

التعميم الأول:

$$\begin{aligned} \text{Var}(x+y) &= E[(x+y) - E(x+y)]^2 - 5 \\ &= E[(x+y) - E(x) - E(y)]^2 \\ &= E[(x - E(x)) + (y - E(y))]^2 \\ &= E[(x - E(x))^2 + (y - E(y))^2 + 2(x - E(x))(y - E(y))] \\ &= E[(x - E(x))^2] + E[(y - E(y))^2] + 2E[(x - E(x))(y - E(y))] \\ &= \text{Var}(x) + \text{Var}(y) + 2\text{Cov}(x, y) \end{aligned}$$

نفس الخطوات في حالة $\text{Var}(x-y)$ التعميم الثاني:

1- حساب القيم التالية:

$$\begin{aligned} * E(x) &= \frac{\sum x_i}{n} = \frac{80}{10} = 8 \\ * E(y) &= \frac{\sum y_i}{n} = \frac{100}{10} = 10 \\ * \text{Var}(x) &= \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{366}{9} = 40,67 \\ * \text{Var}(y) &= \frac{\sum (y_i - \bar{y})^2}{n-1} = \frac{76}{9} = 8,45 \\ \text{Cov}(x, y) &= \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1} = \frac{57}{9} = 6,33 \end{aligned}$$

$$\text{Var}(c) = E(c - E(c))^2 - 1$$

= 0 ، $E(c) = c$ 3

$$\text{Var}(c) = E(c - c) = E(0) = 0$$

$\text{Var}(c) = 0$

$$\text{Var}(c x_i) = E[c x_i - E(c x_i)]^2 - 2$$

$E(c x_i) = c E(x_i)$ لدينا

$$\begin{aligned} &= E[c x_i - c E(x_i)]^2 \text{ إذن} \\ &= E[c(x_i - E(x_i))]^2 \\ &= c^2 E(x_i - E(x_i))^2 \\ &= c^2 \text{Var}(x_i) \end{aligned}$$

$\text{Var}(c x_i) = c^2 \text{Var}(x_i)$

$$\text{Var}(c+x) = \text{Var}(c) + \text{Var}(x) = 0 + \text{Var}(x)$$

$\text{Var}(c+x) = \text{Var}(x)$

$$\begin{aligned} \text{Var}(c-x) &= \text{Var}(c) + \text{Var}(-x) \\ &= 0 + \text{Var}(-x) \\ &= (-1)^2 \text{Var}(x) \end{aligned}$$

$\text{Var}(c-x) = \text{Var}(x)$

n	x	y	$(x_i - \bar{x})^2$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
1	0	9	6400	1	8
2	1	10	6241	0	0
3	2	11	6084	1	-6
4	5	12	9	4	-6
5	7	10	1	0	0
6	9	5	1	25	-5
7	10	6	4	16	-8
8	11	10	9	0	0
9	15	12	49	4	14
10	20	15	144	25	60
Σ	80	100	366	76	57

$$\begin{cases} \bar{x} = 8 \\ \bar{y} = 10 \end{cases} \quad \text{لدينا:}$$

- بيان $\text{Cov}(x, y) \neq 0$ ، إذن المتغيرين غير مستقلين فيما بينهما.
التقرين الثالث:

- تقدير العلاقة الخطية:

$$S_i = \alpha + \beta y_i$$

$$\hat{\beta} = \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2} \quad \text{لدينا}$$

$$\hat{\beta} = \frac{\Sigma x_i y_i - n \bar{x} \bar{y}}{\Sigma x_i^2 - n \bar{x}^2}$$

$$\hat{\beta} = \frac{\Sigma (y_i - \bar{y})(S_i - \bar{S})}{\Sigma (y_i - \bar{y})^2}$$

$$\hat{\beta} = \frac{\Sigma y_i S_i - n \bar{y} \bar{S}}{\Sigma y_i^2 - n \bar{y}^2}$$

$$\hat{\beta} = \frac{33 - 5(8 \times 0,76)}{338 - 5(8)^2}$$

$$\hat{\beta} = \frac{2,6}{18} = 0,14$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x} \quad \text{لدينا}$$

$$\hat{\alpha} = \bar{S} - \hat{\beta} \bar{y}$$

$$\hat{\alpha} = 0,76 - (0,14 \times 8)$$

$$\hat{\alpha} = -0,36$$

$$\hat{S}_i = -0,36 + 0,14 y_i \quad \text{رؤيتنا}$$

t	x _t	y _t	x _t y _t	x _t ²
1	8000	7389,99	59119920	64000000
2	9000	8169,65	73526850	81000000
3	9500	8831,71	83907245	90250000
4	9500	8652,84	82201980	90250000
5	9800	8788,08	86123184	96040000
6	11000	9616,21	105778310	121000000
7	12000	10593,45	127121400	144000000
8	13000	11186,11	145419430	169000000
9	15000	12758,09	191371350	225000000
10	16000	13869,62	221913920	256000000
Σ	112800	99855,75	1176477589	1336540000

$$\bar{x} = \frac{\sum x_t}{n} = \frac{112800}{10}$$

$$\bar{x} = 11280$$

$$\bar{y} = \frac{\sum y_t}{n} = \frac{99855,75}{10}$$

$$\bar{y} = 9985,575$$

تقدير معاملات التوزيع :

$$\hat{y}_t = \hat{\alpha} + \hat{\beta} x_t$$

$$\hat{\beta} = \frac{\sum x_t y_t - n \bar{x} \bar{y}}{\sum x_t^2 - n \bar{x}^2}$$

$$\hat{\beta} = \frac{1176477589 - 10 \times 11280 \times 9985,75}{1336540000 - 10(11280)^2}$$

$$\hat{\beta} = \frac{50104729}{64156000} \Rightarrow \hat{\beta} = 0,78$$

$$\hat{\alpha} = \bar{y} - \hat{\beta} \bar{x}$$

$$\hat{\alpha} = 9985,575 - 0,78 \times 11280$$

$$\hat{\alpha} = 1187,175$$

$$\hat{y}_t = 1187,175 + 0,78 x_t$$