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Université Aboubakr Belkaïd– Tlemcen –
Faculté de TECHNOLOGIE

**Biomedical Engineering Department**

1st year Common Core in biomedical Engineering

# Tutorials on Electrical Circuits


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## Preface

This handout is a learning guide for first-year Biomedical Engineering students at Abou Bekr Belkaid University, Tlemcen. It covers the fundamental principles of electrical circuits, which are essential for future studies in biomedical engineering.

This document serves as a supplement to the **“Electrical Circuits” courses**, offering additional exercises to enhance understanding. It includes comprehensive course notes, a variety of exercises with detailed solutions, and engaging homework assignments.

We extend our gratitude to **Debbal Sidi Mohammed** and **Habibes Naima** for their careful review and corrections, ensuring the accuracy of this work.

We wish you success in your studies and hope this resource helps you master electrical circuits.

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# Tutorial 1

## **Course Notes:** *Charge, Current, Energy & voltage*

### *Charge, Current*

- Charge, symbolized by Q and measured in coulombs, serves as an attractive force [1].
- Electrons bear a negative charge, while protons are positively charged, both exhibiting a charge magnitude of approximately 1.602E−19 coulombs.
- This implies that one coulomb equals the charge carried by roughly $6.242*10^{18}$ electrons.
- The opposite charges attract but the like charges repel [3].
- Charge can be transferred, and the rate of this movement is termed current, denoted by I in amperes.
- One ampere equals one coulomb per second, or about $6.242*10^{18}$ electrons will pass over a wire in 1 second [2].

### *Current I*

Electrical current I represent the flow of the electric charges in a conductor. The unit is amperes (A).

It means the rate at which the electric charges passes through the conductor.

The formula is:

$$I = {}^{Q}/_{t}$$

*I* : current (A)
*Q* : charge (C)
*t* : time (s)

**Example**

A battery delivers 4 coulombs of charge in half a second. What is the current?

**Solution**

I = Q / t = 4 / 0.5 =4 / 0.5  = 8 A

**Example**

We have a device that delivers 50 mA. Calculate the charge moved in 2 seconds and the total number of electrons involved.

**Solution**

1) Charge : $Q=I * t = 50\ (mA) * 2\ (s) = 100\ mC = 0{,}1\ C$

2) 1 C = $6.242\ 10^{18}$ electrons, so the equivalent electrons = Q * $6.242\ 10^{18}$

Equivalent of electrons =100 * $6.242\ 10^{18}$ =$6.242\ 10^{17}$ electrons

### *Energy & Voltage*

- Energy, denoted as E and measured in joules (J), represents the capacity to perform work.
- Moving a charge, like separating an electron from an atom, requires energy.
- This energy is reflected in the concept of electric potential difference, known as voltage.
- Voltage V is measured in volts (v).

$$V = E/Q$$

V : voltage (v)
E : Energy (J)
Q : Charge (C)

Unlike current, voltage always involves a difference between two points with a reference point like earth ground.

For example, $V_{AB}$ indicates the voltage difference between points A and B.

$$\mathbf{V_{AB}=V_A-V_B}$$

**Example**

A charge of 50 coulombs is moved from the point A to B requiring 150 joules of energy. Calculate the voltage between A and B.

**Solution**

*Since the charge is moved from point A to point B, the potential at B is higher than A.*

As a result, the voltage, which is the difference in potential between the two points, is directed from B to A.

VBA = E / Q = 150J/50C = 3V

***Power P***

Power (P) is the rate at which the energy will be consumed.

$$\boldsymbol{P = {}^{E}\!/_{t}}$$

The formula expresses the relationship between power P (W), energy E (J), and time t (s).

**Example**

A device consumes 100 joules of energy in 0.2 seconds. Calculate its power consumption.

**Solution**

P = E / t = 100J / 0.2s = 500 W

Power can be calculated by multiplying the current flowing through a circuit by the voltage across it.

We have : I = Q / t & V = E / Q ,

so V * I = (E / Q) * (Q / t) = E / t = P

**P =V * I**

**Example**

A 9-volt battery provides a current of 0.5 amperes. Calculate the power it delivers in watts.

