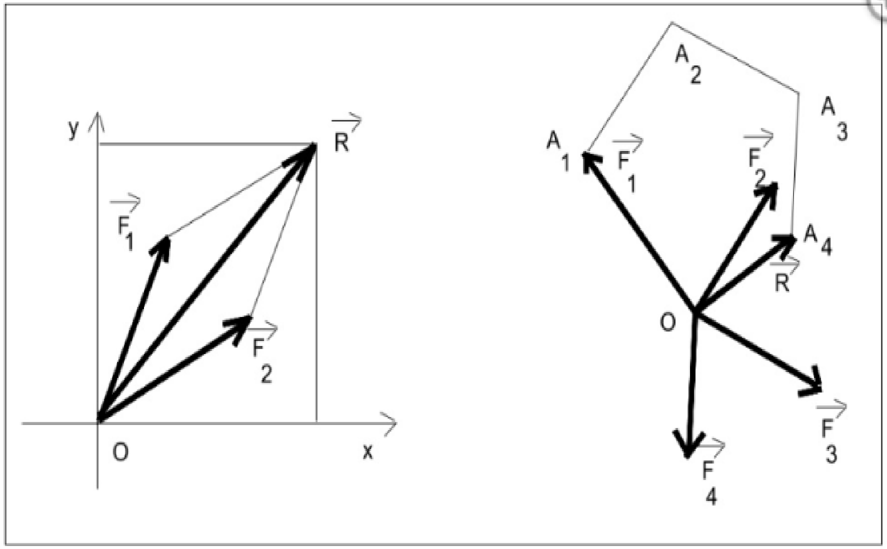
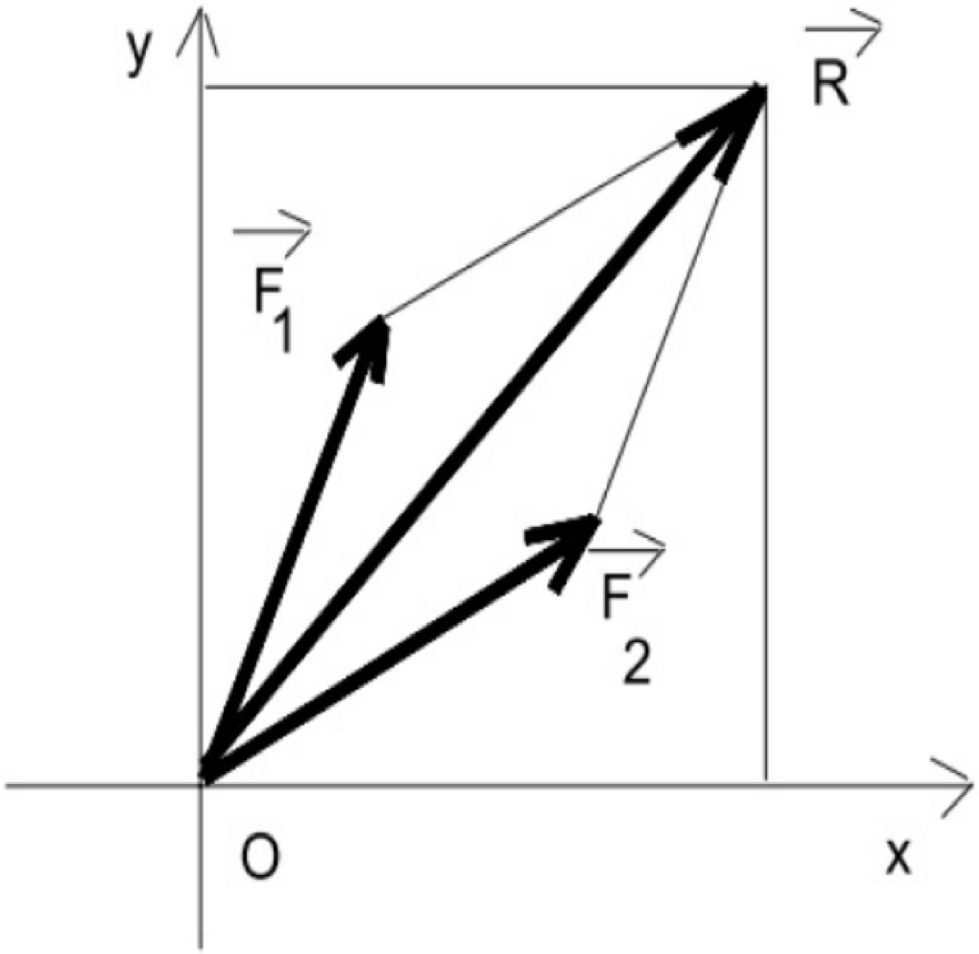
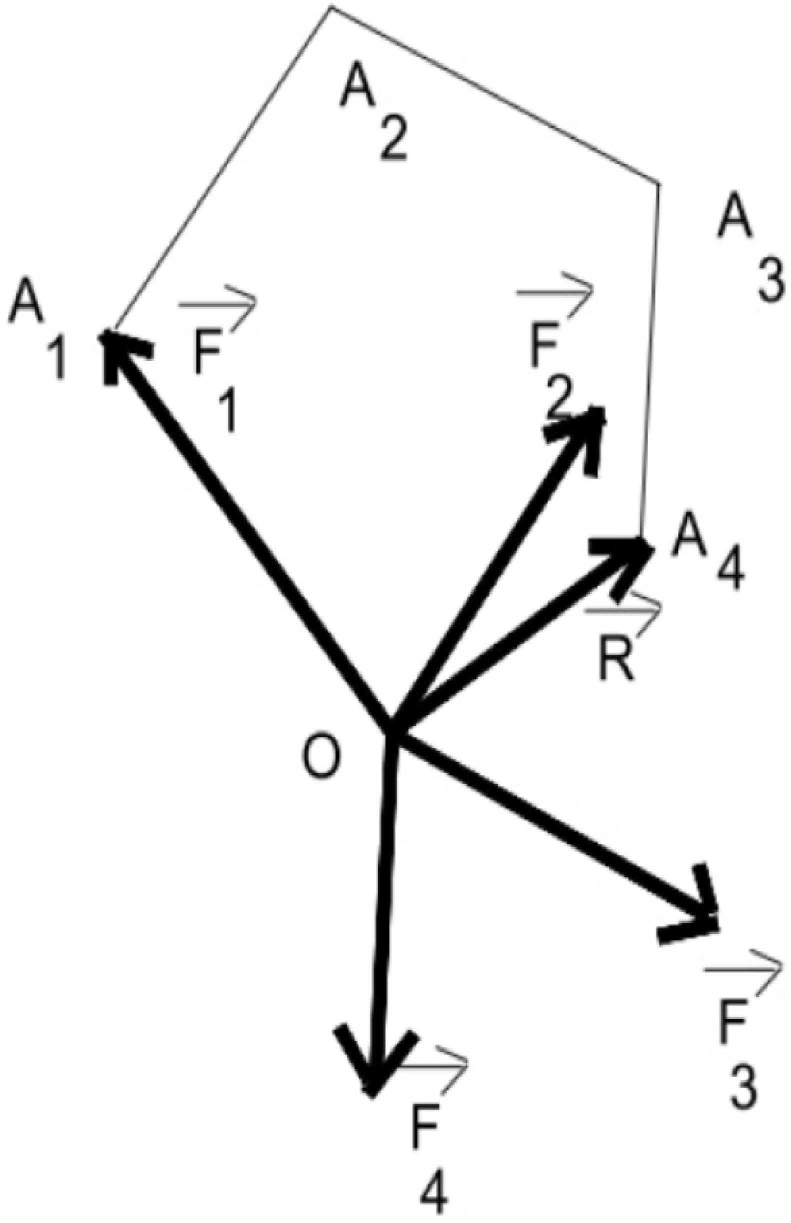


