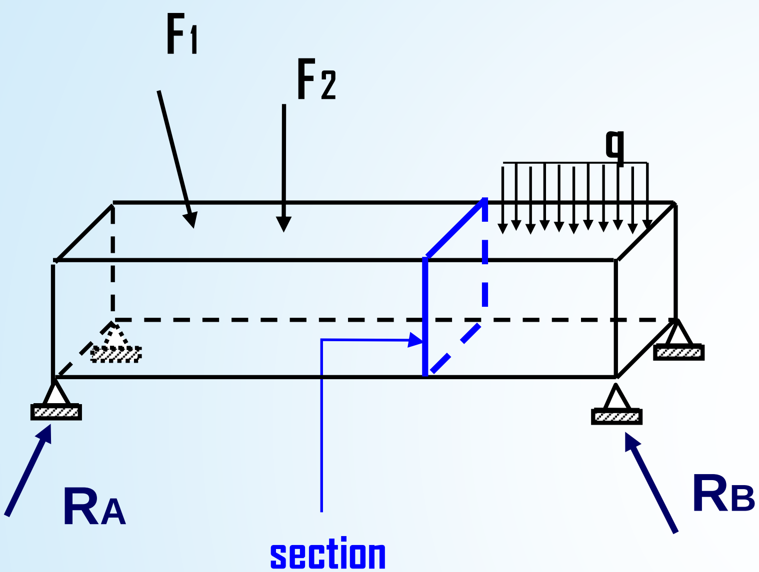
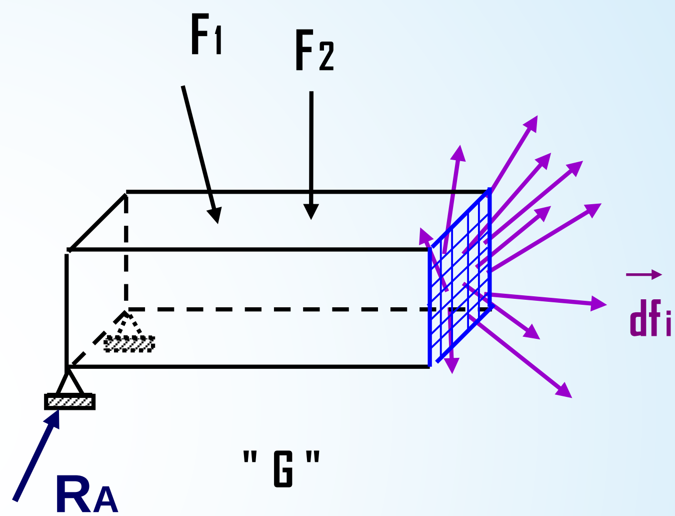
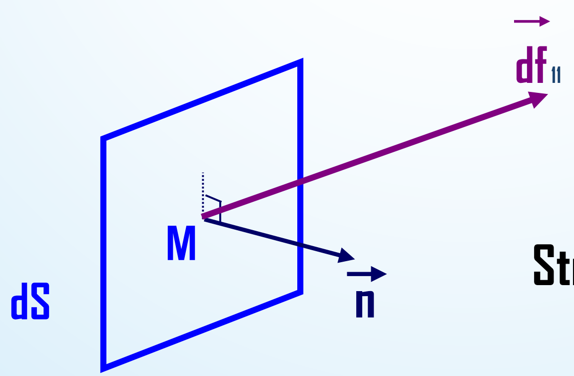
Chapter 2