**Solution**

P = V * I = 0.5A * 9volts = 4.5 W

## *Homework's Assignment*

**Exercise No. 1**

A sphygmomanometer is powered by a "6V | 1200mAh" battery. The current required for its operation is 80 mA. Calculate:

1. The duration for which the sphygmomanometer will operate.
2. The power of the sphygmomanometer.
3. The energy provided by the battery.

**Exercise No. 2**

A battery powers a muscle stimulator at 54 mW for 25 hours with an operating current of 12 mA. Calculate:

1. The charge of the battery.
2. The supply voltage.
3. The energy of the muscle stimulator.
4. If the operating current is 4 mA, what will be the operating duration?

## *Solution of Homework's Assignment*

**Exercise 1**

According to the statements, we have : The voltage V=6v, the charge Q=1200 mAh, and the electric current is 80 mA.

1) The duration for which the sphygmomanometer will operate is :
   $I = Q/$ , $I$=1200 (mAh)/80 (mA), $I$=15 hours.
2) The power of the sphygmomanometer is:
   $P=V.I$, $P$=6 * 80.10^-3), $P$=0.48 W.
3) Battery energy is:

   V= E/Q , E = V*Q = 6 * 1200 *10^-3 * 3600 = 25920 J
   Or
   P = E/t , E = P * t = 0.48 * 15*3600 = 25920 J

**Exercise 2**

**Based on the statements provided:**

The power P= 54 mW, the time t =25 hours, and the electric current is 12 mA.

1. **Electric Charge:**
   $I = Q / t$ , so Q = I * t = 12 mA * 25 hours =300 mAh
   Q = 300 ($10^{-3}$ * 3600) = 1080 Coulombs

2. **Supply Voltage:**
   P =V * I , so V = P / I ⇒ V = 54 × $10^{-3}$ W / 12×$10^{-3}$ A
   V = 4.5 V

3. **Energy of the Muscle Stimulator:**

*Method 1:*

V=E/Q, so E=V*Q

E=4.5V×1080C ⇒ E=4860Joules

*Method 2:*

P=E/t , so E=P*t

E=54×10−3×25×3600 ⇒ E=4860Joules

4. **Operating Duration if** I=4mA**:**
   I=Q/t , so t =Q/I ⇒ t =(1080C) /(4×10−3)
   t=270000 s
   t=75hours

## Problem 1

A patient is equipped with an implantable cardioverter-defibrillator (ICD). This device uses a 3.7 V, 3000 mAh lithium-ion battery. During a discharge, the ICD delivers 36 J of energy in 5 ms.

Calculate:

1. The current intensity during the discharge.
2. The voltage across the battery during the discharge (assume a constant voltage for simplicity).
3. The theoretical number of discharges the battery can provide, assuming an 80% efficiency for the conversion of chemical energy to electrical energy.
4. Estimate the battery life in years, assuming an average discharge frequency of once per month.
5. Discuss the factors that can influence the actual battery life, in addition to the discharge frequency.

# Tutorial 2

## **Course Notes** *Combining Resistors, Kennelly theorem, Ohm's law, Power balance*

Schematic symbols:

| Voltage source | Current source | Earth ground | Resistance |
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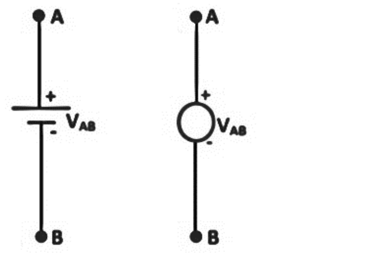
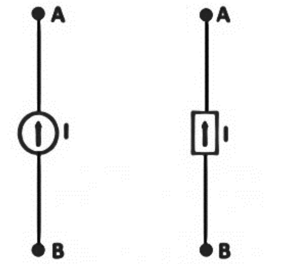
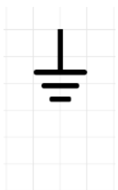
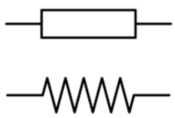
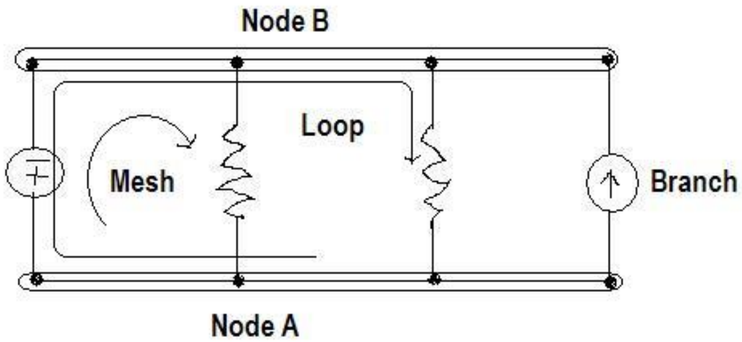
|   |   |  |  |

