Transformer

Introduction

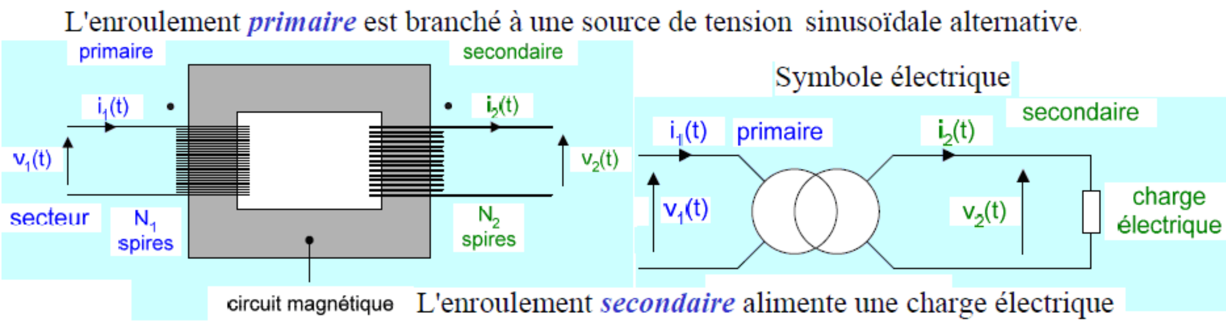
The transformer provides an alternating voltage change with excellent efficiency (up to 99%). Since losses in copper cables due to the Joule effect (**$P_{EJ} = RI^2$**) are proportional to the square of the current, electrical energy can only be **transmitted** at high voltage (**HV** of the order of 220kV). The voltage supplied by generators (5 to 10 kV) must be raised before they can be transmitted.
Then, as the HT is dangerous, it will be necessary to lower it thanks to the transformers (220V) for use ...

Constitution

The single-phase transformer is made up of two independent windings entwined in a common magnetic circuit: an alternating magnetic flux flows through the magnetic circuit.
The winding with the most turns is called high voltage (it's made of thinner wire) and the other is called low voltage.

- To minimize (electrical) **eddy-current** losses, **laminated magnetic circuits** are made from thin metal sheets (approx.0,4 mm thick), insulated from each other by varnish.

- To reduce (magnetic) **hysteresis** losses, these laminations are made of **silicon-added iron** (grain-oriented)...



A current of the same type flows through the primary winding, creating a flux through the magnetic circuit that generates a fem of the same type in both windings. A voltage therefore appears at the terminals of the secondary winding. It's thanks to the excellent magnetic (iron) connection between the coils that there's a mutual induced fem (coupling coefficient K close to 1).

Using Lentz's law, we can write :

$$v_1(t) = N_1 \frac{d\Phi}{dt} \quad \text{et} \quad v_2(t) = N_2 \frac{d\Phi}{dt} \qquad \text{d'où la relation :} \quad \frac{v_2(t)}{v_1(t)} = \frac{N_2}{N_1} = \mathbf{m} = \text{rapport de transformation}$$

The ideal transformer

No loss: $S_1 = S_2 \rightarrow P_1 = P_2$ and $Q_1 = Q_2 \rightarrow P(_1)^2 + Q_1^2 = P_2^2 + Q_2^2 \rightarrow S_1 = V_1.I_1 = S_2 = V_2.I_{(2}$

$$\longrightarrow \quad \text{rapport de transformation} \quad m = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Efficiency is determined by $\eta = P_2 / P_1 = 1 = 100\%$.

- Two main types of transformers :

- voltage step-up (current step-down): $m > 1$ $N(2) > N_1$
- voltage step-down (current step-up): $m < 1$ $N(2) < N_1$

No-load voltage

No load on secondary: $I_2 = 0$ but $I_{1v} \approx 0$;
V_2 depends on the current I_2 delivered to the load.