Stresses and strains

I – CONCEPTS OF STRESSES

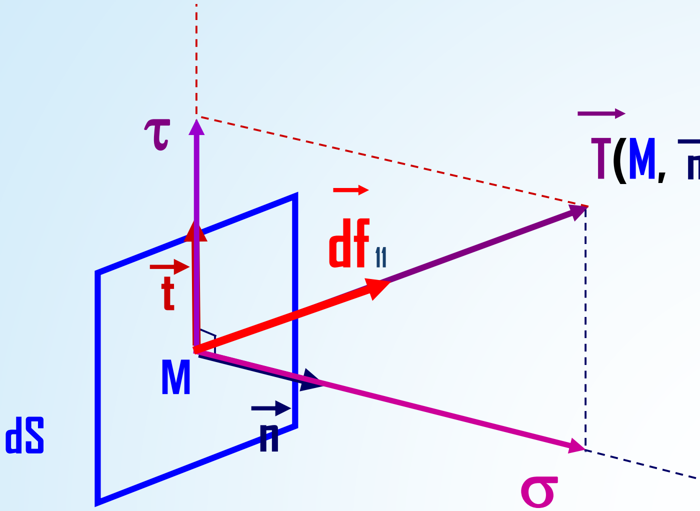






Stress vector

2- Normal stress and tangential stress



$$\vec{T}(M, \vec{n}) = \frac{\vec{df}}{dS}$$

$$\vec{T}(M, \vec{n}) = \sigma.\vec{n} + \tau.\vec{t}$$

σ : Normal stress

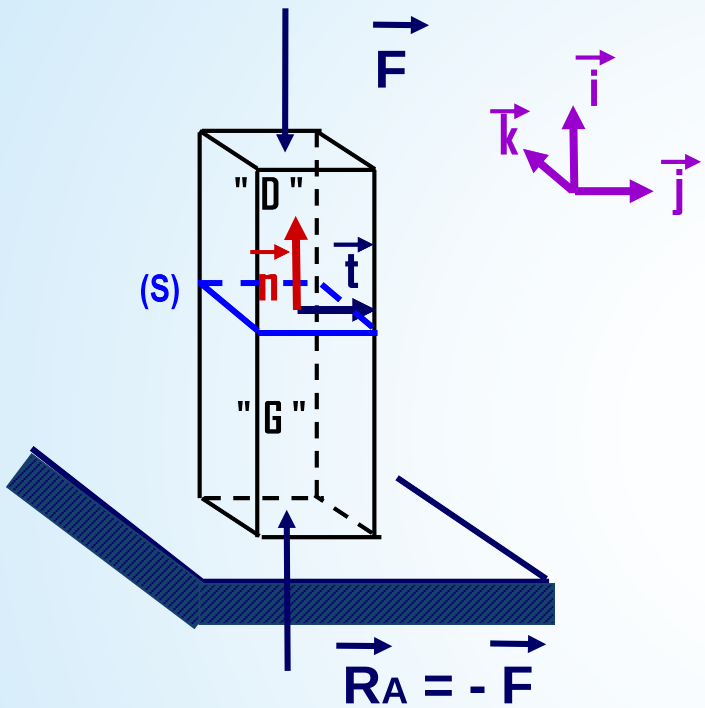
τ : tangential stress

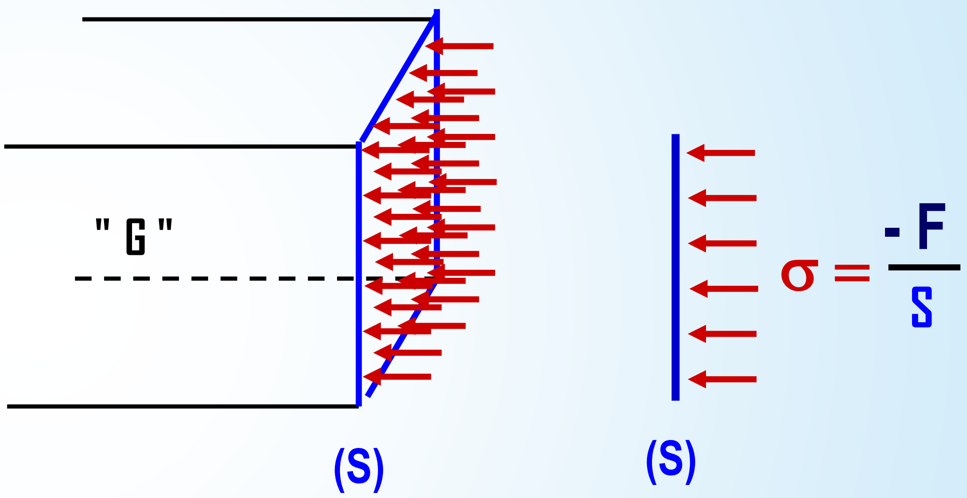
$$\|\vec{T}\| = \sqrt{\sigma^2 + \tau^2}$$

- The unit of stress is the ratio of a force to a unit area (N/mm2 = MPa).

3- The different types of constraints (Stresses)

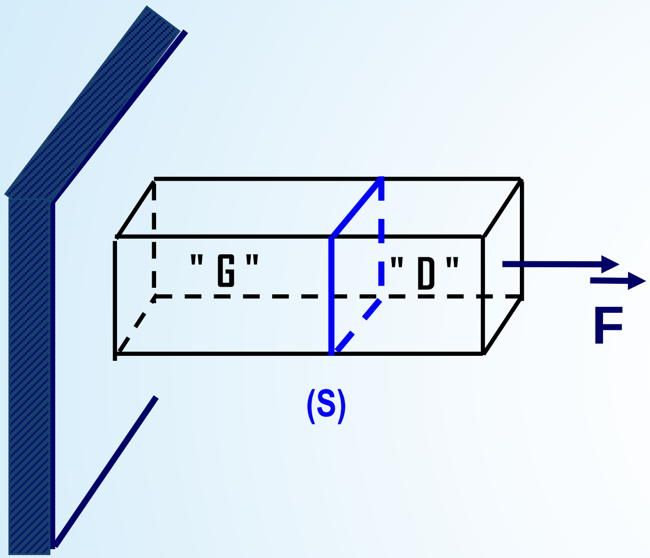
a) Compressive stress

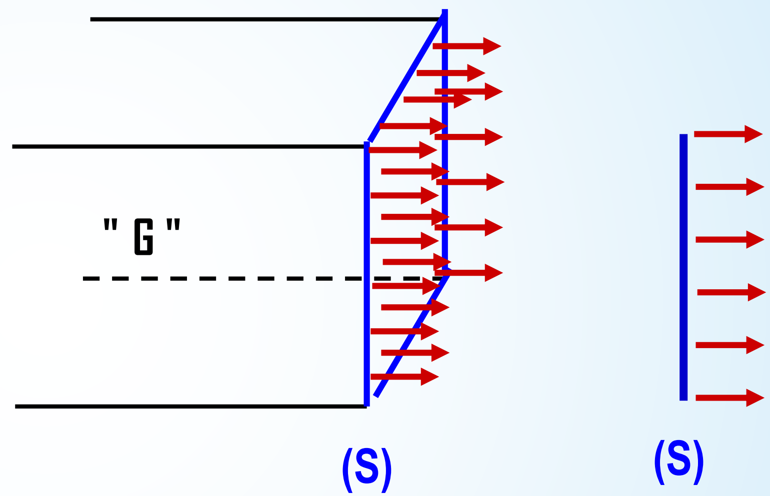




Field of constraints

b) Tensile stress

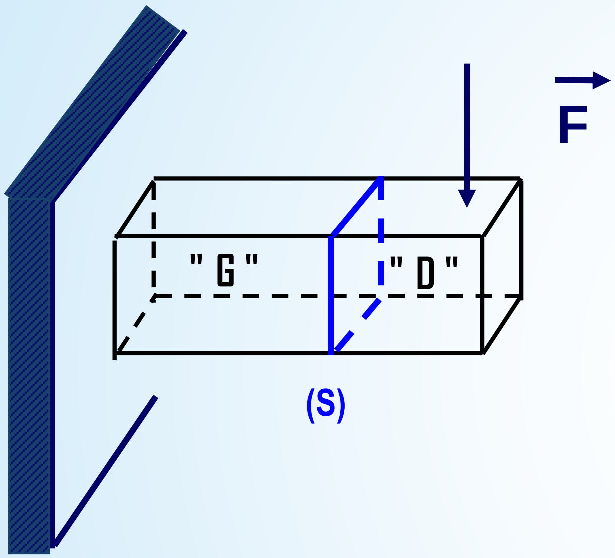


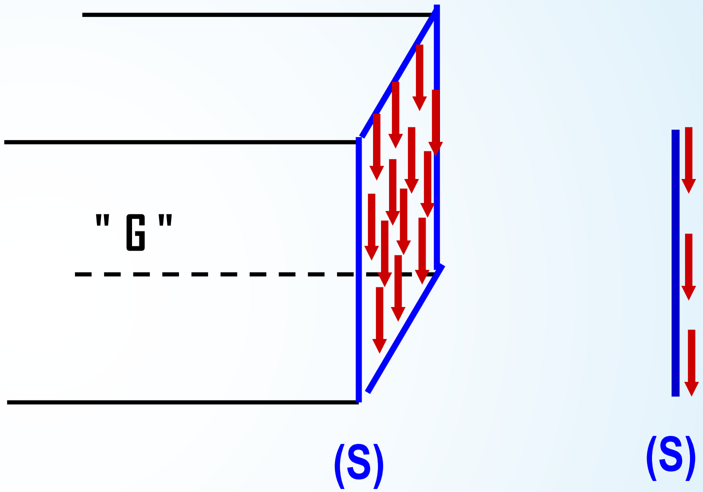


$$\sigma = \frac{F}{S}$$

Field of constraints

c) Shear stress :