To analyse electrical circuits, it's crucial to grasp key concepts like mesh, nodes and branches,




**Circuit:** it's a closed path through which electricity flows. It can contains various electrical components like voltage or current sources and resistors. Circuits can be simple or complex, depending on how many components they contain.

**Branch:** it's a single component, like a battery or resistor, with its own terminals.

**Node:** A node is a point where two or more branches connect.

**Loop:** A closed path within a circuit is a loop.

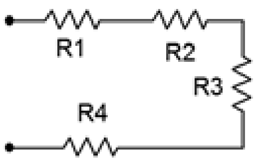
**Mesh:** A loop that does not have any other loops inside. All meshes are loops, but not all loops are meshes.

## 1) Combining Series resistors

The total resistance is found by summing the individual resistors:

$$R_{Total}=R_1+R_2+R_3+\cdots+R_N$$

Several resistors R1, R2, R3 and R4, with values of 12 ohms, 39 ohms, 56 ohms, and 47 ohms. Calculate the total resistance of this combination.




R = R1+R2+R3+R4 = 12+39+56+47 = 154 ohms

## 2) Combining Parallel resistors

The total resistance is found by :

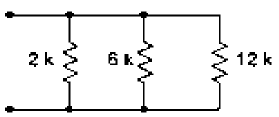
$$1/R_{parallel} = 1/R_1 + 1/R_2 + 1/R_3 + \cdots + 1/R_N$$

$$R_{parallel} = 1 / ( 1/R_1 + 1/R_2 + 1/R_3 + \cdots + 1/R_N )$$

*Simplification by using **the Product-Sum Rule** in case of 2 parallel resistors*

