



## Matrix Worksheet 02

**Exercise 01** : In each case assume that  $A$  is a square matrix that satisfies the given condition. Show that  $A$  is invertible and find a formula for  $A^{-1}$  in terms of  $A$ .

a.  $A^3 - 3A + 2I = 0$ .

b.  $A^4 + 2A^3 - A - 4I = 0$ .

**Exercise 02:** : Let  $A$  be a matrix

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$$

1. Verify that  $A^2 - 3A + 2I = 0$ .
2. Deduce that  $A$  is invertible and determine  $A^{-1}$

**Exercise 03** : Let  $M$  be a  $2 \times 2$  square matrix:

$$M = \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}$$

1. Determine the matrices :  $M^2$  and  $M^3$ ?
2. Has  $M$  an inverse matrix ? if it is yes, find  $M^{-1}$

**Exercise 04** : For the following matrices, perform a test for invertibility and, if possible, compute the inverse matrix.

$$\begin{aligned} A &= \begin{pmatrix} -5 & 7 \\ 8 & 10 \end{pmatrix} & B &= \begin{pmatrix} 2 & -2 \\ 0 & 1 \\ 3 & -6 \end{pmatrix} & C &= \begin{pmatrix} -1 & -4 \\ 1 & -2 \end{pmatrix} & D &= \begin{pmatrix} 3 & 4 \\ 1 & 9 \end{pmatrix} & N &= \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix} \\ E &= \begin{pmatrix} 0 & 0 & 2 \\ 3 & 1 & -1 \\ 2 & 2 & 4 \end{pmatrix} & F &= \begin{pmatrix} -1 & -1 & 2 \\ 0 & 2 & 2 \\ -3 & 2 & 5 \end{pmatrix} & M &= \begin{pmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{pmatrix} & G &= \begin{pmatrix} 4 & 2 & 5 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

**Exercise 05** : let  $a$  be a non nul real and  $N$  a matrix

$$N = \begin{pmatrix} 2+a & 4 \\ 1+a & 3 \end{pmatrix}$$

1. Give the values of  $a$  real such as  $N$  must be an invertible
2. Find the invertible matrix  $N^{-1}$  when  $a = 1$ ,  $a = 2$  and  $a = -3$