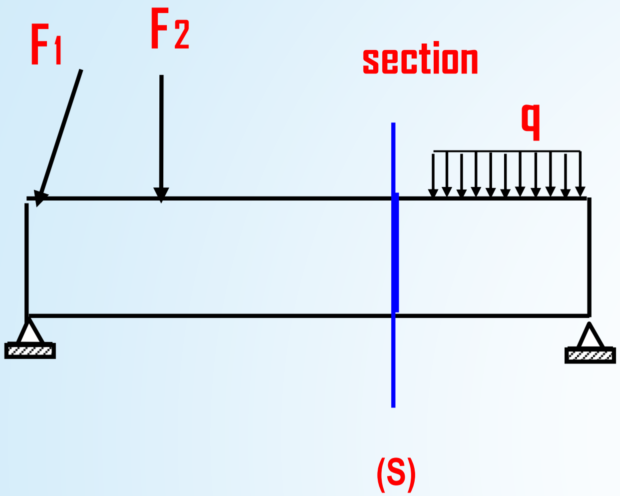


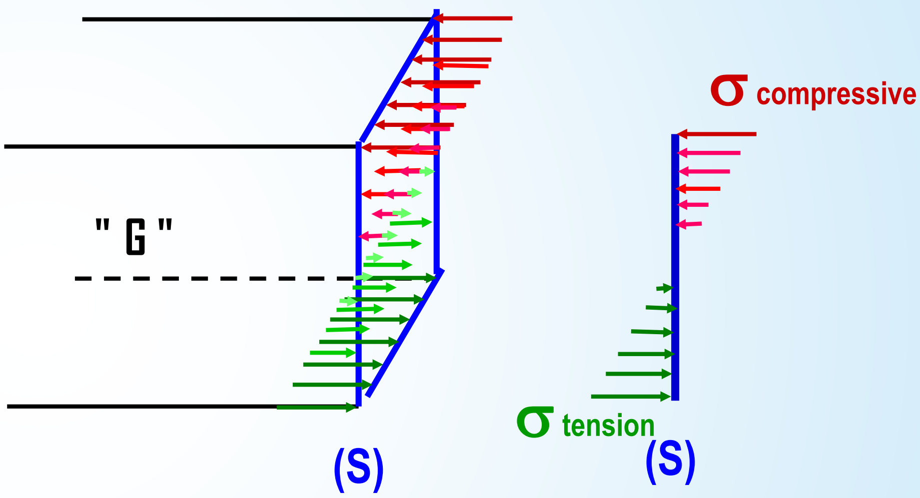


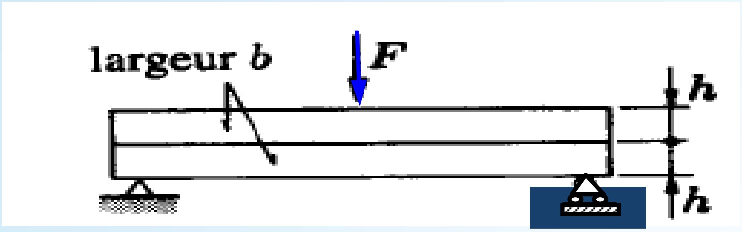
$$\tau = \frac{F}{S}$$

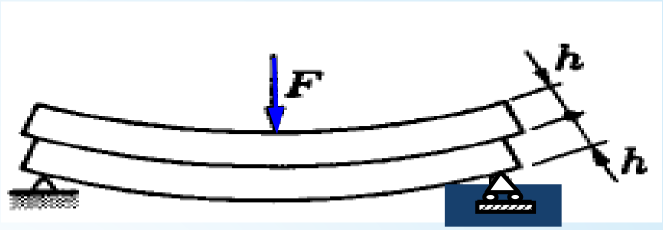
Field of constraints

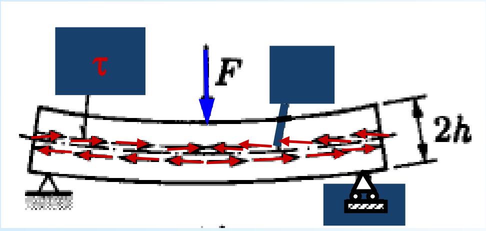
d) Bending stresses :











: Appearance of tangential stresses at the level of fibre contact

<u>Demonstration of shear stresses</u>

Bending stresses :
- Normal stresses
  - of Compression
  - of tensile
- Tangential stress

What is the purpose of stress calculations?

We must check that the stresses generated by external loads do not exceed the permissible stress limit for the material.

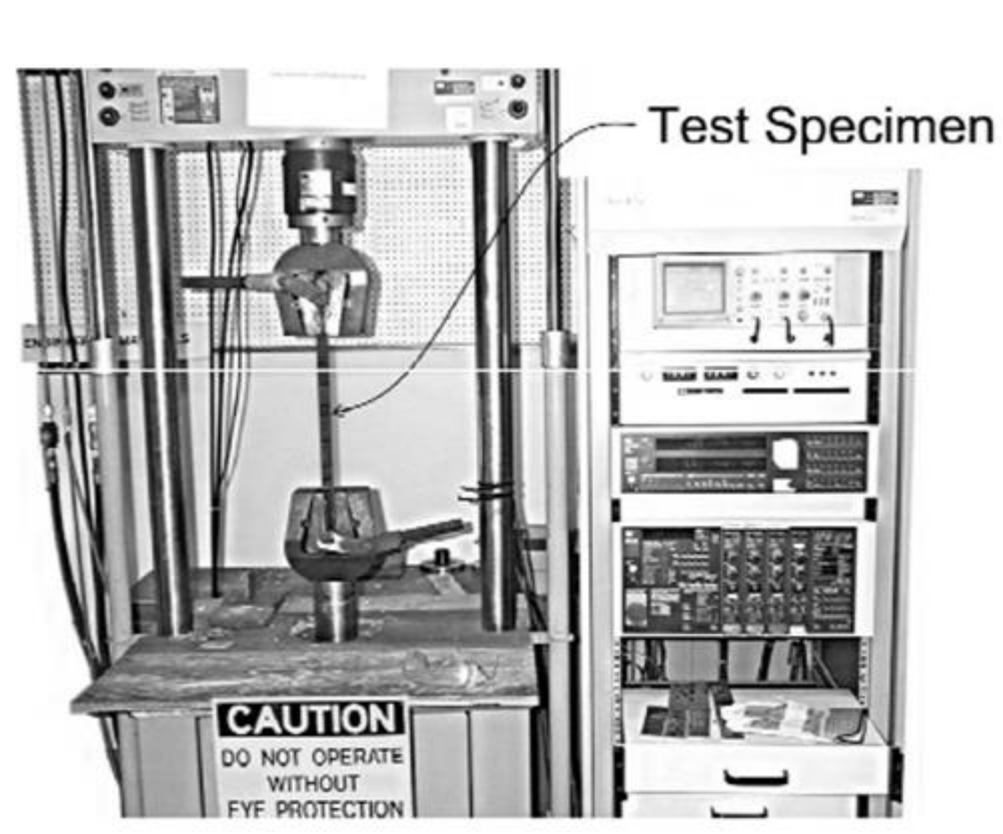
$$\sigma = \frac{F}{S} \leq \sigma_{ad}$$