$$R_{Parallel} = 1 / ( 1/R_1 + 1/R_2 ) = R_1R_2 / (R_1+R_2)$$

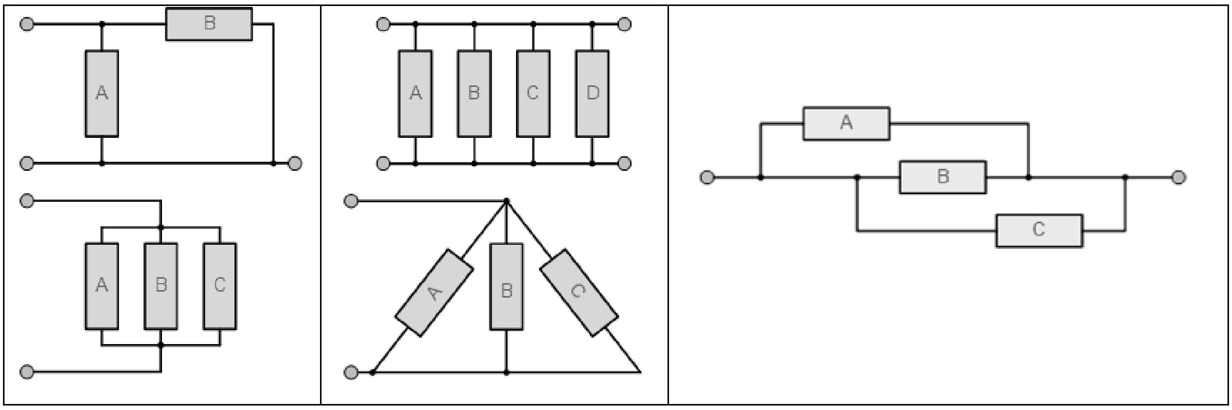
Calculate the R equivalent



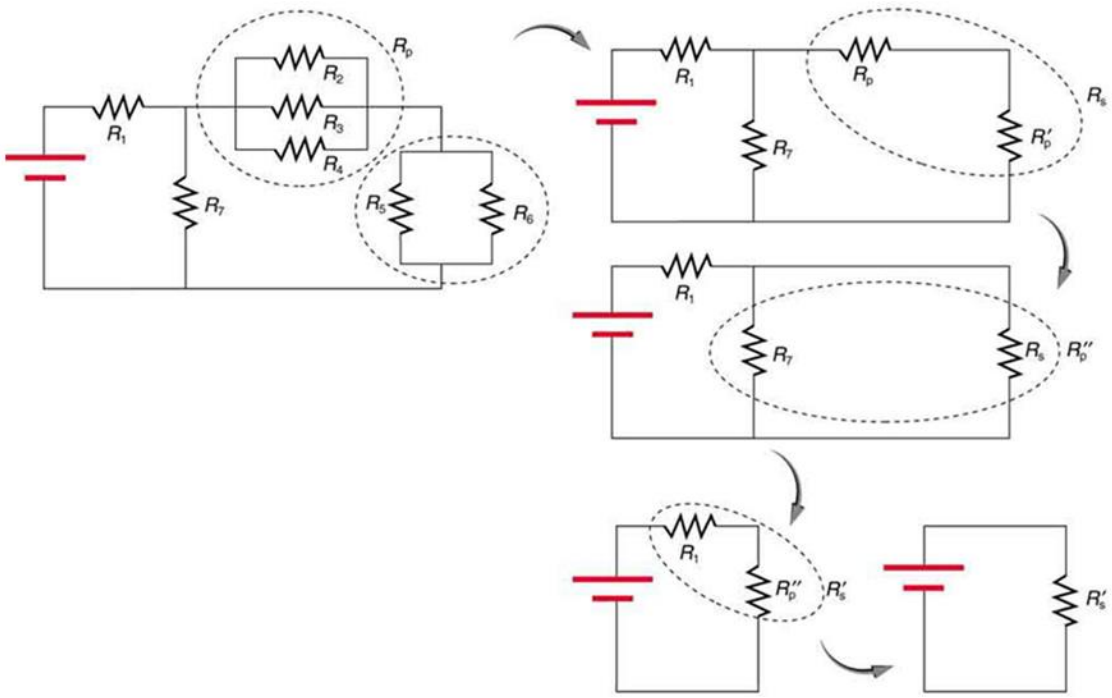

R = 1/(1/2+1/6+1/12) = 1,33k

Or
$R_{23}=R_2R_3/R_2+R_3$=(6*12)/6+12) = 4k
R = $R_1R_{23}/R1+R_{23}$=(4*2)/4+2) = 1,33k

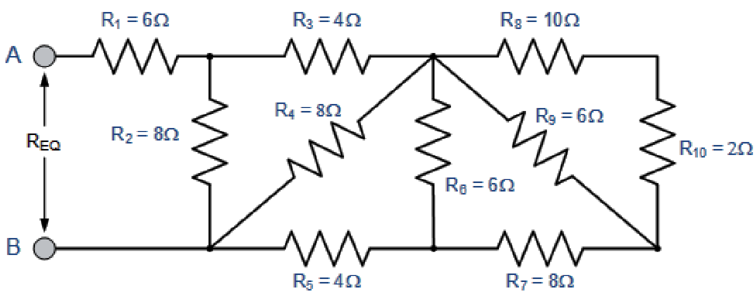
***Examples of Parallel network topology:***




***Example of simplifying a circuit***



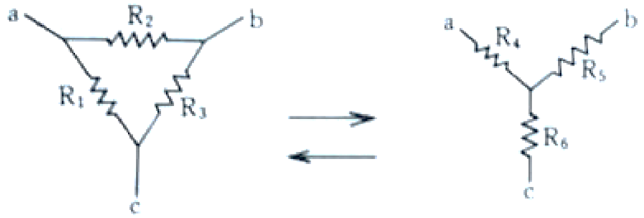

**Homework's Assignment:** Calculate the equivalent resistor




*Result to be found:* ***Req=10Ω***

### 3) Kennelly's theorem

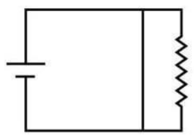
Kennelly's theorem allows us to transform a circuit containing 3 resistors branched in a delta (or triangle Δ) configuration into a new configuration of 3 resistors connected in a star (or Y).



