# The Matrices :

## 2- Elementary Operations with Matrices

Let A be a matrix of dimension: $n_1 \times p_1$ and B Another matrix of dimension: $n_2 \times p_2$ .

## 1) Equality: $(=$ المساواة $)$

We say that two matrices are equal if:

- They have the same dimension
- Any term $a_{ij} = b_{ij}$ for each i, j.

**Exercise :** Let :

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -1 & 2 \end{pmatrix} \quad et \;\; B = \begin{pmatrix} a+2 & -2 & 0 \\ 1+d & 4-c & 2 \end{pmatrix}$$

Find the values of th reals : a, b and c such that the matrices A and B are equals

### Solution :

For A and B to be equal, it is necessary that:

- $dim(A) = (2,3) = dim(B)$
- The termes satisfy

$$a + 2 = 1 \quad \Longrightarrow a = -1$$
$$4 - c = -1 \quad \Longrightarrow c = 5$$
$$1 + d = 3 \quad \Longrightarrow d = 2$$

## 2) Transposition

We call the transpose of $A$, the matrix denoted by ${}^tA$ $(ou\ bien\ A^T)$, obtained by writing the rows of $A$ as columns.

**Example :**

The matrix transpose of $A = \begin{pmatrix} \mathbf{1} & \mathbf{4} & \mathbf{0} \\ 3 & -1 & 2 \end{pmatrix}$ $is$ ${}^tA = \begin{pmatrix} \mathbf{1} & 3 \\ \mathbf{4} & -1 \\ \mathbf{0} & 2 \end{pmatrix}$

## 3) Addition and Subtraction (+, -):

In order to compute the sum $A + B$, the following condition must be satisfied:

$$size(A) = 2 \times 3 = size(B).$$

Additionally, $A + B$ is computed by adding the terms of $A$ to the elements of $B$ located at the same position.

**Exercise :**

**Let be :**

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} -3 & 1 & 4 \\ -2 & 1 & -5 \end{pmatrix} \quad and \quad M = \begin{pmatrix} 3 & 0 \\ -3 & \frac{1}{2} \\ 2 & 2 \end{pmatrix},$$

**Compute the matrices** $A + B$ *and* , $A + M$

**Solution :**

**Since** $dim(A) = dim(B) = 2 \times 3$, then we can calculate $A + B$ **, indeed:**

$$A + B = \begin{pmatrix} 1-3 & -2+1 & 0+4 \\ 3-2 & -1+1 & 2-5 \end{pmatrix} = \begin{pmatrix} -2 & -1 & 4 \\ 1 & 0 & -3 \end{pmatrix}$$

For $A + M$, we cannot compute this sum because

$$dim(A) = 2 \times 3 \neq dim(B) = 3 \times 2$$

Therefore, the sum $A + M$ does not exist.

**Remark :**

In the case where we want to find the difference of the two matrices, $A$ *and* $B$, we follow the same process, but we need to use element-wise subtraction.

## 4) Product ( $\times$) الجداء :

**Definition 07 (scalar multiplication) :**

Let $k$ be a real number (a constant). The product $k.A$ is the matrix obtained by multiplying each element of matrix $A$ by $k$

**Exemple :** Let

$$A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & -1 & 2 \end{pmatrix},$$

The scalar multiplication of matrix is 3A and ,

$$3.A=\begin{pmatrix}1\times 3 & -2\times 3 & 0\times 3\\ 3\times 3 & -1\times 3 & 2\times 3\end{pmatrix}=\begin{pmatrix}3 & -6 & 0\\ 9 & -3 & 6\end{pmatrix},$$

**<u>Définition</u> 08 : (Matrix Multiplication) :**

Let $m, n\ and\ p$ be integers. Let $A$ be a matrix of size $m \times n,$ and $B$ be a matrix of size $(n \times p)$.

The matrix product $A \times B$ is a matrix of dimension $(m, p)$ obtained by calculating the product of $A$ by the columns of $B$.

$$AB=\begin{pmatrix}a_{11} & 0 & \cdots & 0\\ a_{21} & a_{22} & \cdots & 0\\ \vdots & \vdots & & \vdots\\ a_{i1} & a_{i2} & \cdots & a_{in}\\ \vdots & \vdots & & \vdots\\ a_{m1} & a_{m2} & a_{m1} & a_{mn}\end{pmatrix}\begin{pmatrix}b_{11} & b_{12} & \cdots & b_{1j} & \cdots & b_{1p}\\ b_{21} & b_{22} & \cdots & b_{2j} & \cdots & b_{2p}\\ \vdots & \vdots & & \vdots & & \vdots\\ a_{n1} & a_{n2} & \cdots & b_{nj} & \cdots & b_{np}\end{pmatrix}=$$

$$\begin{pmatrix}c_{11} & c_{12} & \cdots & c_{1j} & \cdots & c_{1p}\\ c_{21} & c_{22} & \cdots & c_{2j} & \cdots & c_{2p}\\ \vdots & \vdots & & \vdots & & \vdots\\ c_{i1} & c_{i2} & \cdots & c_{ij} & \cdots & c_{ip}\\ \vdots & \vdots & & \vdots & & \vdots\\ a_{m1} & a_{m2} & & a_{mj} & & c_{mp}\end{pmatrix}=C\ (dimension\ m\times p)$$

With

$$c_{ij}=a_{i1}b_{1j}+a_{i2}b_{2j}+a_{i3}b_{3j}+\cdots+a_{n1}b_{nj}$$

$$=\sum^{n} a_{ik}b_{kj}\ (i=1,2,\ldots,m; j=1,2,\ldots,p)$$

**<u>Example</u> :**

Let the matrices :

$$A=\begin{pmatrix}1 & -2 & 0\\ 3 & -1 & 2\end{pmatrix},\qquad B=\begin{pmatrix}2 & 1\\ 4 & -1\\ -1 & 5\end{pmatrix}\quad and\quad C=\begin{pmatrix}1 & 4\\ 1 & -5\end{pmatrix}$$

We notice that:

- Since the number of columns of $A = 3 =$ the number of rows of $B$, then we can compute the matrix product $AB$ .
  Moreover, the size of the resulting matrix $AB$ is $2 \times 2$, and the calculation gives:
  $$AB=\begin{pmatrix}-6 & 3\\ 0 & 14\end{pmatrix}$$

- We cannot compute the matrix product $AC$ because the number of columns of $A$ is 3, while the number of rows of $C$ is 2, so:
the number of columns of A is not equal to the number of rows of B.

$$\text{the number of columns of } A \neq \text{ the number of rows of } B$$

Therfore $AC$ doesn't exist.

**Remarks :**

- The matrix product is not commutative. In other words, $AB \neq BA$.
- There is no specific method for computing the power of a matrix. $A^n$. It is computed using matrix multiplication as follows:

$$A^n = \underbrace{A.A.A \dots .A}_{n\ fois}$$

And this can only be done if matrix $A$ is square.

**Properties**

- ${}^t(A.B) = {}^tB.{}^tA$
- $A.I_n = A$
- $k.A = A.k \quad with \quad k \in \mathbb{R}.$