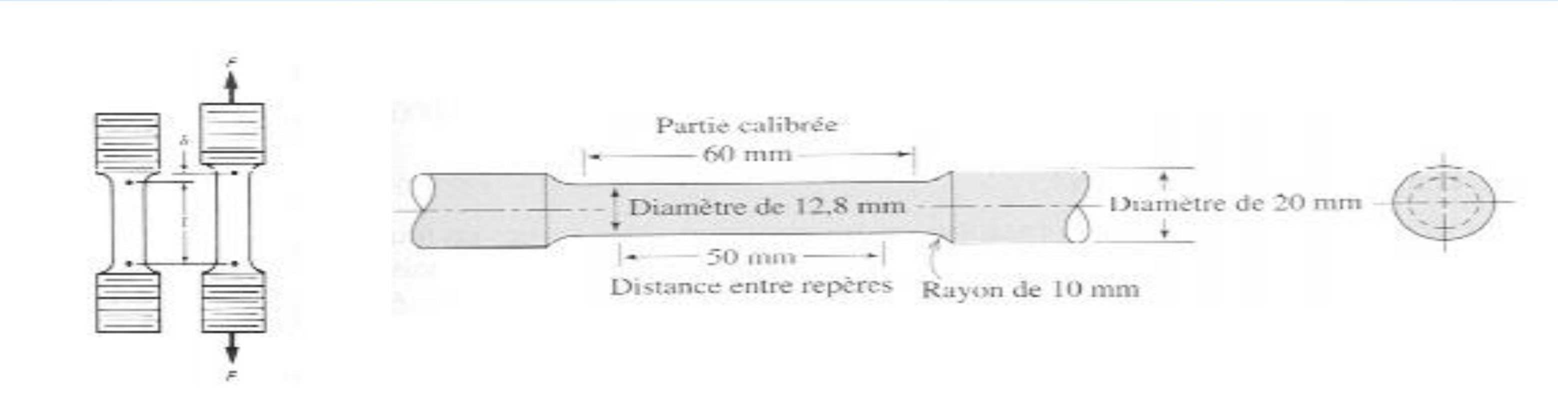
σ_{ad} : stress above which the piece is subject to deterioration of its mechanical and dimensional characteristics, or even failure.