Rapport de transformation à vide : $m_v = \frac{V_{2vide}}{V_1} \approx \frac{N_2}{N_1}$

The voltage at the transformer's secondary when it is at no load is frequently given as : $V_{2v} = mV_1$

The real transformer

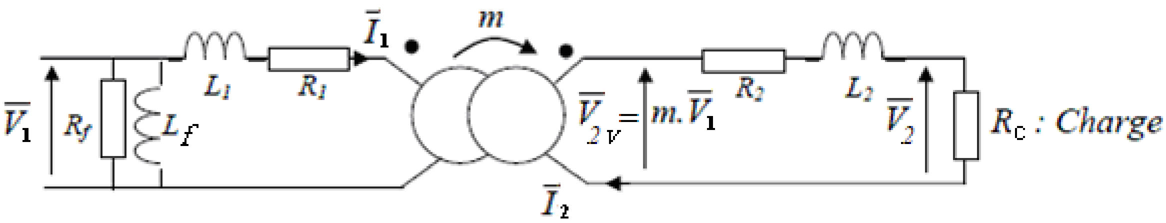
In reality, $P_2 < P_1$: yield < 1
as there are **losses in the winding**:

- Joule losses in windings **= $P_{(EJ)}$; (**Series winding resistances: R_1 and R_2)

- magnetic losses in series winding leakage inductances: L_1 and L_2

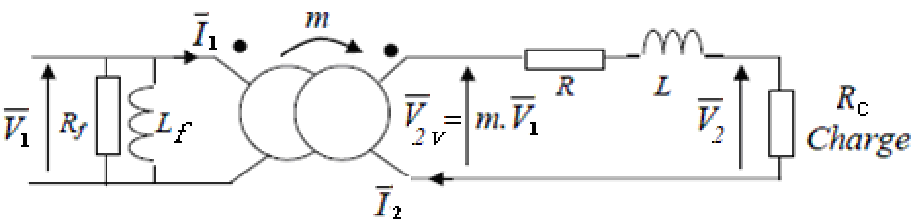
And **losses in the iron** = **$P_{(fer)}$ = $P_{(H)}$ + P_{CF}** = heating of the magnetic circuit, equivalence is made with a resistance R_f of the metal mass parallel to the input + magnetization of the magnetic circuit, equivalence is made with an inductance of the metal mass called "magnetizing inductance" $L_{(f}$

Starting from an ideal transformer, the equivalent diagram of the real transformer is shown:



This diagram is not practical for quickly characterizing a transformer, so

→ we use the so-called "**secondary equivalent diagram**" shown below:



On peut écrire : $\overline{Z} = R + jX$

$\overline{V}_2 = m\overline{V}_1 - \overline{Z}.\overline{I}_2$ et : $\overline{I}_2 = \frac{\overline{I}_1}{m}$

Where $\underline{Z}$ is the **secondary** impedance of an ideal transformer with a transformation ratio of m.

R = **$R_S = R_2 + m^2.R_1$** = winding resistance
$X_S = X = L\omega$; L = **$L_S = L_2 + m^2.L_1$** = **secondary** winding inductance

Determining equivalent elements

These elements are determined from the two "tests" called "no-load test" and "running test".

- No-load test :

The transformer is not connected to any load (Rc infinite; $I_2 = 0$) and supplied from the primary.
(Voltmeter (V_1) + ammeter (I_1) + wattmeter on primary (P_1) and voltmeter on secondary are fitted.

We measure P_1 and $S_1 = V_1.I_1$, knowing that: $P_{(1)} = V_1^2/R_{(f)}$ (eddy currents) and $Q_1 = V_{(1)}^2/(L_f.\omega)$ (hysteresis).

On calcule alors directement : $R_f = \dfrac{V_1^2}{P_1}$ et $L_f = \dfrac{V_1^2}{\omega\sqrt{(S_1^2 - P_1^2)}}$

- Short circuit test :
(Voltmeter (V_1) + ammeter (I_1) + wattmeter are fitted on the primary side (P_1) and ammeter on the secondary side (I_2). The transformer is short-circuited on the secondary and supplied with **reduced voltage** on the primary: V_1 is increased from 0 until I_2 is nominal (V_1 is low, so R_f and L_m can be neglected).

We measure P_1 and $S_1 = V_1.I_1$ s

Knowing that $P_1 = R.I_{(2)}^2$ and $Q_{(1)} = L\omega.I_2^2$

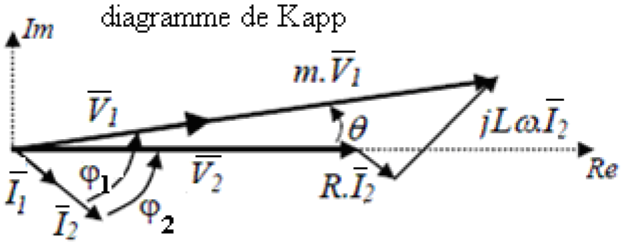
On calcule alors directement : $R = \dfrac{P_1}{I_2^2}$ et $L = \sqrt{\dfrac{(S_1^2 - P_1^2)}{\omega I_2^2}}$

NB: I_2 must not exceed the rated current = maximum current supported by the secondary winding.