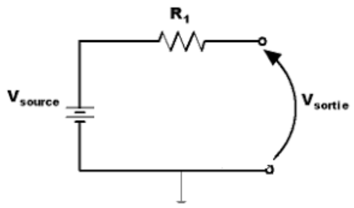

| Delta to Star Transformation (Δ → Y) | Star to Delta Transformation (Y → Δ) |
|---|---|
| • R4 = (R1 * R2) / (R1 + R2 + R3)<br>• R5 = (R2 * R3) / (R1 + R2 + R3)<br>• R6 = (R1 * R3) / (R1 + R2 + R3) | • R1 = (R4 * R5 + R5 * R6 + R6 * R4) / R5<br>• R2 = (R4 * R5 + R5 * R6 + R6 * R4) / R6<br>• R3 = (R4 * R5 + R5 * R6 + R6 * R4) / R4 |

### 4) Open & short circuit

Short-circuit (closed circuit)



Open circuit



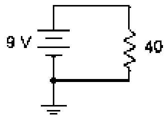

- An open circuit refers to an element with infinite resistance, resulting in the absence of electric current flow within it.
- Conversely, a short circuit represents a conductor with zero resistance, such as a conductive material.
- the case of a short circuit, the voltage across its terminals remains consistently zero

*5) The rule : "Ohm's law"*

**V = R * I**

V: voltage (volts) I: current (amperes) & R: resistance (ohms)

A 40 ohms resistor is supplied by a 9 volts battery.
Calculate current and determine the power in the resistor.




P = V * I

P= 0225 * 9

P= 2,025 Watts

I = V / R

I = 9 / 40

= 0,225 A

**6) Power balance**

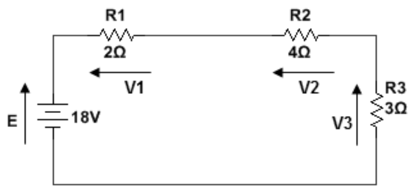
Electrical circuits contain 2 types of components.

- **Generators:** such as batteries, direct current or alternating voltage generators, etc.
- **Receivers:** such as lamps, motors, and resistors.

The principle of power balance states that the **total power supplied** to the circuit must equal the **total power consumed** by the components within it.

$$\sum P_{generators} = \sum P_{receptors}$$

**Example**: We aim to calculate the power distribution in this circuit




A 2A current, induced by the generator, flows through the 3 resistors.
$P_{generator} = V * I = 18 * 2 = 36$ watts
$P_{receptors} = P_{R1} + P_{R2} + P_{R3}$

$P_{R1} = V_1 * I = R_1 * I^2 = 2*4 = 8$watts
$P_{R2} = V_2 * I = R_2 * I^2 = 4*4 = 16$watts
$P_{R3} = V_3 * I = R_3 * I^2 = 3*4 = 12$watts