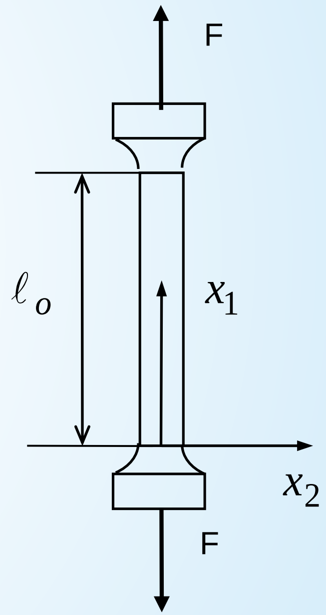
σ_{ad} is experimentally determined

MECHANICAL PROPERTIES OF MATERIALS

1- Tensile test





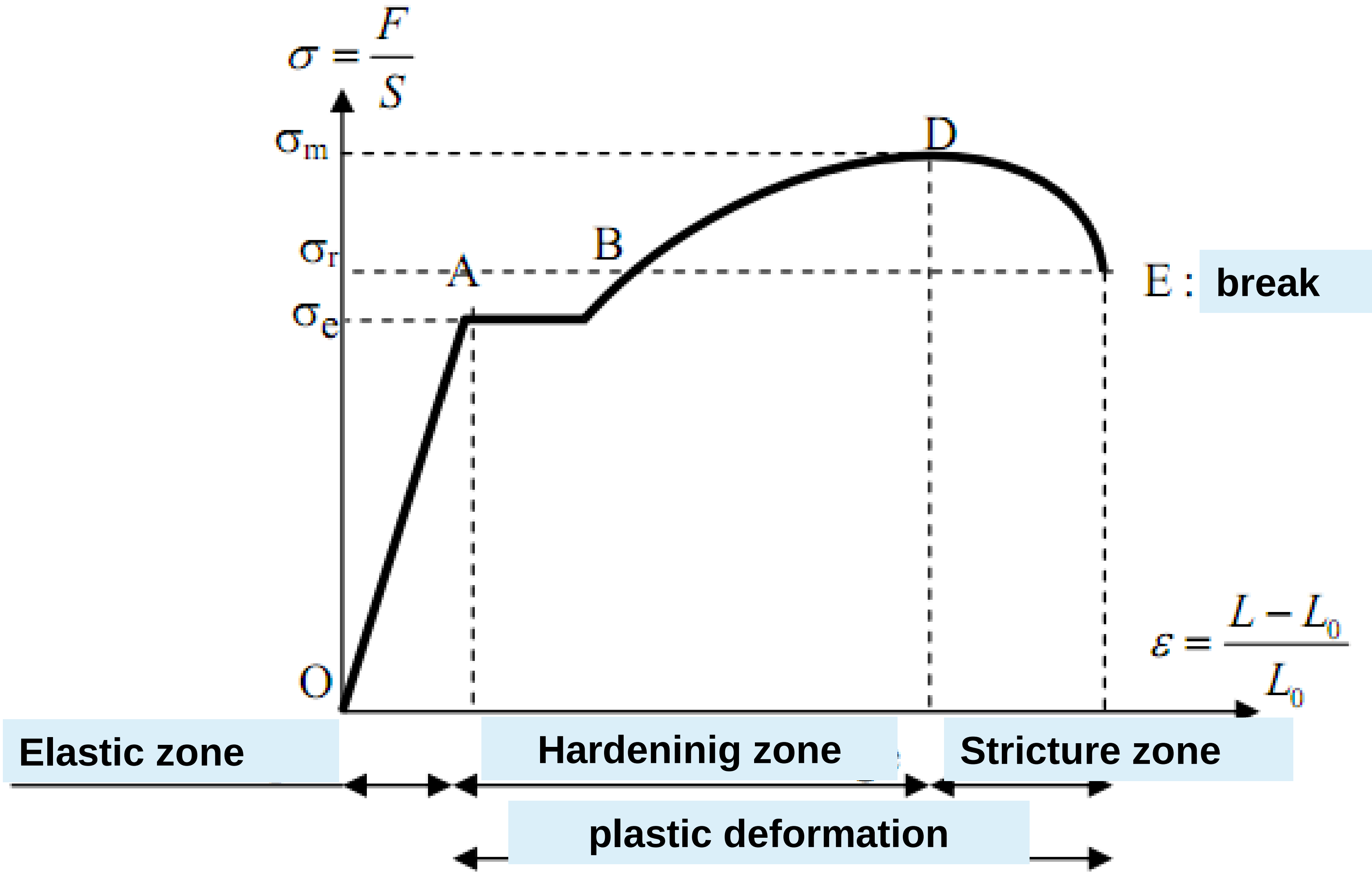


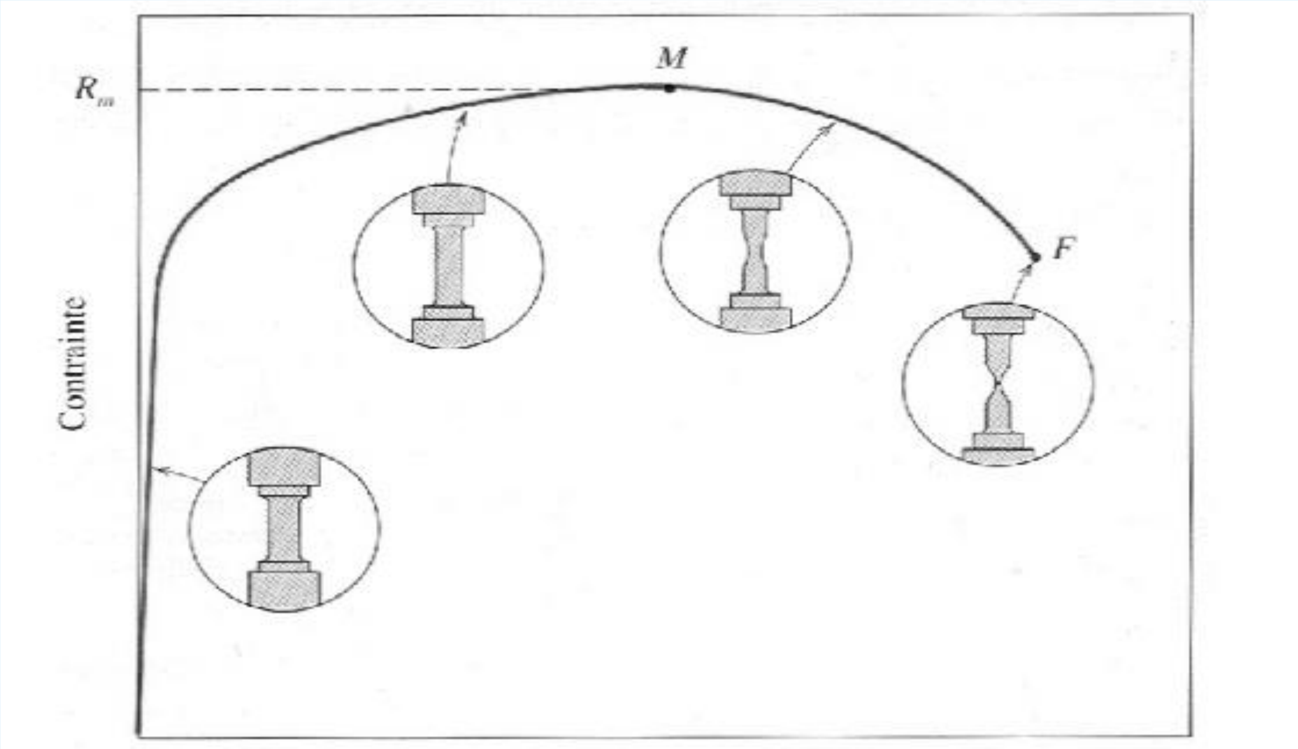
The specimen is subjected to an increasing force F, the relative elongation $\Delta\ell$ is measured and the test is continued until failure.

- The state of stresses and strains :

$$\sigma_1 = \frac{F}{S} \qquad \varepsilon_1 = \frac{\Delta\ell}{\ell_o}$$

Strain-stress diagram for steel





Stress-strain diagram for aluminium

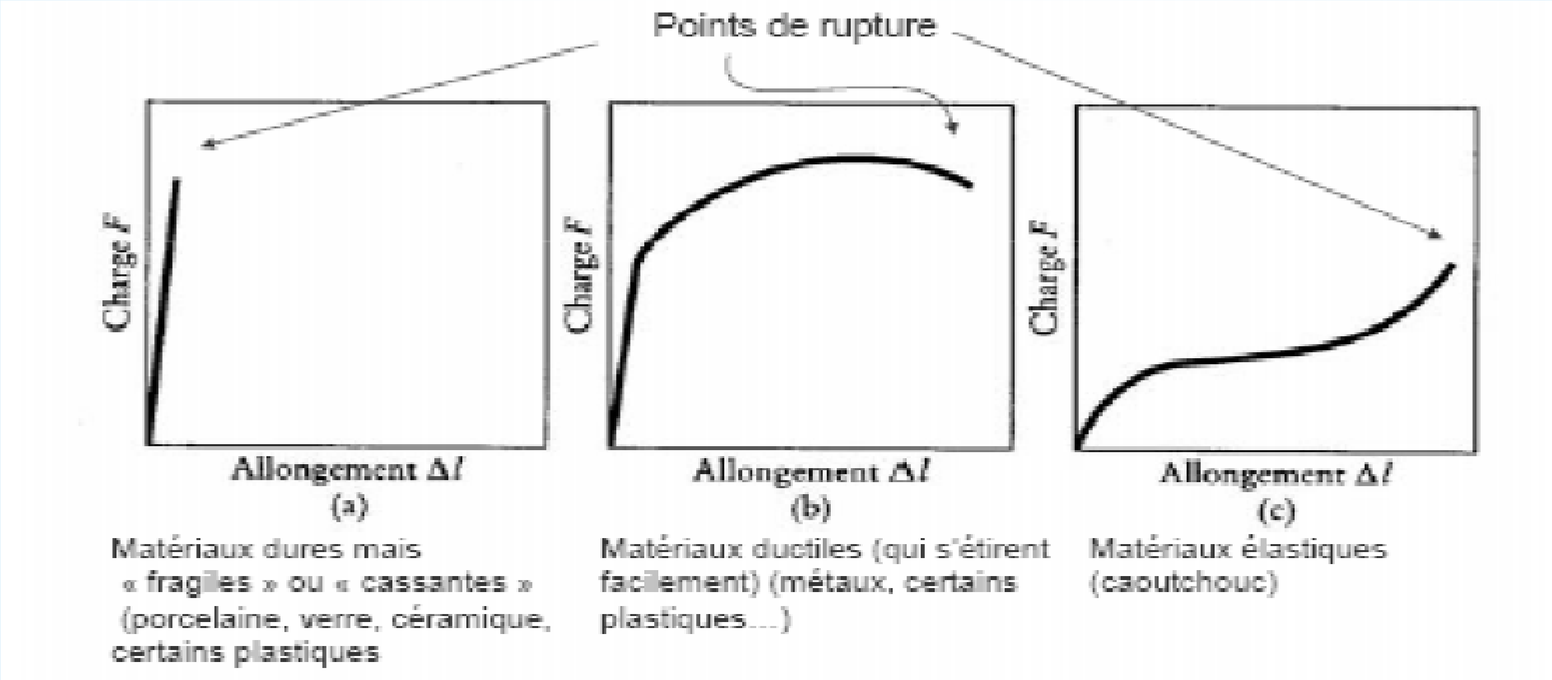
2- Ductile and fragile materials

-The behaviour of steel can be described in a stress-strain diagram composed of two phases: elastic behaviour + plastic behavior

- ⇒ **Ductile behavior**

- For glass, once the elastic limit is exceeded, breakage occurs:

- ⇒ **Fragile behaviour**



Stress-strain diagram for fragile, ductile and elastic materials

3- Admissible stress - Concept of safety factor

$$\sigma = \frac{F}{S} \leq \sigma_{ad} = \frac{\sigma_e}{f_s}$$

σ_{ad} : stress above which the component suffers deterioration of its mechanical and dimensional characteristics, or even breakage

- σ_e is determined experimentally.

For safety reasons, the stress must remain below an admissible limit stress.