Representation of voltages and currents in the complex plane

In order to carry out calculations on the equivalent diagram of a real transformer, it is usual to represent its voltages and currents in the complex plane. A typical representation is shown below:

This is the Kapp diagram = Fresnel representation of the equivalent diagram seen from the secondary:



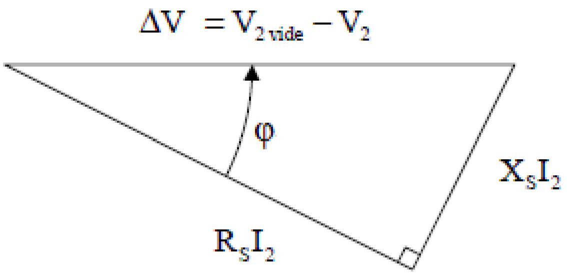
$$m\overline{V_1} = \overline{V_2} + \overline{Z}.\overline{I_2}$$

$$\overline{Z} = R + jL\omega$$

Le plus important à retenir est qu'il existe une chute de tension entre $\overline{V}_2$ et $m.\overline{V}_1$

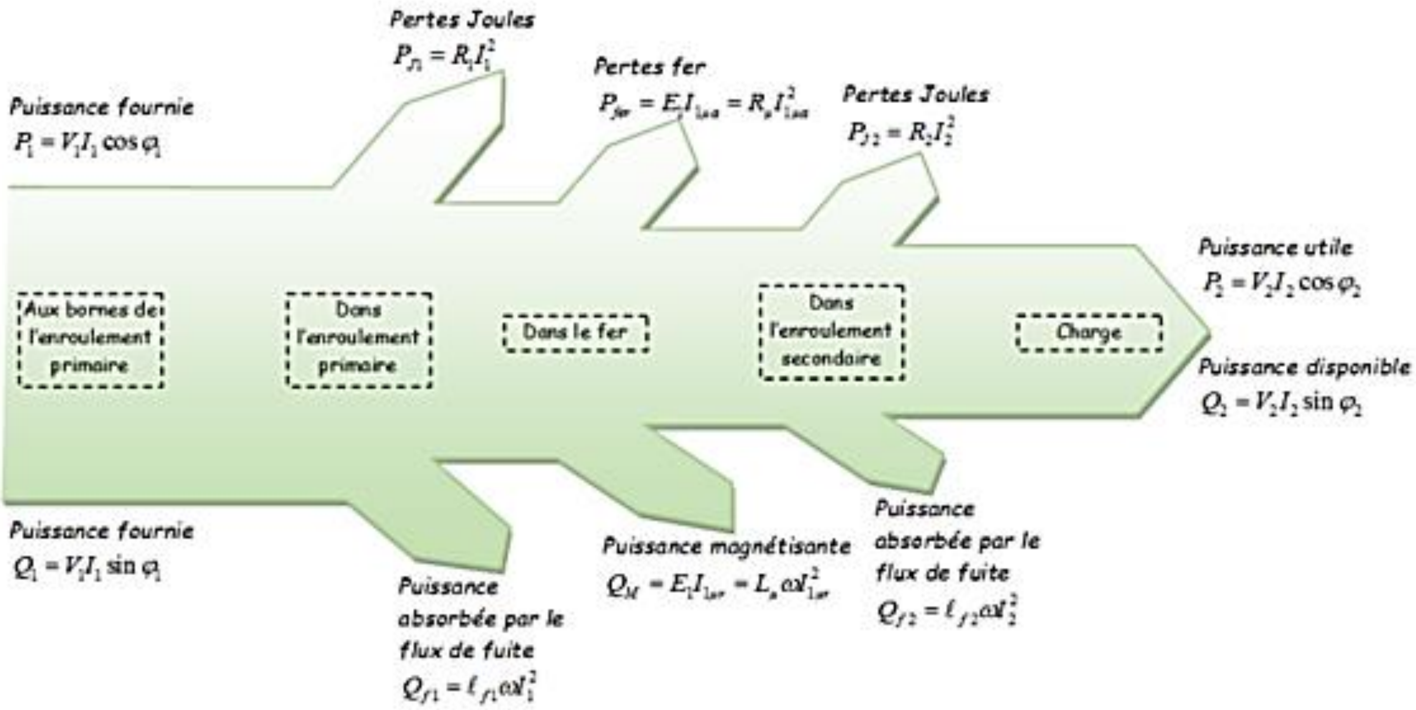
chute de tension en charge au secondaire = $\overline{\Delta V} = \overline{Z}\,\overline{I}_2$

