Aboubakr Belkaid University of Tlemcen. **(2024/2025)**

Faculty of Sciences **Master, all specialties**

Mathematics department **Technical English 2**

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Lesson N°7 : Improve writing skill

Short vs long answers

Mathematical Expressions – English/French

The following table provides a selection of useful expressions commonly used in mathematical writing and reasoning. These phrases help articulate logical connections between statements, clarify assumptions, and structure proofs or derivations. Each expression is accompanied by its French equivalent to support understanding and facilitate bilingual learning. Students are encouraged to become familiar with this vocabulary to improve both their reading comprehension and their ability to express mathematical ideas clearly in English.

English Expression	French Translation
It follows from hypothesis 1 that ...	Il résulte de l'hypothèse 1 que… / Il s'ensuit que…
We deduce from relation 2 that ...	On peut donc déduire de la relation 2 que…
Conversely, 3 implies that ...	Inversement / réciproquement, 3 implique que…
Equality (1) holds, by Proposition 2.	L'égalité (1) est vraie, par la proposition 2.
By definition, The following statements are equivalent.	Par définition, les deux lignes (informations) ci-après sont équivalentes.
Thanks to expression (H), the properties (1) and(2) of the above set are equivalent to each other.	Grâce à l'expression (H), les propriétés (1) et (2) de l'ensemble cité plus haut sont équivalentes.
Function f has the following properties.	La fonction f a les caractéristiques / propriétés suivantes
Theorem 1 holds unconditionally.	Le théorème 1 est toujours vrai.
This result is conditional on Axiom A.	Ce résultat est dépendant de l'Axiome A.
This is an immediate consequence of Theorem 3.	Ceci est une conséquence immédiate du théorème 3.
Note that (2) is well-defined, since ...	Il est à noter que (2) est bien défini, puisque…
As x satisfies the relation (2), formula (1) can be simplified as follows.	Comme x satisfait la relation (2), la formule (1) peut être simplifiée comme suit.
We conclude (the argument) by combining inequalities (2) and (3).	On conclut (l'argument) par combinaison des inégalités (2) et (3).
(Let us) denote by X the set of all ...	Notons par X l'ensemble de tout…
It is enough to show that ...	C'est suffisant de montrer que…

We are reduced to proving that ...	On est réduit / limité à prouver que…
The main idea is as follows.	L'idée principale est comme suit.
We argue by contradiction. Assume that P exists.	On argumente par l'absurde… supposant que P existe.
The formal argument proceeds in several steps.	L'argument progresse en plusieurs étapes.
Consider first the special case when ...	Il faut premièrement considérer le cas spécial quand…
..., which proves the required claim.	…, ce qui prouve la revendication / hypothèse requise.
We use induction on n to show that ...	On utilise la récurrence sur n pour montrer que…
On the other hand, ...	D'autre part, …
..., which means that ...	…, ça veut dire que…
In other words, ...	Autrement dit, …

Answer with "true" or "false". In the "false" case, give a counter example or an explanation.

Long answer	Short answer
1) In R, if A is upper bounded, then $Max\ A$ exists. False, because in IR, if A is upper bounded, then sup A exists but not necessarily Max A. As a counter example, consider A=[0,1[$\sqsubset IR$, one has sup A=1, But Max A does not exist.	1) False
2) In IR, if A is upper bounded, and B is lower bounded then (A+B) is bounded. False, because sup(A+B)=supA+supB and inf(A+B)=infA+infB So, when supB does not exist then sup(A+B) does not exist either. And when infA does not exist then inf(A+B) does not exist either.	2) False
3) In IR, if Sup A exists and Max B exists then Sup(AUB) exists. True, because	3) True