σ_e : elastic limit

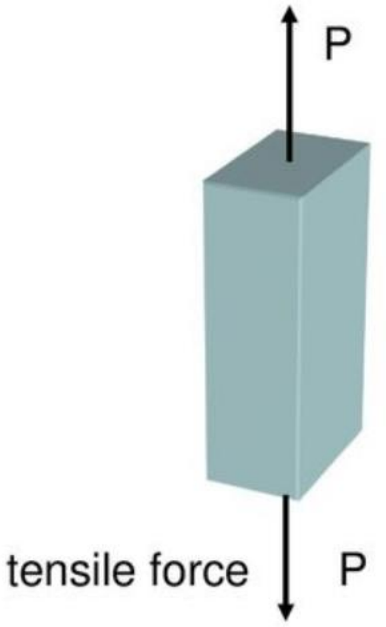
f_s : safety factor.

Exple: f_s =1,5 (concrete) ; f_s =1,15 (steel)

1. Stress

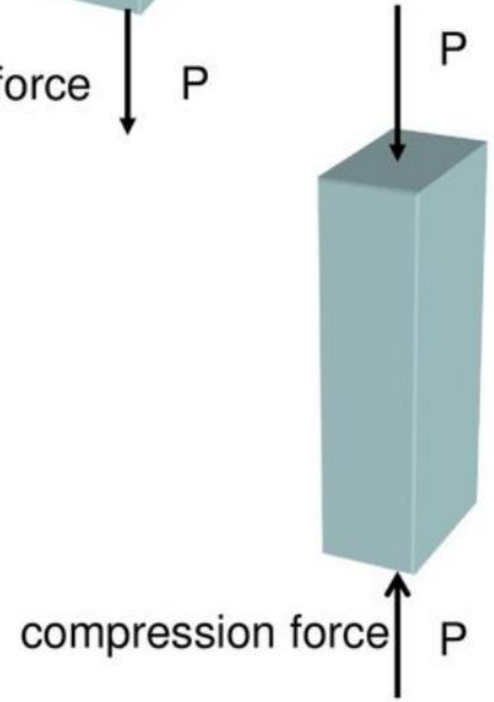
Tensile stress = force (pull) / Cross-sectional area