We show by making the approximations: θ is small (θ ≈ **0; φ = φ1 = φ2) that we can write:**



$$\Delta V = mV_1 - V_2 \simeq R I_2 . \cos\varphi + L . \omega . I_2 . \sin\varphi$$

Transformer efficiency = active power ratio



The **classic** transformer **efficiency** is easily expressed as a function of the data at constant current I_2 :

$$\eta = \frac{P_u}{P_u + P_{pertes}} = \frac{R_u . I_2^2}{R_u . I_2^2 + R . I_2^2 + V^2 / R_f} = \frac{V_2^2 / R_u}{V_2^2 / R_u + R . I_2^2 + V^2 / R_f} = \frac{P_2}{P_1} = \frac{V_2 I_2 \cos\varphi_2}{V_2 I_2 \cos\varphi_2 + R_s I_2^2 + p_{fer}}$$

But for distribution transformers, the load is variable. Efficiency changes over time.

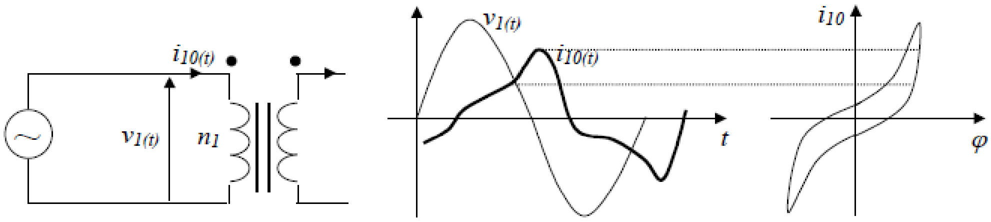
We therefore define **energy efficiency**, over a defined time T, as the quotient of the energy used over the total energy consumed during this time.

$$\eta_e = \text{rendement énergétique sur le temps T} = \frac{\textit{Energie utile}}{\textit{Energie utile} + \textit{Energie perdue}}$$

NB: In most cases, the calculation is based on a time T = 24 hours.

No-load current

If we look through an oscilloscope at the current drawn by a transformer with its secondary open, we see a waveform as shown in the diagram below:



This current is fully justified by the presence of the magnetic circuit's "hysteresis cycle", also shown in the diagram. At no load, the primary current saturates the magnetic circuit. Flux saturation requires high ampere-turns, which justifies the high current.

Hysteresis, on the other hand, imposes current asymmetry. Remember that, at no load, the transformer is a highly non-linear dipole.

NB: This current has its fundamental at 50Hz and odd harmonics at 150, 250, 350Hz, etc...

Three-phase transformer :

In order to transform the amplitude of the voltages of a three-phase system, 3 single-phase transformers are theoretically required, the phases of which will be coupled, depending on the constraints, in star or delta configuration.

In reality, we use a single magnetic circuit on which all 6 windings are wound. This is known as a three-phase transformer.

It is also possible to couple the primary and secondary differently, for example to create a local neutral or provide a phase shift between certain voltages.

Yield :

Trois enroulements au primaire (un par phase).

Trois enroulements au secondaire (un par phase).

$$\boxed{\eta = \frac{\sqrt{3}U_2 I_2 \cos\varphi_2}{\sqrt{3}U_2 I_2 \cos\varphi_2 + 3R_s I_2{}^2 + p_{fer}}}$$

Transformer exercises

Exercise 1

Draw up the transformer's no-load power balance.
Deduce that no-load power consumption is approximately equal to iron losses.

