1. Functions of two variables.

Definition: A **multivariate function** has several different independent variables : $x, y, z, t, \dots$. It also called : **multivariable function**, or **function of several variables**

Examples:

- $f(x,y) = 2xy^2 - 3yx + 5x^2 - 12y$
  f is function of two variables x and y.
- $f(x,y,z) = xyz + x^3 + 2y^2xz^2 - 3z + 5y + 1$
  here, f is a function of three variables: x, y and y.

Definition: (1st order partial derivatives)

The partial derivative of a function represents the derivative of the function with respect to one of its variables and the authors variables are considered as a constant. The symbol of partial differentiation is ∂.

- When f is a function of two variables x and y we have two partial derivatives: $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.
- When f is a function of three variables x, y and z, there are three partial derivatives: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$.

To read this symbol, we say for

- $\frac{\partial f}{\partial x}$: "the partial derivative of f with respect to x"
- $\frac{\partial f}{\partial x}$: "the partial derivative of f with respect to y"

How to calculate The partial derivative of a function ?

To calculate the first-order partial derivatives, we apply this step:

1) Identify the variables.
2) Treat the rest of the variables as constants.
3) Apply fundamental derivative rules to different this function

Example1: let the function $f(x,y) = 3x + 4y - 2$

we can represent the partial derivatives:

- with respect to x as $\frac{\partial f}{\partial x} = 3$ (y is considered as constant, $(4y)' = 0$)
- with respect to y as $\frac{\partial f}{\partial y} = 4$ (x is considered as constant, $(3x)' = 0$)

Example 2: let the function $f(x,y,z) = xyz + x^3 + 2y^2z^3 - 3z + 5y + 1$

the partial derivatives of f are:

$$\frac{\partial f}{\partial x} = yz + 3x^2 \qquad \frac{\partial f}{\partial y} = xz + 4yz^3 + 5 \qquad \text{and} \qquad \frac{\partial f}{\partial z} = xy + 6y^2z^2 - 3$$

Definition: (Gradient) The **gradient** of a function f, denoted as Δf is the collection of all its partial derivatives into a vector.

f is a function of tow variables	f is a function of three variables
$\Delta f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}$	$\Delta f = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$

Definition (The second-order partial derivatives)

The second-order partial derivatives of function f of two variables are defined by :

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) \quad , \quad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) \quad and \quad \frac{\partial^2 f}{\partial x\,\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$

Wich determines a square matrix called the Hessian matrix denoted by:

$$H = \begin{pmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x\,\partial y} & \frac{\partial^2 f}{\partial y^2} \end{pmatrix}$$

Remark: let f be a multivariable function, in order to get the maxima and minima point we mut find the solution of the following equation :

$$\Delta f = 0$$