Sup (AUB)=max (supA, supB). It exists since sup A exists and supB=maxB exists.	
4) Sup $(IR\backslash IN)$exists in R. False, because $(IR\backslash IN)$ is an infinite union of intervals of the type:]n, n+1[, n∈ IN. It is therefore an infinite set (non-upper bounded) of IR. Or we may argue by contradiction: suppose that $\exists\ \alpha = sup(IR\backslash IN)$, then α is an upper bound of $(IR\backslash IN)$, so $\forall x \in (IR\backslash IN), x \leq \alpha$ and since IR is Archimedean then: $\exists N \in IN / \alpha < N$, which means that : $\forall \beta \in]\alpha, N[, \alpha < \beta$. This is a contradiction with the fact that α is an upper bound of $(IR\backslash IN)$.	4) False
5) Max(Z/IN) does not exist. False, because Max(Z/IN)=Max $(\{\ldots, -2, -1\})$ =-1.	5) False
6) Let E= $\left\{(-\ 1)^{m} + (-\ 1)^{n} :\ n\ ,\ m\ \ \epsilon\ IN\right\}$ Sup E exists and Max E does not exist. False, because E= $\{-\ 2, 0, 2\}$ So, Max E=2 and then Sup E=2 exists.	6) False

TD N°7 : Short vs long answers

Answer with "true" or "false". In the "false" case, give a counter example or an explanation.

1) If (U_n) is bounded and $V_n \to 0$, then $(U_n.V_n) \to 0$.	1)
2) If $\| V_n \| \to L$, then $V_n \to L$.	2)
3) If $V_n \to L$, then $\| V_n \| \to \| L \|$.	3)
4) If $\forall$ n $\in IN$, $U_n \leq W_n \leq V_n$ and [(U_n) and $(V_n) \to L$] ; then $(W_n) \to L$.	4)
5) If (U_n) and (V_n) diverge, then $(U_n + V_n)$ diverges.	5)
6) If (U_n) converges and (V_n) diverges, then $(U_n . V_n)$ Converges.	6)
7) Every bounded sequence is convergent.	7)
8) If (U_n) is decreasing and lower bounded by 0 then it converges to 0.	8)

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TD N°7 solution: Short vs long answers

Answer with "true" or "false". In the "false" case, give a counter example or an explanation.

1) If (U_n) is bounded and $V_n \to 0$, then $(U_n . V_n) \to 0$. Mainly because $\lvert Un.Vn \rvert \leq M.\lvert Vn \rvert$	1) True
2) If $\lvert V_n \rvert \to$ L, then $V_n \to$ L. Counter example Consider the sequence (V_n)/ $V_n = (-1)^n$, then $\lvert V_n \rvert \to 1$ but (V_n) is not convergent.	2) False
3) If $V_n \to$ L, then $\lvert V_n \rvert \to \lvert$L$\rvert$. Mainly because $\lvert \lvert Vn \rvert - \lvert l \rvert \rvert \leq \lvert Vn - l \rvert$	3) True
4) If $\forall$ n $\in IN$, $U_n \leq W_n \leq V_n$ and [(U_n) and (V_n) $\to$ L] ; then (W_n) $\to$ L. This is true according to the three sequences theorem.	4) True
5) If (U_n) and (V_n) diverge, then $(U_n + V_n)$ diverges. Counter example Take (U_n)=n and (V_n) $=-$n.	5) False

Then ∀ n ∈IN , $(U_n + V_n) = 0$, i.e. $(U_n + V_n) \to 0$.	
6) If (U_n) converges and (V_n) diverges, then $(U_n . V_n)$ Converges. Counter example Take $(U_n)=\frac{1}{n} \to 0$ and $(V_n) = n^2 \to +\infty$ Then ∀ n ∈IN^* , $(U_n . V_n)$ =n i.e. $(U_n . V_n) \to +\infty$.	6) False
7) Every bounded sequence is convergent. Counter example take $(U_n)=(-1)^n$. (U_n) is bounded but it doesn't converge.	7) False
8) If (U_n) is decreasing and lower bounded by 0 then it converges to 0. Counter example take $(U_n)=1+\frac{1}{n}$ (U_n) is decreasing (since $U_{n+1} - U_n = \frac{-1}{n(n+1)} < 0$) and lower bounded by 0 (since ∀ n ∈IN^*, 0<$1+\frac{1}{n}$), but it converges to 1.	8) False