Figure 1.3


y
$\vec{R}$
$\vec{F}_1$
$\vec{F}_2$
O
x


A_2
A_3
A_1
$\vec{F}_1$
$\vec{F}_2$
A_4
$\vec{R}$
O
$\vec{F}_3$
$\vec{F}_4$

Equilibrium of a solid subject to parallel forces

Forces of the same direction

The resultant of two parallel forces F_A and F_B of the same direction is a force parallel to these two forces, of the same direction as them, and of intensity equal to the sum of their intensities (figure 1.4).

$$\vec{F}_A + \vec{F}_B = \vec{R}$$

Equilibrium of a solid subject to parallel forces

Forces of the same direction

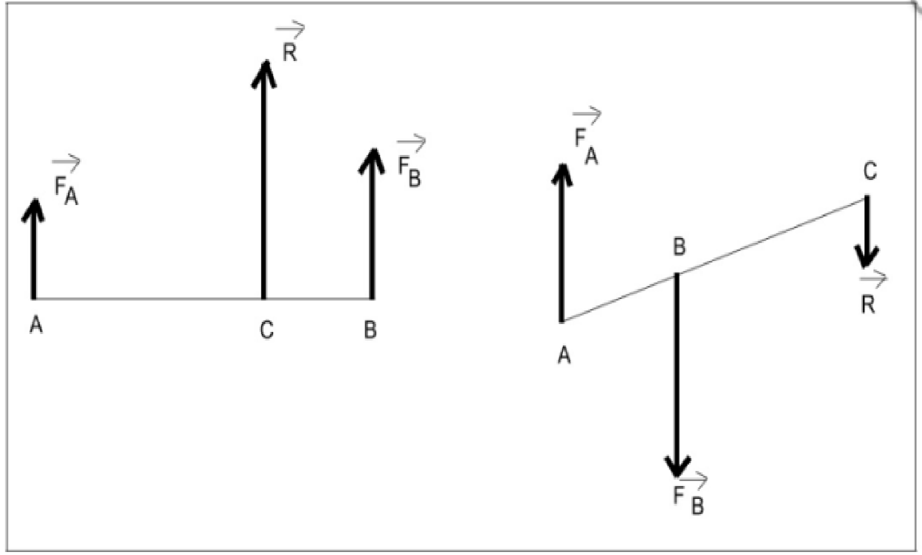
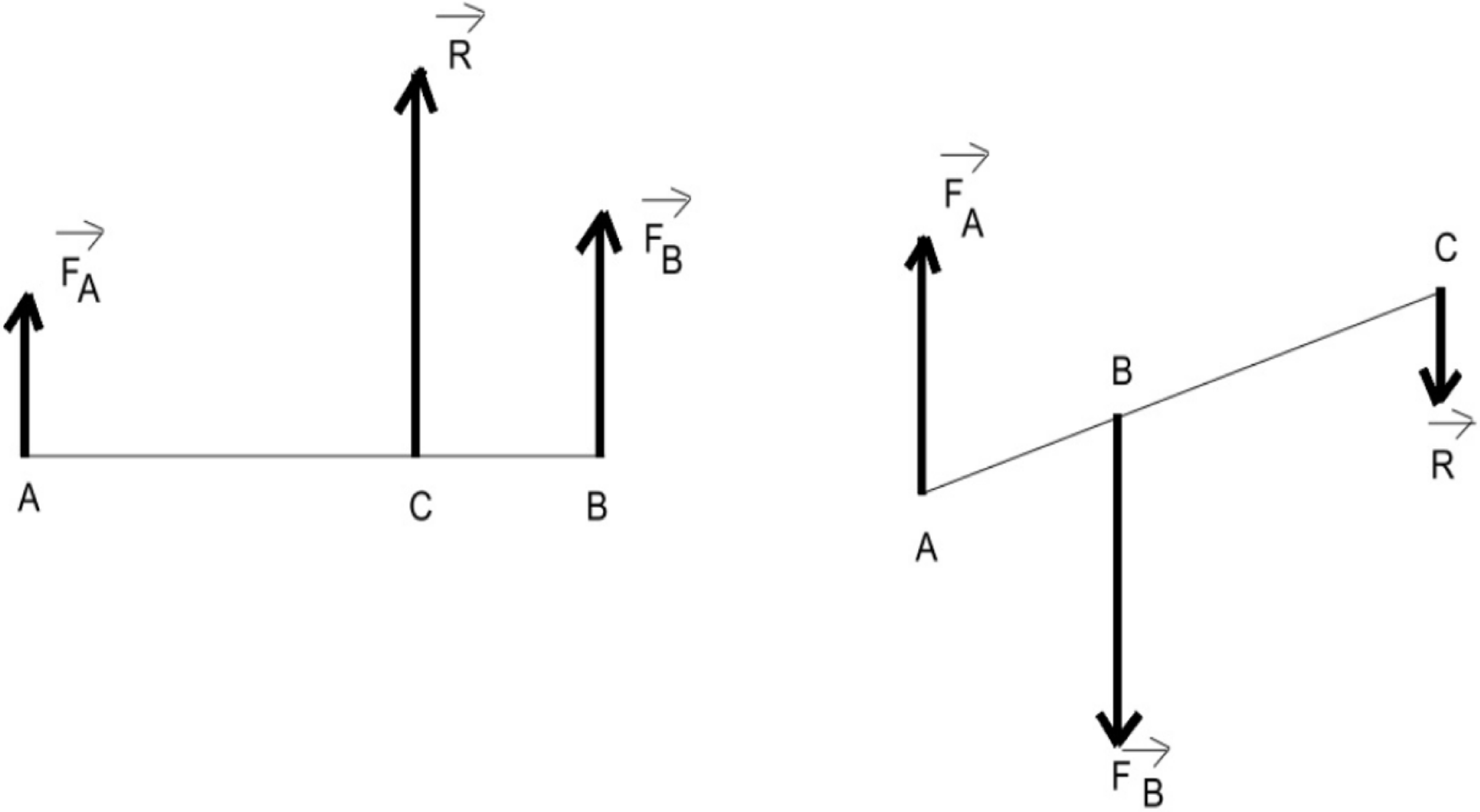