Answer key 1

$P_1 = P_2$ + Joule losses + Iron losses   At no load ($I_2 = 0$), the power supplied to the secondary is zero: $P_2 = 0$
P_1 at no load = Joule losses + Iron losses At no load, a transformer consumes very little current:
Joule losses are therefore negligible compared to Iron losses P_1 at no load ≈ iron losses

Exercise 3

A single-phase transformer has the following characteristics: 230 V / 24 V 50 Hz 63 VA
1- Calculate rated primary current I_{1N} and rated secondary current I_{2N}.
2- When a transformer is energized, a very large inrush current (of the order of 25 I_{1N}) is produced for about ten milliseconds. Evaluate the inrush current.

Answers 3

1- $I_{1N} = S_N/V_{1N} = 63/230 = 0{,}27$ A $I_{2N} = S_N/V_{(2N)} = 63/24 = 2{,}6$ A
2- power-up current $25\times 0.27 = 6{,}8$ A

Exercise 5

A distribution transformer has the following nominal characteristics:
$S_2 = 25$ kVA, $P_{(Joule)} = 700$ W and $P_{(iron)} = 115$ W.
1- Calculate the nominal efficiency for :
- a resistive load
- an inductive load with a power factor of 0.8

2- Calculate the efficiency for : - a resistive load consuming half the rated current.

Corrected 5

1- A resistive load $P_2 = S_2 \cos\varphi_2$ $\cos\varphi_2 = 1$ $P_{2N} = 25000\times 1 = 25$ kW
$P_1 = P_2$ + Joule losses + Iron losses = 25000 + 700 +115 = 25.815 kW; Nominal efficiency: $P_2/P_1 = 96.8\%$.
- an inductive load with a power factor of 0.8;   Efficiency = $(25000\times 0.8)/(25000\times 0.8 + 700 + 115) = 96.1\%$.

2- - A resistive load consuming half the rated current $P(2) = S_2 \cos\varphi_2$; $I(2) = I_{2N}/2$
therefore : $P_2 \approx P_{(2N)}/2 \approx$ 12.5 kW
Joule losses are proportional to the square of currents (Joule's Law) Pjoules = $700\times (1/2)^2 = 175$ W
Yield = (12500x0.5) / [(12500 x 0.5) + 175 + 115] = 97.7%.
Yield = (6250) / [(6250) + 175 + 115] = (6250) / [6540] ≈ 95.6

Exercise 6

A single-phase transformer has the following characteristics:
- nominal primary voltage: $V_{1N} = 5375$ V / 50 Hz - number of turns ratio: $N_2/N_1 = 0.044$
- primary winding resistance: $R_1 = 12\ \Omega$ - secondary winding resistance: $R_2 = 25$ mΩ
- primary leakage inductance: $L_1 = 50$ mH   - secondary leakage inductance: $L_{(2)} = 100$ µH

1- Calculate no-load voltage at secondary.

2- Calculate the secondary winding resistance R_S.

3- Calculate the secondary leakage inductance L_S. Deduct the leakage reactance XS.

The transformer flows into a resistive load $R_c = 1\ \Omega$.
4- Calculate the voltage across the secondary V_2 and the current flowing through the load I_2.

Fixed 6

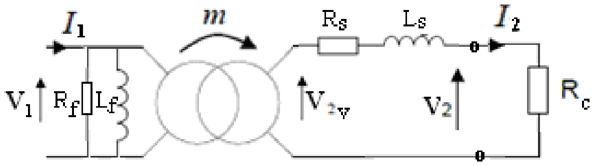
1- Calculate the no-load voltage at the secondary: V20 = $5375\times 0.044 = 236.5$ V

2- $R_S = R_2 + R_1\, m_{(v)^2} = 0.025 + 12\times 0.044^2 = 48.2$ mΩ

3- $L_S = L_2 + L1\, m_{(v)^2} = (100\cdot 10^{-6}) + (50\cdot 10^{-3}\cdot 0.044^2) = 197$ µH $_{X(S)} = L_S\omega = 197\cdot 10^{(-6)}\, 2\cdot\pi\cdot 50 = 61.8$ mΩ

4- Calculate the voltage across the secondary V_2 and the current flowing through the load I_2.