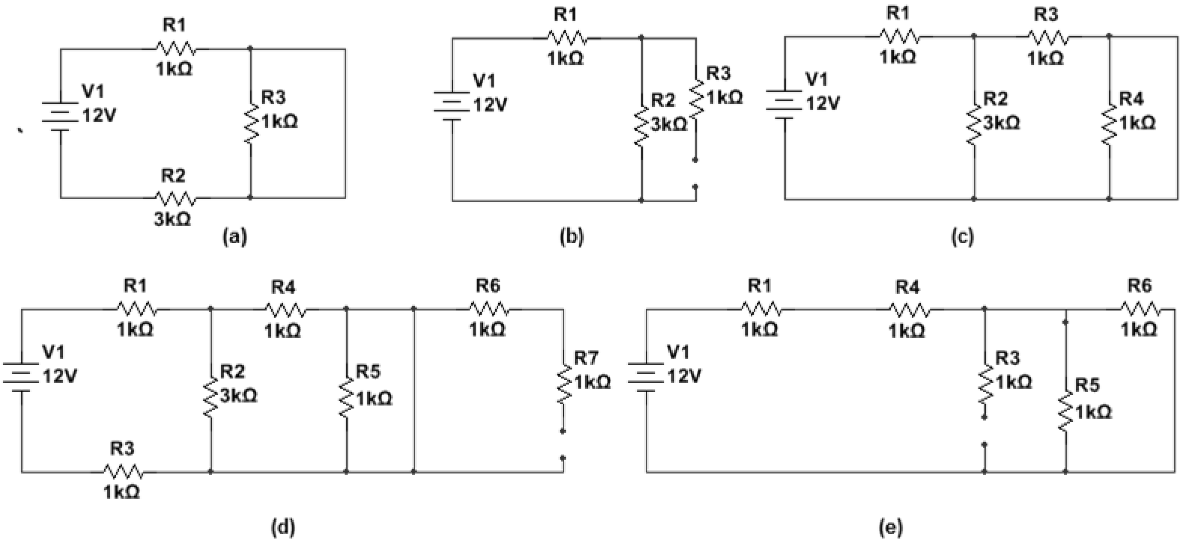
$\sum P_{generators} = \sum P_{receptors}$

## Exercises (Homework's Assignment)

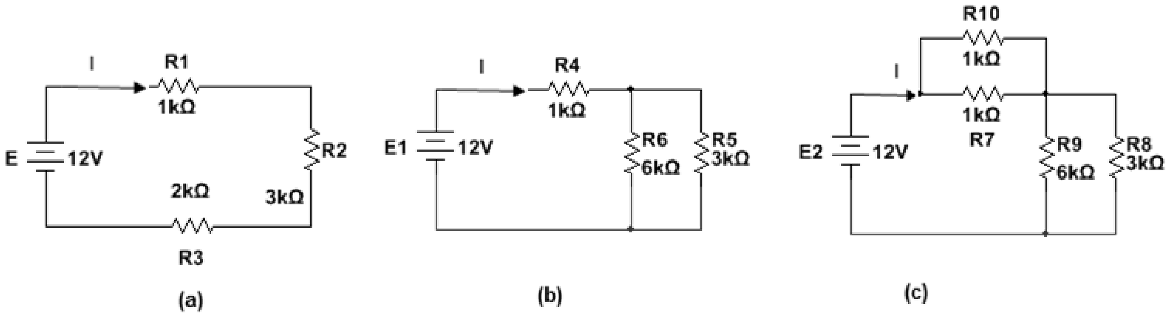
**Method for Simplifying a Circuit (Calculation of Equivalent Resistance)**

a) Remove resistances that are in series with open circuit or in parallel with short circuit.
b) Calculate the equivalent resistance of each branch.
c) Conclude the overall circuit's equivalent resistance.
d) Use of Ohm's Law to find current or voltage of the generator supplying the circuit.

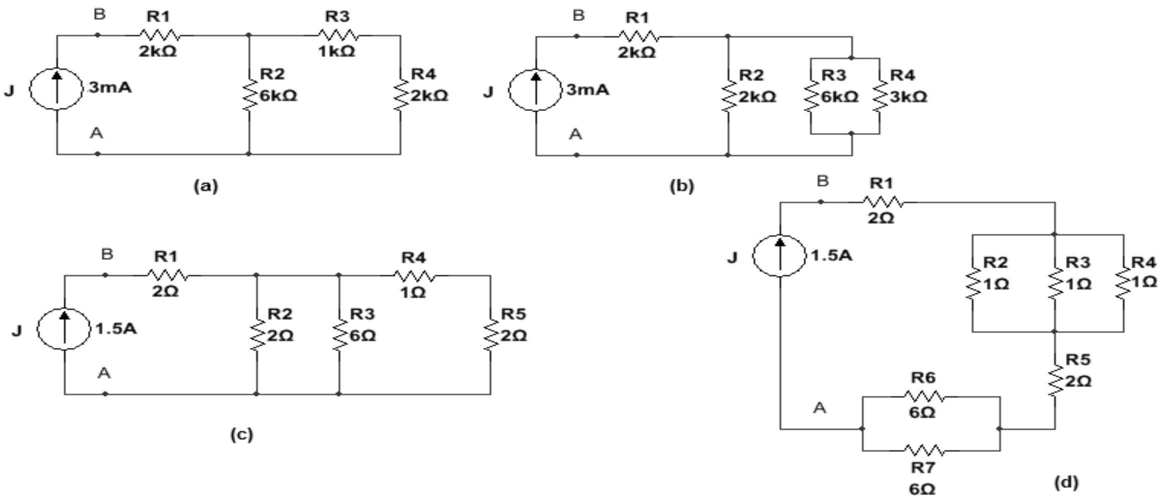
**Exercise 1** Calculate the equivalent resistance