Figure 1.4

On the other hand, the point of application of the resultant $\vec{R}$ is a point C located on the segment AB, between A


R
F A
F B
A
C
B
F A
C
B
R
A
F B

Equilibrium of a solid subject to parallel forces

Forces of the same direction

Figure 1.4

On the other hand, the point of application of the resultant $\vec{R}$ is a point C located on the segment AB, between A and B, such as:

$$\vec{F}_A \times CA = \vec{F}_B \times CB$$

Equilibrium of a solid subject to parallel forces

Forces of the same direction

Parallel forces in opposite directions

Two forces $\vec{F}_A$ and $\vec{F}_B$ parallel and opposite in direction (figure 1.3) admit a resultant $\vec{R}$ parallel to these forces, in the direction of the greater force, and of intensity equal to the difference in their intensities:

$$\vec{R}=\vec{F}_B-\vec{F}_A$$

On the other hand, the point of application of the resultant $\vec{R}$ is a point C located on the line AB, outside the

Equilibrium of a solid subject to parallel forces

Forces of the same direction

Parallel forces in opposite directions

$$R = F_B - F_A$$

On the other hand, the point of application of the resultant $\vec{R}$ is a point C located on the line AB, outside the segment AB, on the side of the greatest component, and such as :

$$\left|\vec{F}_A \times CA\right| = \left|\vec{F}_B \times CB\right|$$

Equilibrium of a solid subject to parallel forces

Forces of the same direction

Parallel forces in opposite directions

Properties of a center of gravity

The center of gravity G of a solid, as the point of application of its weight, has the same properties as a center of parallel forces.

Types of forces in strength of materials

in theory, we consider forces located in a plane, this plane generally being the vertical plane of symmetry of the structure under study (for example, the plane of symmetry of a beam with a Te-shaped cross-section, as shown in figure 1.5.

Types of forces in strength of materials

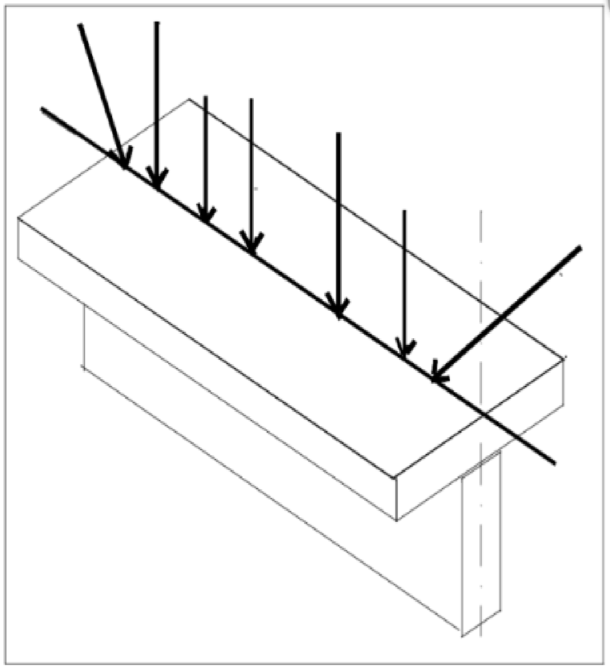
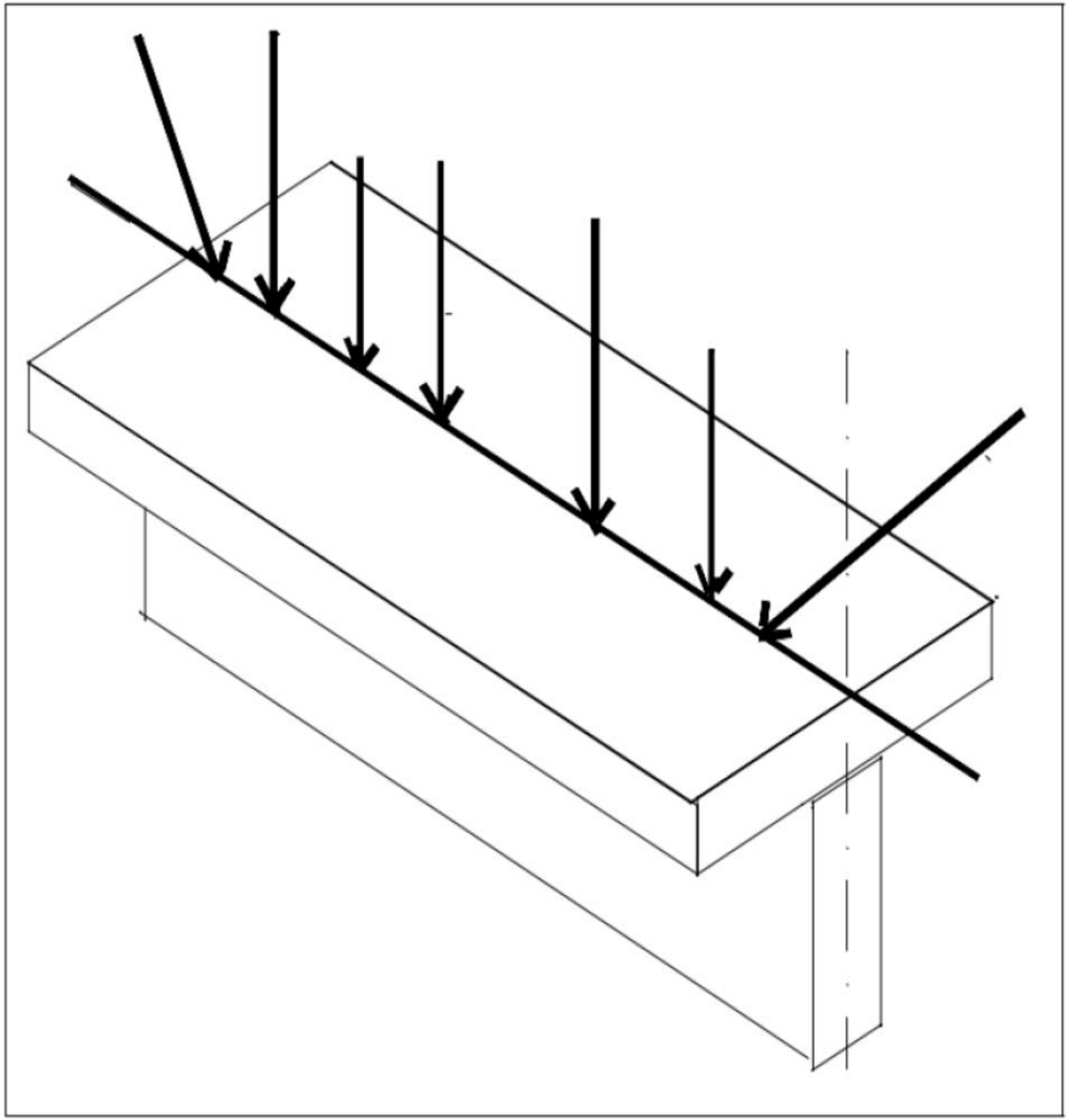