$$\sigma_t = \frac{P}{A}$$

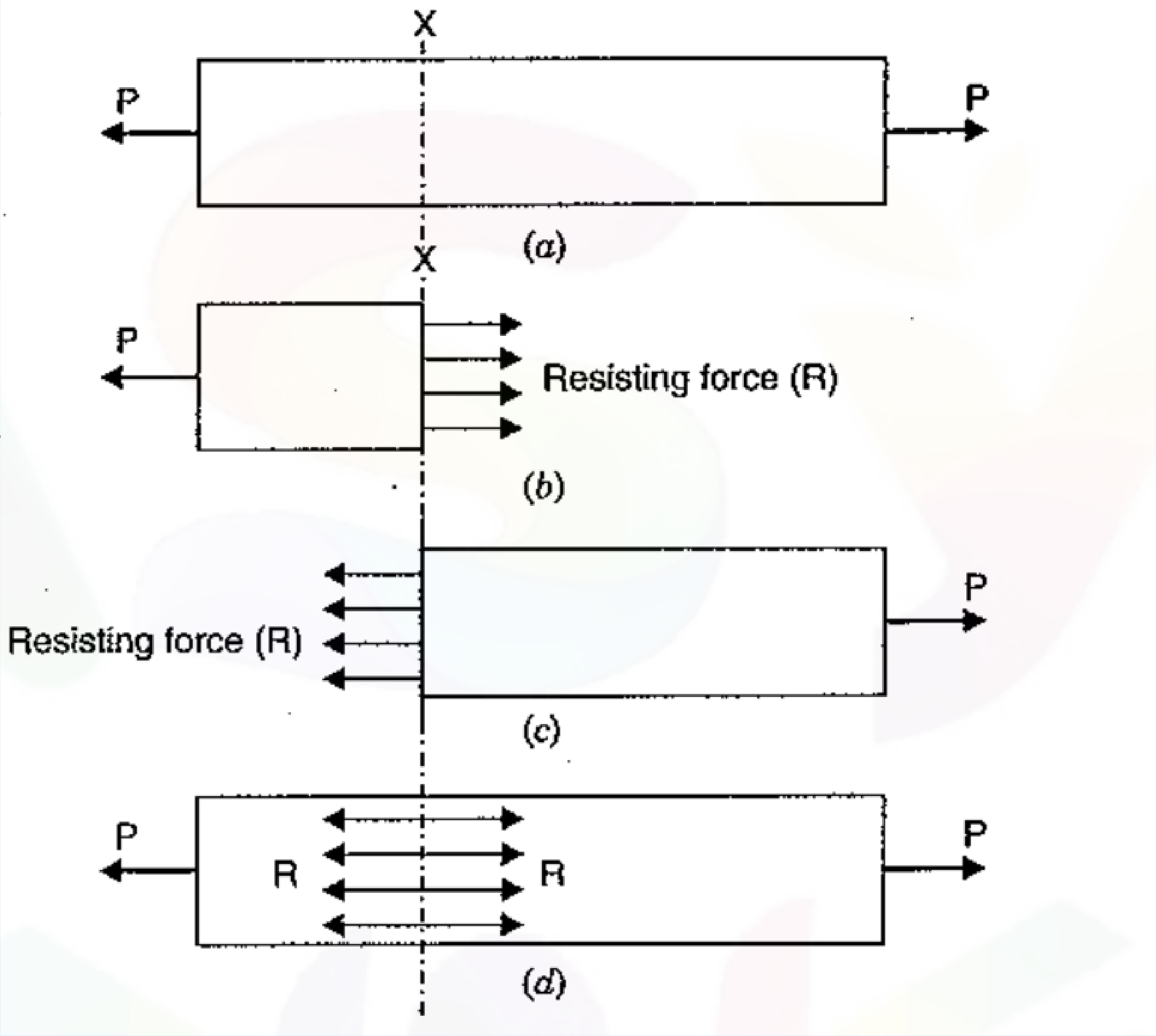


Compression stress = force (push) / Cross-sectional area

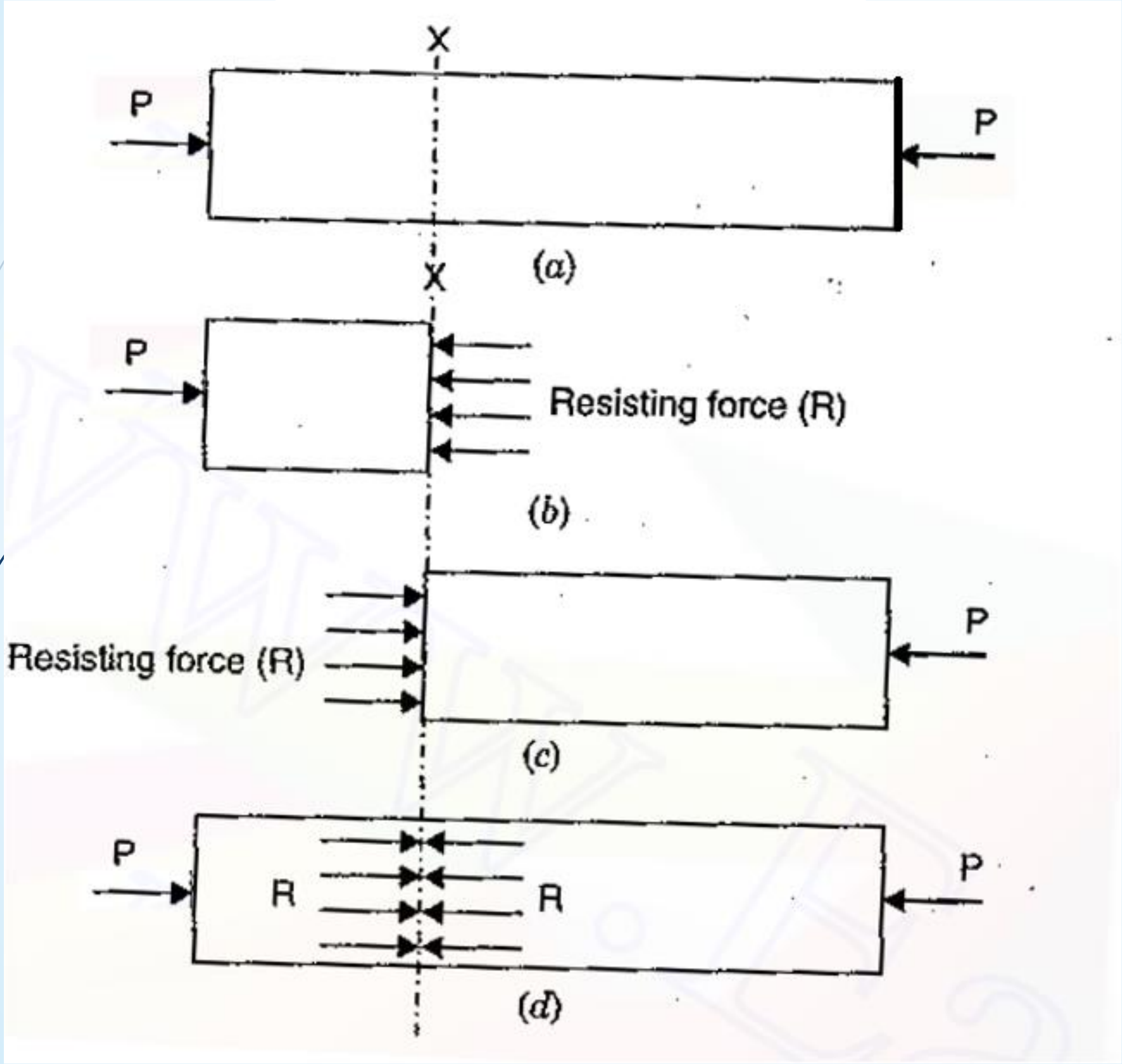
$$\sigma_c = \frac{P}{A}$$



Tensile stress



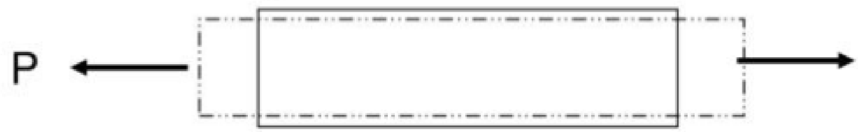
Compression stress



2. Strain

$$\text{strain} = \frac{\text{change in length } (x)}{\text{original length } (l)}$$

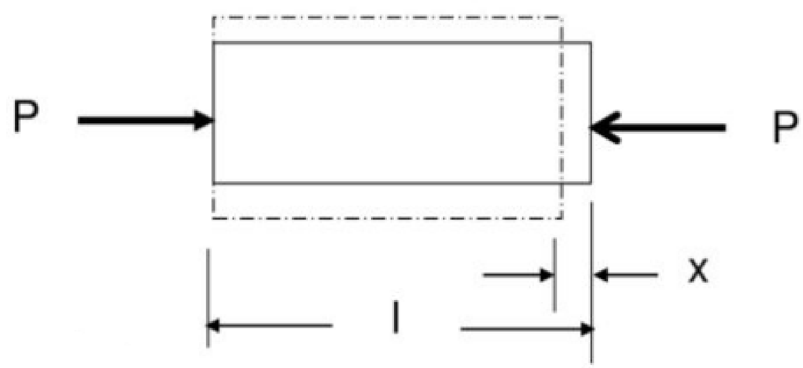
$$\varepsilon = \frac{x}{l}$$



Bar in tension

$$\text{tensile strain } \varepsilon_t = \frac{x}{l}$$

$$\text{compressive strain } \varepsilon_c = \frac{x}{l}$$



3. Hook’s law, Principal of superposition

Hook’s law

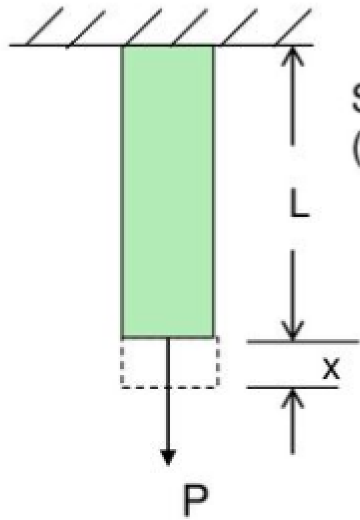
stress (load) α strain (extension)

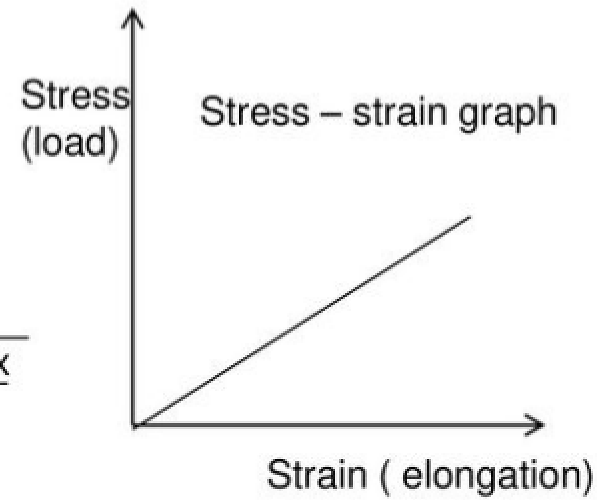
$$\text{modulus of elasticity} = \frac{stress}{strain}$$