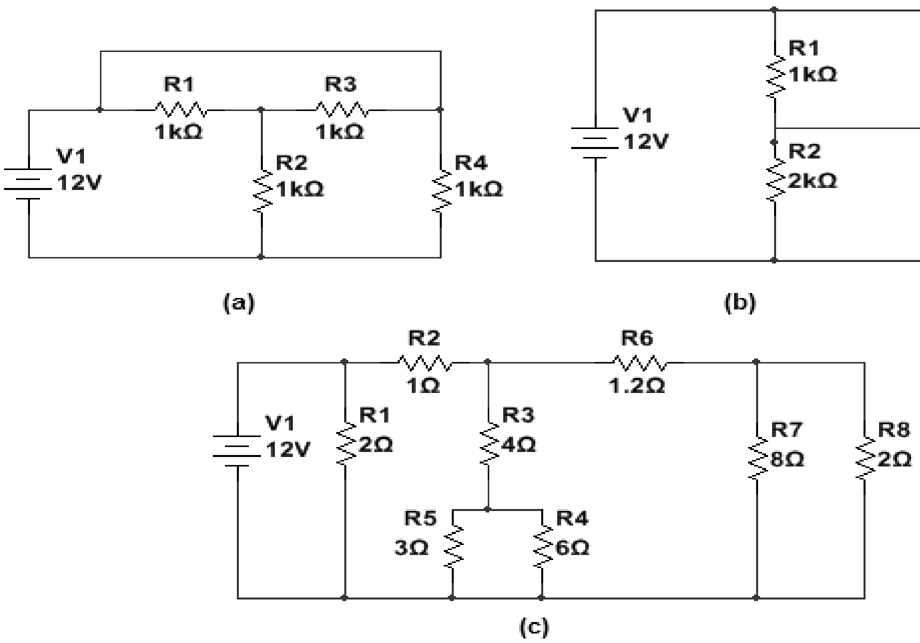

**Exercise 2** Calculate I



**Exercise 3** Calculate the electrical voltage between the two points A and B in each of the circuits

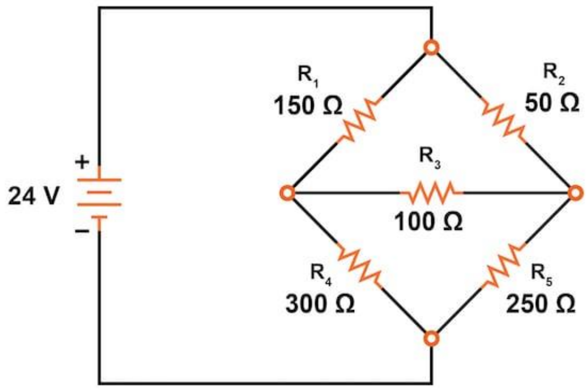



**Exercise 4** Simplify the electrical circuits and then calculate their respective equivalent resistance



Problem 2

Consider the following electrical circuit:



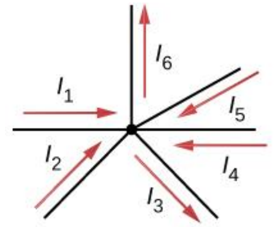

1. Simplify the circuit:
   - Use series and parallel resistor combinations wherever possible.
   - Apply Kennelly's theorem to simplify any delta or star configurations.
   - Redraw the simplified circuit diagram.
2. Calculate the total equivalent resistance of the circuit.
3. Determine the total current flowing through the circuit.
4. Calculate the voltage drop across each resistor.
5. Calculate the power dissipated by each resistor.
6. Verify the power balance principle by showing that the total power supplied by the voltage source equals the sum of the power dissipated by all resistors.
7. If a resistor (e.g., R3) fails open circuit, how will it affect the current flowing through the other resistors? Explain your reasoning.
8. If a resistor (e.g., R2) fails short circuit, how will it affect the voltage across the other resistors? Explain your reasoning.