Figure 1.5

The forces applied to structures can be :



Types of forces in strength of materials

Figure 1.5

The forces applied to structures can be :

- **Concentrated forces** (for example, the reaction of a joint, or the action of a vehicle wheel). In reality, these forces are applied to small surfaces, but for calculation purposes they are usually assimilated to point forces. (figure 1.6)

Types of forces in strength of materials

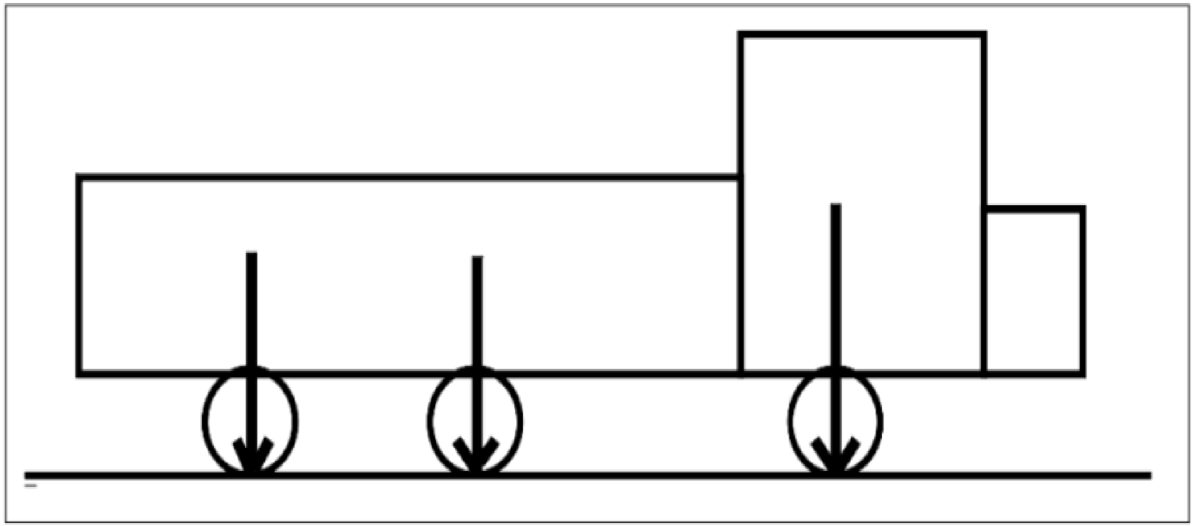


Figure 1.6

- Distributed forces for example beam's self-weight or additional load corresponding to a layer of snow

Types of forces in strength of materials

Figure 1.6

- **Distributed forces** for exemple beam's self-weight or additional load corresponding to a layer of snow. (figure 1.7)