$$E = \frac{\sigma}{\varepsilon}$$

Substituting $\sigma = P / A$ and $\varepsilon = x / L$

$$x = \frac{P L}{A E}$$

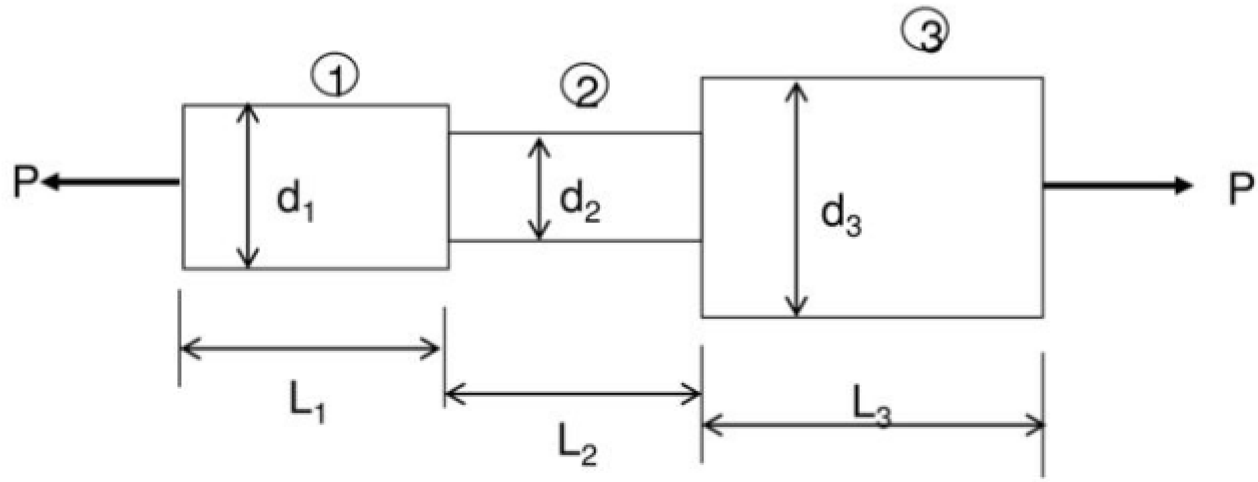




Principal of superposition

The effect of a system of forces acting on a body is equal to the sum of the effects these same forces applied.

4. Varying cross-section and loads



Loads $P = P_1 = P_2 = P_3$

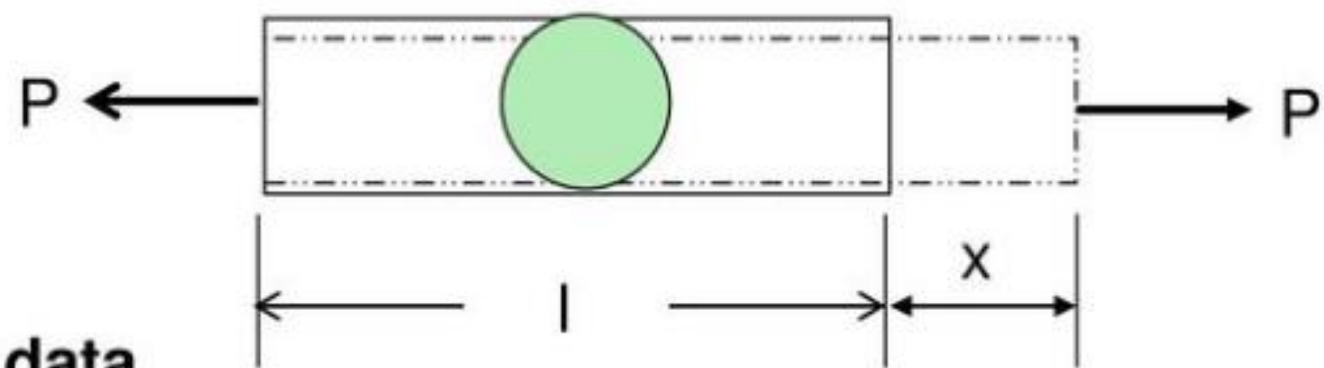
$P = \sigma_1 A_1 = \sigma_2 A_2 = \sigma_3 A_3$

The changes in length $x_1 = \varepsilon_1 L_1$, $x_2 = \varepsilon_2 L_2$, $x_3 = \varepsilon_3 L_3$

$$x_1 = \frac{P L_1}{A_1 E}, \quad x_2 = \frac{P L_2}{A_2 E}, \quad x_3 = \frac{P L_3}{A_3 E}$$

The total changes of length = $x_1 + x_2 + x_3$

Example 1



Given data

Tensile load P = 15,000 N
Steel rod diameter, d = 2 cm

To find

stress

Calculation

$$\sigma_t = \frac{P}{A}$$

$$\sigma_t = \frac{15000}{(\pi/4) \times 2^2}$$

$$= 4777 \quad N/cm^2$$

Example 2

Given data

A wire , Tensile strain $\varepsilon = 0.0002$

Elongation $x = 0.75$ mm

To find

Length of wire l

Calculation

$$\varepsilon = + \frac{x}{l}$$

$$+0.0002 = + \frac{0.75 \text{ mm}}{l}$$

$$l = 3750 \text{ mm}$$

$$\text{length of wire} = 3.75 \text{ m}$$

- Exercise 1:
- A steel bar with a 10 mm square cross-section is subjected to a tensile force of 10 kN. Knowing that the modulus of elasticity of steel is 200 GPa, calculate the deformation of the bar.

Solution exercise 1

- The cross-sectional area of the bar is $A = 10 \times 10 = 100\ mm^2 = 0.0001\ m^2$.
- The force applied is $F = 10$ kN = 10 000 N.
- The modulus of elasticity of the steel is $E = 200$ GPa $= 200 \times 10^9$ Pa.
- The deformation of the bar is given by the following formula:
- $\varepsilon = F / (A \times E)$
- $\varepsilon = 10{,}000 / (0.0001 \times 200 \times 10^9) = 0.0005$
- The deformation of the bar is therefore 0.0005.

Exercise 2:

A steel beam of 3 meters long and 20 mm square, is subjected to a compression force of 50 kN. Knowing that the modulus of elasticity of steel is 200 GPa, calculate the elongation of the bar.

Solution exercise 2

- The beam cross-section is A = 20 x 20 = 400 mm²
-

The applied force is F = 50 kN = 50,000 N.

The modulus of elasticity of the steel is E = 200 GPa = 200 000 MPa.

The initial length of the beam is L = 3 m.

The deformation of the beam is given by the following formula:

$\varepsilon = F / (A \times E)$

$\varepsilon = 50\,000 / (400 \times 200\,000) = 0.000625$

The deformation of the beam is therefore 0.000625.

The change in beam length is then given by the formula:

$\Delta L = \varepsilon \times L$

$\Delta L = 0.00625 \times 3 = 0.001875$ m

The change in beam length is therefore 0.001875 m.

- Exercise 3:

A steel bar of 2 meters long and 15 mm square is subjected to a tensile force of 80 kN. Knowing that the modulus of elasticity of steel is 200 GPa, calculate the elongation of the bar.

Solution exercise 3

- The cross-sectional area of the bar is A = 15 x 15 = 225 mm²
- The force applied is F = 80 kN = 80,000 N.

The modulus of elasticity of the steel is E = 200 GPa = 200 000 MPa.

The initial length of the bar is L = 2 m.

The elongation of the bar is given by the following formula:

$\Delta L = (F \times L) / (A \times E)$

$\Delta L = (80{,}000 \times 2000) / (225 \times 200\,000) = 3{,}555$ mm

The elongation of the bar is therefore 3,55 mm.