Schéma électrique équivalent : f = 50 Hz



Total complex impedance: **Z** = $(R_S+Rc) + jX_S$ Total impedance: $Z = [(R_S+Rc)^2 + X_S^2]^{1/2}$

Courant au secondaire $I_2 = \dfrac{V_{2vide}}{Z} = \dfrac{V_{2vide}}{\sqrt{(R_s+R)^2+X_s^2}} = 225{,}2\ A$

Ohm's law: $V_2 = Rc \cdot I_2 = 225.2$ volts

Another method:

$\Delta V = V_{2V} - V_2 \approx (R_S.\cos\varphi_2 + X_S.\sin\varphi_2).I_2$ The load is resistive: $\cos\varphi_2 = 1$ ($\sin\varphi v = n0$) Hence$\Delta_{V(2)} \approx {}_{R(S)}.I_2$

On the other hand: $V_2 = Rc.I_2 \rightarrow I_2 \approx {}_{V(2V)} / [(R_S+Rc)^2 + X_{(S)^2}]^{1/2} \approx$ 225.6 A hence : $V_2 \approx$ 225.6 V **Repeat calculation**

Exercise 7

A single-phase control and signalling transformer has the following characteristics:

230 V/ 24 V 50 Hz 630 VA

1- Total losses at rated load are 54.8 W.
Calculate the transformer's rated efficiency for $\cos\varphi_2 = 1$ and $\cos\varphi_2 = 0.3$.

2- Calculate the rated secondary current I_{2N}.

3- No-load losses (iron losses) are 32.4 W. Deduct Joule losses at rated load.
Deduct R_S, the secondary winding resistance.

4- Voltage drop at secondary for $\cos\varphi_2 = 0.6$ (inductive) is 3.5% of rated voltage ($V_2 = 24$ V).
Derive X_S, the secondary leakage reactance.

5- A short circuit occurs 15 metres from the transformer.
Calculate the total resistance R of the cable, knowing that the resistivity of copper is: $\rho = 0.027\ \Omega mm^2/m$.
Deduct the current

Corrected 7

1- $(630\times 1) / [(630\times 1) + 54.8] = 92\%$ $(630\times 0.3) / [(630\times 0.3 + 54.8)] = 77.5$

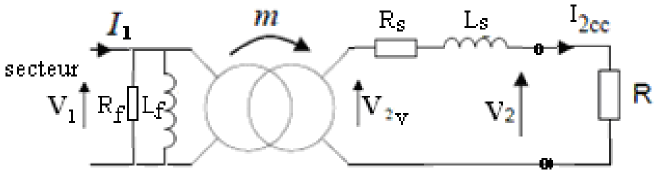
2- Rated secondary current I2N. 630/24 = 26,25 A

3- No-load losses (iron losses) are 32.4 W Power balance: Nominal joule losses: 54.8 -32.4 = 22.4 W
Joule's law: $Rs = {}_{P(jn)} / I_2^2 = 22.4 / 26.25^2 = 32.5\ m\Omega$ $\Leftarrow$ ${}_{P(jn)} =$ $Rs.\ I_2^2$

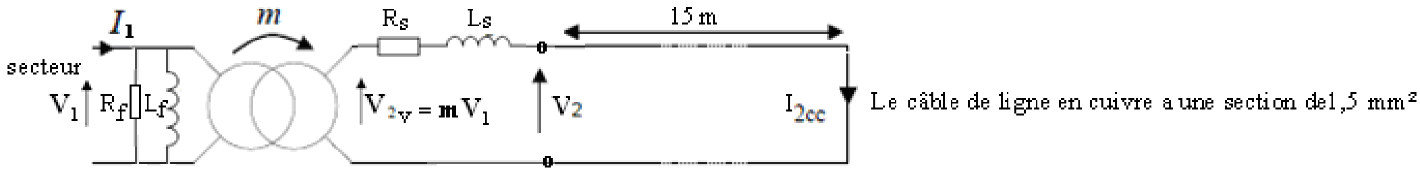
4- Secondary voltage drop:$\Delta V = 0.035\times 24 = 0.84$ V $\Delta V = (R_S.\cos\varphi 2 + X_S.\sin\varphi 2).I_{2N}$
$X_S = (0.84/26.25 - 0.0325\times 0.6) / 0.8 = 15.6\ m\Omega$

5- $R = \rho L/S = 0.027\ 2\times\times 15/1.5 = 540\ m\Omega$

Schéma électrique équivalent



Total complex impedance: $\underline{Z} = (R_S+R) + jX_S$ Total impedance: $Z = [(R_S+R)^2 + X_S^2]^{1/2}$
Short-circuit current : $I_{2cc} = V_2 / R = 24 / 0.54 = 44$ A