Types of forces in strength of materials

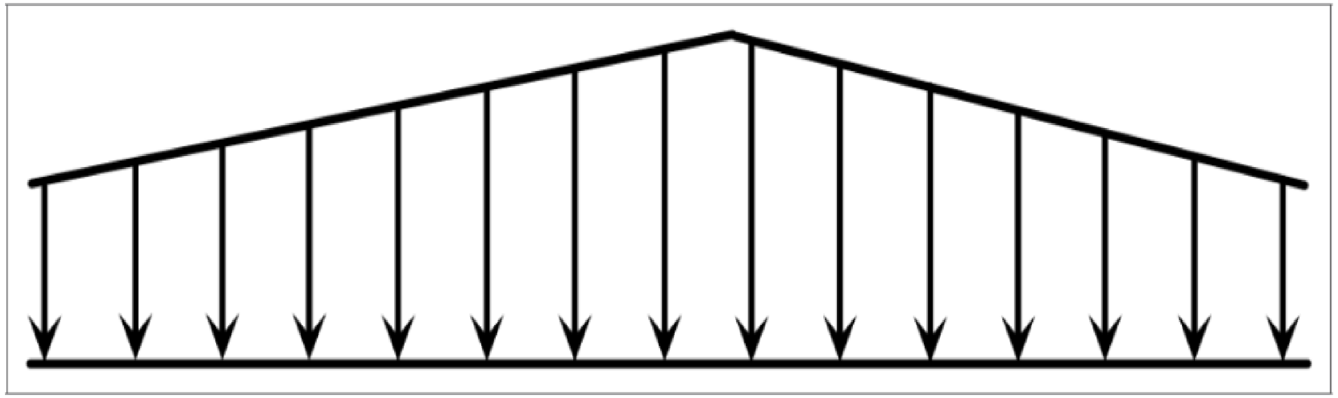


Figure 1.7

Theorem of moments

A solid moving around an axis is in equilibrium when the sum of the moments, relative to this axis, of the forces that tend to make it rotate in one direction is equal to the sum of the moments of the forces that tend to make it rotate in the opposite direction. An application of this theorem can be found in the equilibrium of balances, but also in the equilibrium of certain beams.

Let's do the following experiment

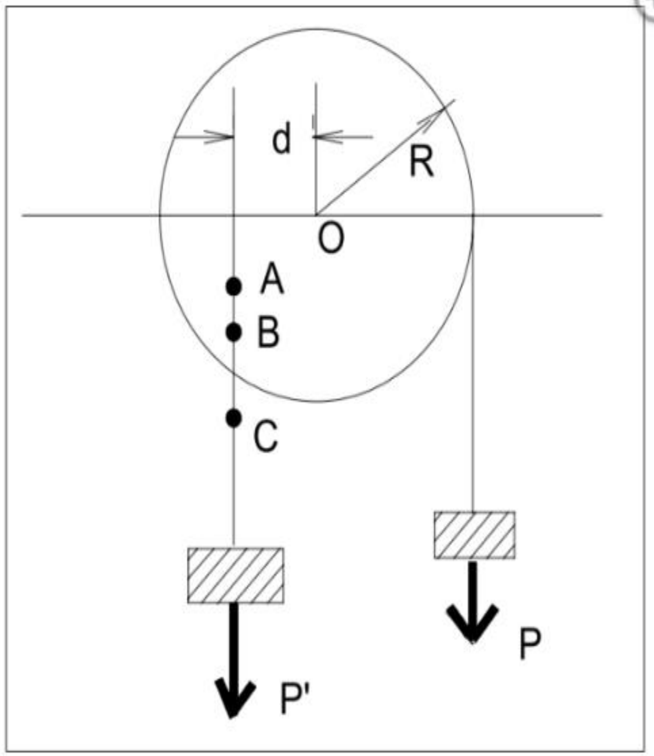
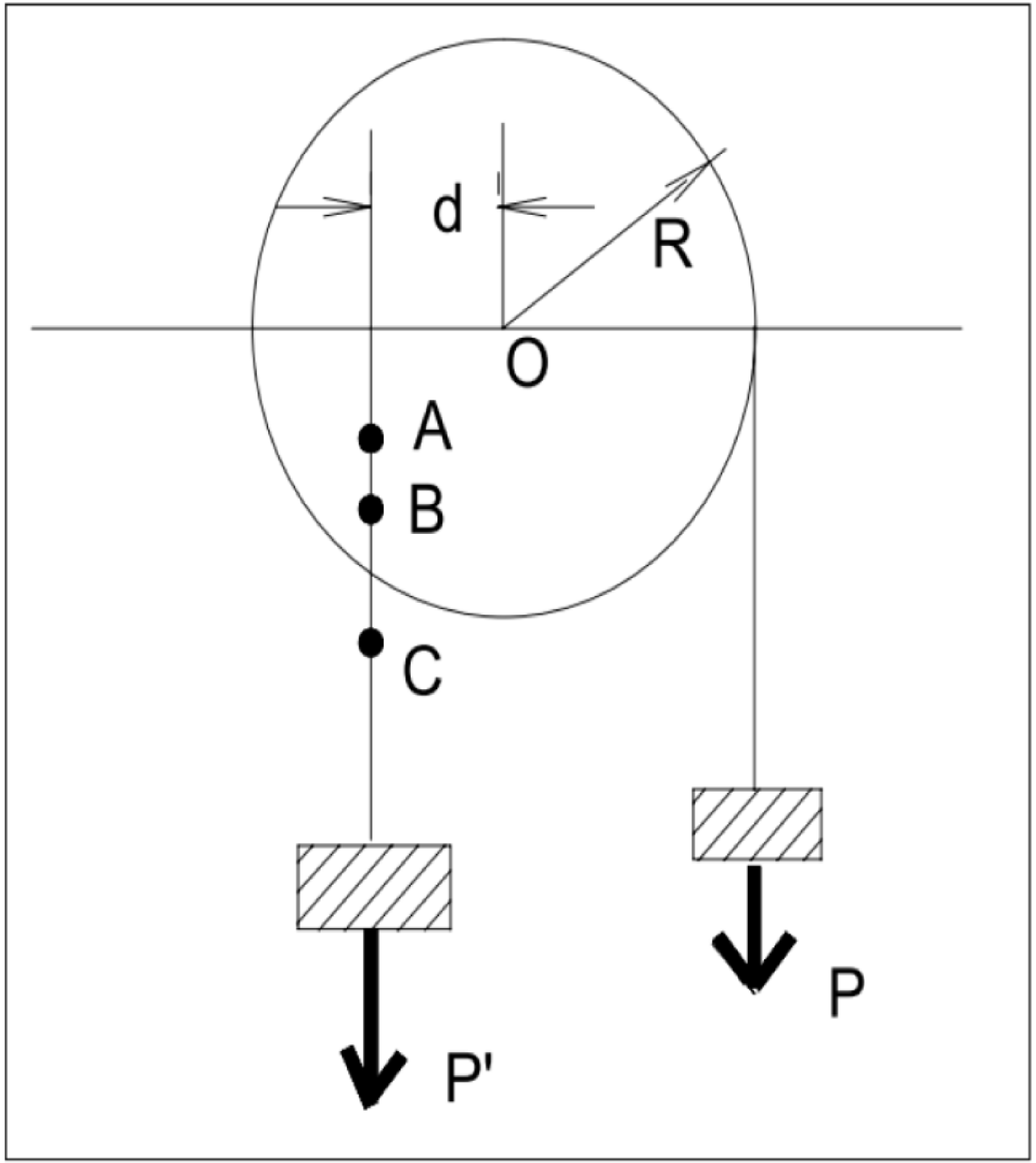


Figure 1.8

We see in figure 1.8 that the product (P x d) is equal to the product (P' x d') . The products


d
R
O
A
B
C
P'
P

do the following experiment

Figure 1.8

ure 1.8 that the product (P x d) is equal to the product (P' x d') . The products

P' x d') represent the moments of the weights relatively to the axis of rotation.

