The Matrices : Definitions

I. Definitions :

- A matrix is a table with n rows and p columns. It represents data that consists of real numbers (called coefficients or terms).
  Matrices are often denoted by uppercase letters such as A, B, C, M, ..., and their coefficients are represented by lowercase letters.

- We denoted by a_{ij} the coefficient of the matrix A, situated on the i-th row of A and at the j-th colunn . Then, the A is Witten by

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1p} \\ a_{21} & a_{22} & \cdots & a_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{np} \end{pmatrix}$$

- The dimension of the matrix (called also the size, the order) : Rows are listed first, followed by columns. is denoted by size (.)

$$\text{size}(.) = \mathit{umber\ of\ rows} \times \quad \mathit{number\ of columns}$$

Exampls :

- The size of the matrix $A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -1 & 2 \end{pmatrix}$ $is\ \ 2 \times 3$
- Let $B = \begin{pmatrix} -1 & 0 & 2 \\ -2 & 0 & 0 \\ 1 & 1 & 3 \end{pmatrix}$ $, the\ order\ of\ B\ is\ 3$

- $Let\ \ C = \begin{pmatrix} -1 & 0 \\ -2 & 4 \\ 1 & 1 \end{pmatrix}$ $and\ \ \text{size}(C) = 3 \times 2$

Definition 02 :

- **A zero matrix** is a matrix where all its coefficients are zeros.

- **A Square matrix** Is a matrix where the number of rows is equal to the number of columns
- A **row matrix** is a matrix where the number of rows is equal to 1. It is also called a "row vector".
- A **column matrix** is a matrix where the number of columns is equal to

Exampls :

- $A = (4 \quad -2 \quad 0)$ is row matrix
- $B = \begin{pmatrix} -1 & 0 & 2 \\ -2 & 0 & 0 \\ 1 & 1 & 3 \end{pmatrix}$ $is\ a\ square\ matrix, the\ size\ is\ \ 3 \times 3,$

  in order to simplify , we can say , $B\ is\ a\ square\ matrix\ order$ 3d' ordre 3.
- $C = \begin{pmatrix} 5 \\ 6 \\ 3 \\ 4 \end{pmatrix}$ $is\ a\ \ column\ matrix.$

***Definition* 03 :**

The main diagonal of a matrix refers to the diagonal that connects the top-left corner to the bottom-right corner. In other words, the main diagonal elements have the same row and column numbers are : a_{11} , a_{22}, a_{33}, … .

Exemple :

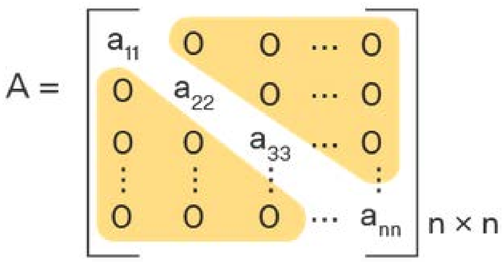
Let

$$A = \begin{pmatrix} 1 & -1 & -3 & -5 \\ 7 & 3 & -3 & 0 \\ 0 & 8 & 2 & 0 \\ -1 & 6 & 0 & -8 \end{pmatrix}$$

The diagonal éléments of A are : $a_{11} = 1$, $a_{22} = 3$, $a_{33} = 2$, and $a_{44} = -8$.

Definition 03 :

The diagonal matrix is a square matrix where all non-diagonal coefficients (those not on the diagonal) are zero.



<u>Example</u> :

Let

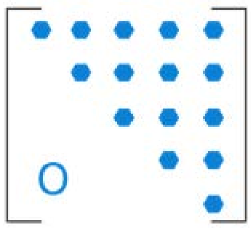
$$A=\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -8 \end{pmatrix}, \qquad B=\begin{pmatrix} 3 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 5 \end{pmatrix},$$

$$C=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad et \qquad M=\begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

A, B and M are a diagonal matrices, but C is not.

<u>Definition</u> 04 :

An **upper triangular matrix** is a square matrix where all coefficients below the diagonal are zero



Upper Triangular Matrix

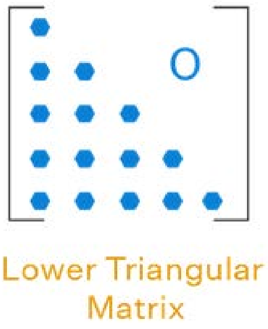
<u>Example</u>: Let the matrices:

$$A=\begin{pmatrix} -1 & 3 & 0 & -1 \\ 0 & 3 & 2 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -8 \end{pmatrix}, \qquad B=\begin{pmatrix} 3 & 1 & 3 \\ 0 & 0 & 2 \\ 9 & 0 & 5 \end{pmatrix},$$

We remark that A is an upper triangular matrix supérieure, however B is not,
Because there is a coefficient below the diagonal that is not zero, which is $a_{31}=9\neq 0$

<u>Definition</u> 05 :

A lower triangular matrix is a square matrix where all coefficients above the diagonal are zero.



Exemple :

Let the matrices

$$A = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ -3 & 0 & 2 & 0 \\ 2 & 0 & 7 & 6 \end{pmatrix}, \quad and \qquad C = \begin{pmatrix} 4 & 0 & 0 \\ 8 & 1 & 4 \\ 9 & -2 & 0 \end{pmatrix},$$

We remark that A is a lower triangular matrix; however, C is not, because there is a coefficient above the diagonal that is not zero, namely $a_{23} = 4 \neq 0$.

Définition 06 :

An **identity matrix** is a diagonal matrix where all the diagonal elements are equal to 1. It is denoted by I_n where n is the order of the matrix.

Example :

Let

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

We notice that:

- I_3 is an identity matrix of order 3(in $\mathbb{R}^3$
- I_2 is an identity matrix of order 2 (in $\mathbb{R}^2$).