Exercise 9
Tests on a single-phase transformer gave :
No-load: $V_1 = 220$ V, 50 Hz (nominal primary voltage); $V_{2v} = 44$ V; $P_{(1v)} = 80$ W; $I_{1v} = 1$ A.
Short-circuit: $V_{1cc} = 40$ V; $P_{1cc} = 250$ W; $I_{2cc} = 100$ A (secondary rated current).
Primary direct current: $I_1 = 10$ A ; $U_1 = 5$ V.
The transformer is considered perfect for currents when these have their nominal values.

1- Determine the no-load transformation ratio m and the number of turns in the secondary, if 500 are counted in the primary.

2- Calculate the primary winding resistance R_1.

3- Check that Joule effect losses can be neglected during the no-load test (to do this, calculate Joule losses at the primary).

4- Assuming that iron losses are proportional to the square of the primary voltage, show that they are negligible in the short-circuit test. Apply numerically.

5- Draw the equivalent diagram of the short-circuit transformer as seen from the secondary.
Deduce the values Rs and Xs characterizing the internal impedance.

Whatever the results obtained previously, for the rest of the problem we'll take Rs = 0.025 Ω and Xs = 0.075 Ω.
The transformer, supplied at its rated voltage on the primary side, delivers100 A on the secondary side with a power factor equal to 0.9 (inductive load).

6- Determine the transformer's secondary voltage. Deduce the power delivered to the secondary.

7- Determine the primary power input (first calculate overall losses).
Deduct the primary power factor and the efficiency.

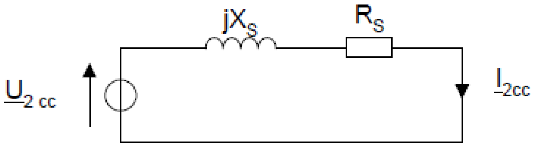
Corrected 9

1- $m = 44 / 220 = 0.2$ $N(2) = 500 \times 0.2 = 100$ turns

2- Primary winding resistance R1: $R_{(1)} = U_{1=} / I_{1=}$ $R_1 = 5 / 10 = 0.5\ \Omega$

3- Joule losses $= R_1.I_1^2 + R_2.I_2^2 = Rs.I_2^2$ **on load**
No-load losses $= R_1.I_{1v}^2 + 0 = R_1.I_{(1v)}^{(2)}$ = Primary losses (because $I_2 = 0$ in the no-load test)
No-load Joule losses $= R_1 I_{(1v)^2} = 0.5 \times 1^2 = 0.5$ W
Therefore: 0.5W << 80 W No-load Joule losses negligible compared with no-load power input
$P_{jv} \approx P_{1v}$ $= 80$ W
Total losses = iron losses + Joule losses≈ iron losses $= V_1^2 / R_{iron}$
hence the no-load test to determine R_{iron} $R_{(iron)} = V_1^2 / P_{(Jv)}$ $R_{iron} \approx 220^2 / 80 = 605\ \Omega$

4- Iron losses = $(40)^2 / 605 = 2.6$ W << 250 W = total losses = iron losses (=0) + joule losses
joule losses = $Rs.I_{2cc}{}^{(2}$
The short-circuit test can therefore be used to determine the equivalent resistance seen from the secondary.
$Rs = \mathbf{P_{1cc} / I_{2cc}}^2 = 250/100^2 = 0.025\ \Omega$



$Zs = mv.V1cc / I_{2cc} = 0.080\ \Omega$

$Z_S = \sqrt{R_S{}^2 + X_S{}^2} \qquad X_S = \sqrt{Z_S{}^2 - R_S{}^2} = 0{,}076\,\Omega$

6-Δ V2 = ($R_S \cos\varphi$ 2 + $X_S \sin\varphi$ 2)I2 = 5.5 V V2 = 44 - 5.5 = 38.5 V P2 = V2 I2 cosφ 2 = 3460 W

7- Overall losses = 80 + 250 = 330 W P1 = 3460 + 330 = 3790 W Efficiency: 3460 / 3790 = 91
P1 = V1 I1 cosφ 1 = V1 mv I2 cosφ 1 Hence: cosφ 1 = 0.86