Let's do the following experiment

Figure 1.8

We see in figure 1.8 that the product (P x d) is equal to the product (P' x d') . The products (P x d) and (P' x d') represent the moments of the weights relatively to the axis of rotation.

Actions and reactions

We consider any point mass in equilibrium:

- If it is not subjected to any action (or force) ;
- Or if the sum of the actions (or forces) applied to it are equal to zero.

for exemple a small ball placed on a horizontal surface stays in equilibrium because the surface in contact with the ball has a reaction R equal and opposite to the weight of the ball (figure 1.9).

Actions and reactions

ball has a reaction R equal and opposite to the weight of the ball (figure 1.9).

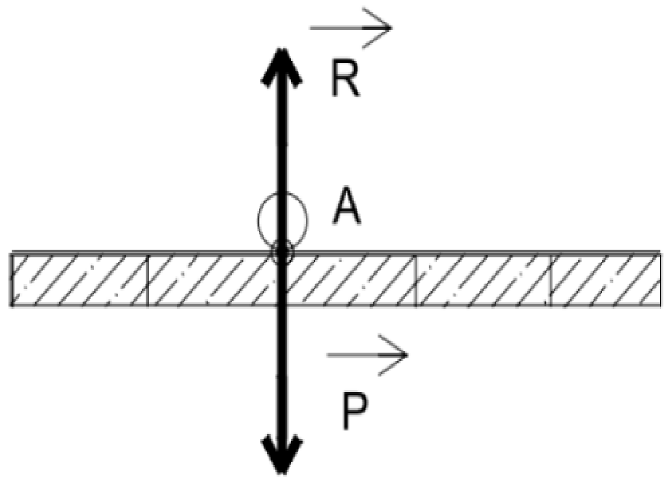


Figure 1.9 Actions and reactions

Equilibrium of a solid

Necessary and sufficient conditions for equilibrium of an undeformable solid are expressed by the following two conditions:

- The general resultant of forces (actions and reactions) applied to this solid is null.
- The resultant moment of all forces (actions and reactions), taken relative to any point, is null.

Equilibrium of a solid

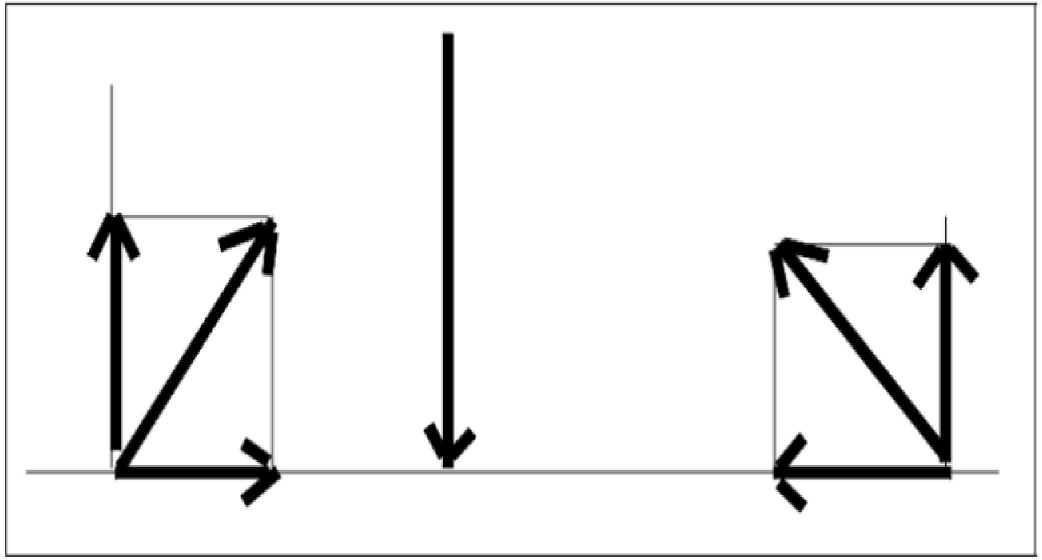


Figure 1.10 Equilibrium of a solid

When the number of unknowns is equal to the number of equations, the system is isostatic. If the number of

Equilibrium of a solid

In the particular situation of forces located in the same vertical plane, these two conditions are expressed by three equations:

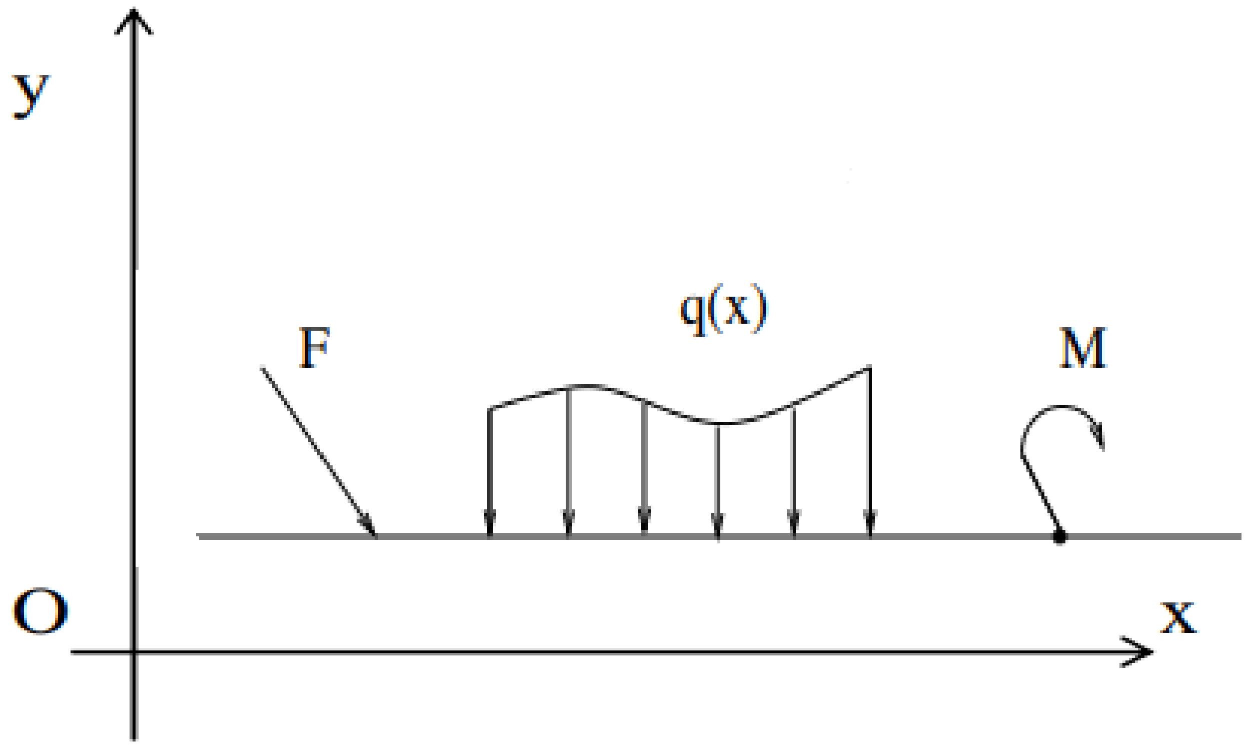
- The sum of the projections of the forces on a horizontal axis Ox of the plane is null.
- The sum of the projections of the forces on a vertical axis Oy of the plane is null.
- The sum of moments taken with respect to any point on the plane is null.

Forces

F: Concentrated force (N)

q(x) : distributed load (N/m)

M : Moment (N.m)



Fundamental principle of statics

Generally, the supporting actions are unknown. To determine them, we need to write down the three equilibrium equations of the system

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

Equilibrium of a solid

Figure 1.10 Equilibrium of a solid

When the number of unknowns is equal to the number of equations, the system is isostatic. If the number of unknowns is greater than the number of equations, it is not possible to solve the problem using the equations of statics alone: the system is hyperstatic.

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Structures isostatiques et hyperstatiques

A structure is in equilibrium when the conditions set out above are met (fundamental equations of static equilibrium).

For a plane structure, there are three equations (3). Let **R** be the number of unknowns for the support reactions of a plane structure loaded in its plane:

- If **$R = 3$**, the static equations can be used to determine the support reactions, and the structure is said to be **externally isostatic**.

- If **$R > 3$**, the number of equilibrium equations is insufficient to determine the support reactions. The structure is said to be **externally hyperstatic** of order R-3, so R-3 additional equations are needed to determine all the reactions.

- If **$R < 3$**, the structure cannot be assured its equilibrium. The structure is said to be **unstable**

Standard supports

Every movement that is blocked has a corresponding support action:

- Any translational movement that is prevented in a given direction will require a supporting action in that direction.

- Any movement around an axis that is blocked corresponds to a Moment around that axis.

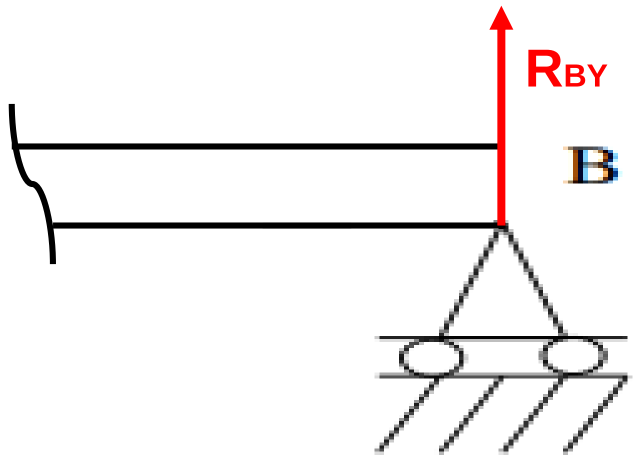
Usual types of connection in civil engineering:

- **Single support**
- **Double support**
- **Embedded**

Standard supports

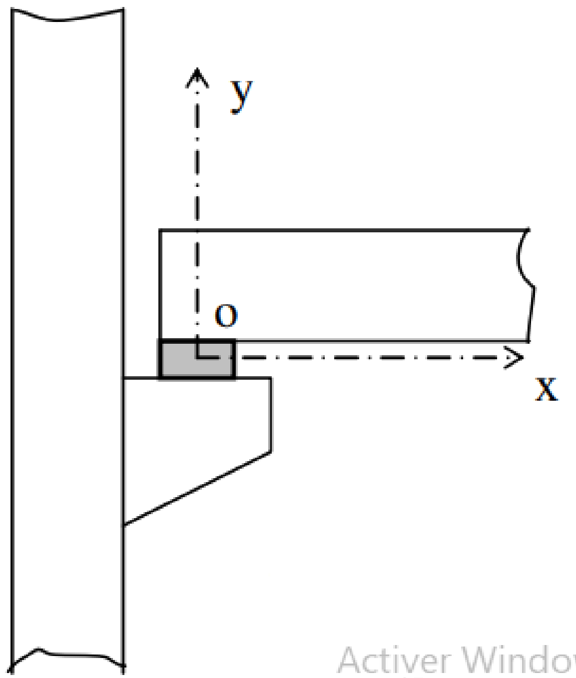
Single support

The single support is a connection that blocks relative movement in the vertical direction Y



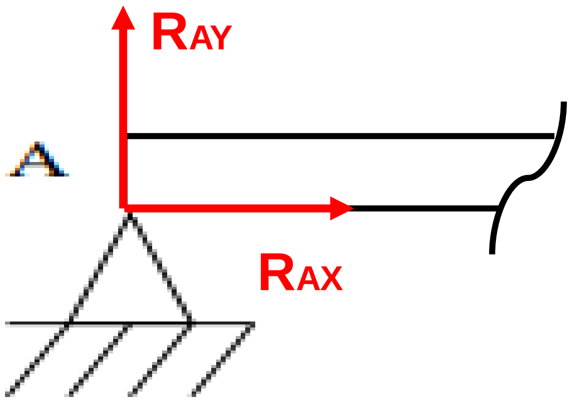
Example :

Beam resting on a bracket attached to a column by means of a neoprene support. The neoprene ensures that the beam can move horizontally and rotate about the centre O of the connection.)



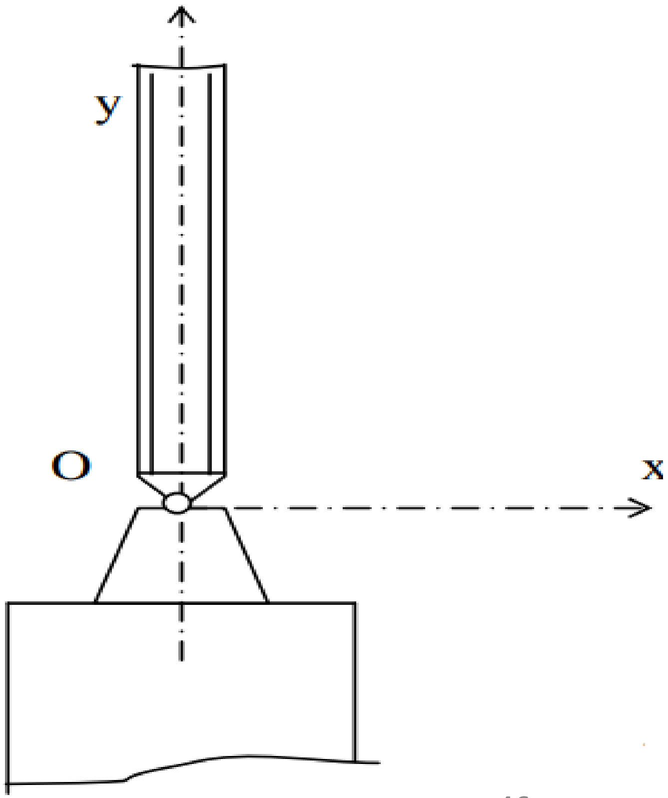
Double support (Articulation)

The double support prevents the beam from moving in both x and y directions, but does not prevent it from turning (rotation).



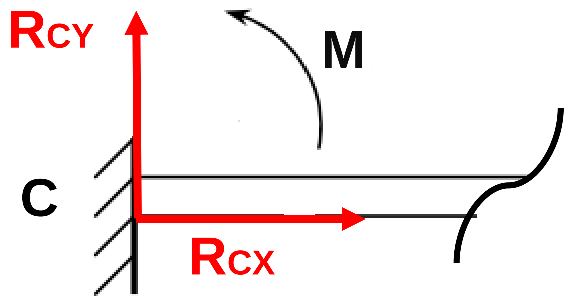
Example :

metal column articulated at the base on a concrete block

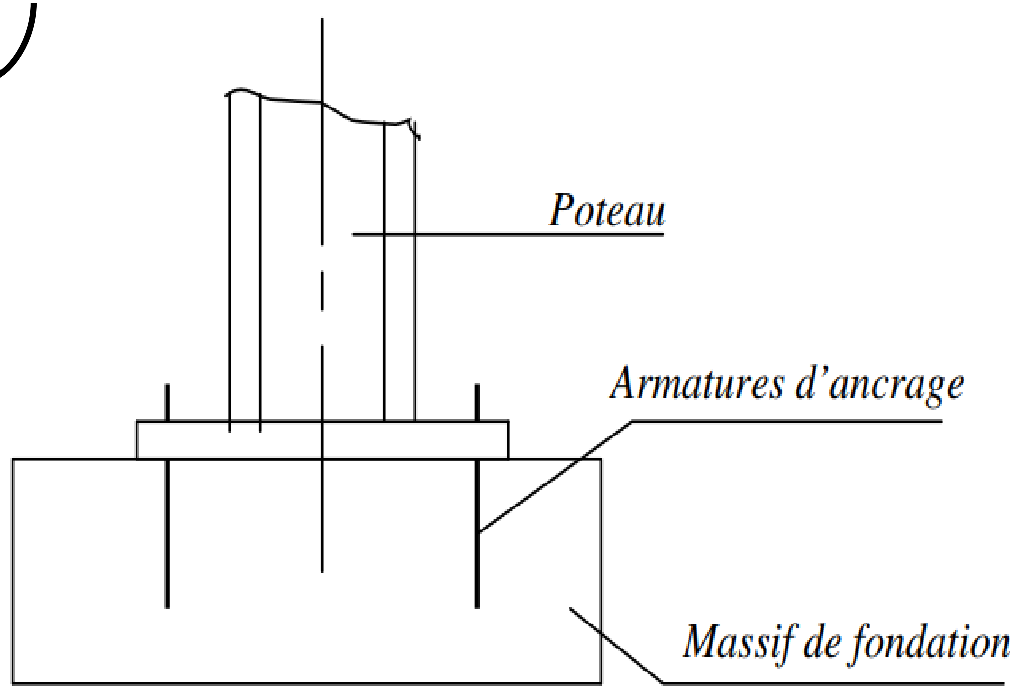


Embedding

An embedment is a connection that eliminates all movement between the connected solids.

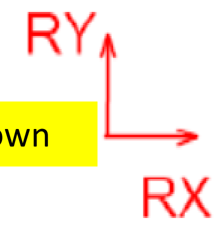
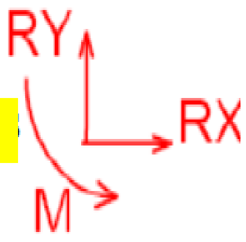


Example :



Standard supports

Examples

type of link	modelling	unknown link
➢ Double support		2 unknown 
➢ Embedding		3 unknown 
➢ Single support		1 unknown 

Standard supports