# Tutorial 3

## Course Notes *Kirchhoff's Law*

***I) Kirchhoff's Current Law KCL (Junction Rule or Node rule):***

Currents entering a junction in a circuit is equal to currents leaving that junction.

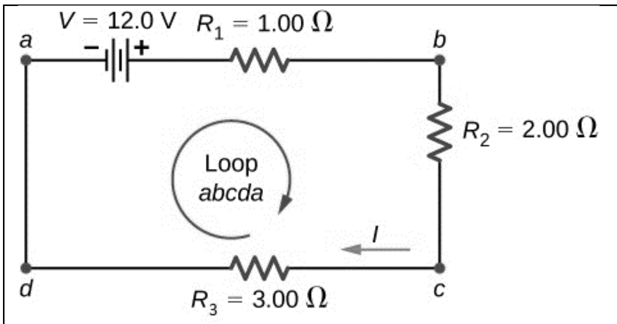



$$\sum I_{in} = \sum I_{out}$$
$$I_1 + I_2 + I_4 + I_5 = I_3 + I_6$$

***II) Kirchhoff's Voltage Law KVL (Loop Rule or Mesh rule):***

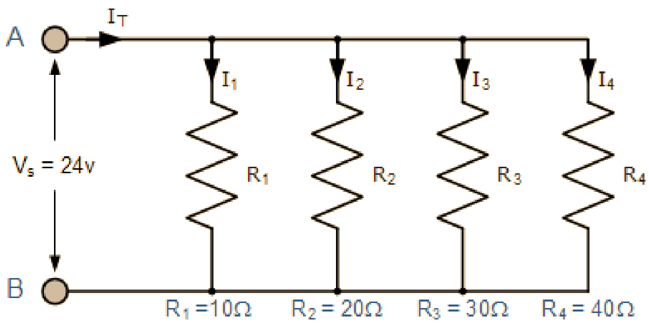
The summation of voltages around a mesh or loop is equal to 0.

$$\sum \mathbf{V = 0.}$$

| | |
|---|---|
|   | $V - V1 - V2 - V3 = 0$<br>$V - R_1 * I - R_2 * I - R_3 * I = 0$<br><br>$I = V / (R_1+R_2+R_3)$<br>$I = 12 / (1+2+3)$<br>$I = 2$ A. |

**Example 1 : (KCL)**

For a network of resistors connected in parallel, find all the currents in the circuits.



$$I_1 = \frac{V_S}{R_1} = \frac{24V}{10\Omega} = 2.4\text{amps} \qquad I_3 = \frac{V_S}{R_3} = \frac{24V}{30\Omega} = 0.8\text{amps} \qquad I_T = I_1 + I_2 + I_3 + I_4$$

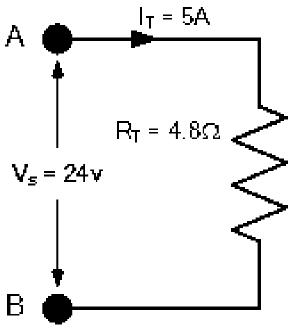
$$I_T = 2.4 + 1.2 + 0.8 + 0.6$$

$$I_2 = \frac{V_S}{R_2} = \frac{24V}{20\Omega} = 1.2\text{amps} \qquad I_4 = \frac{V_S}{R_4} = \frac{24V}{40\Omega} = 0.6\text{amps} \qquad I_T = 5.0\text{ Amps}$$

$I_T$ can also be calculated by :

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad \frac{1}{R_T} = 0.1 + 0.05 + 0.033 + 0.025$$

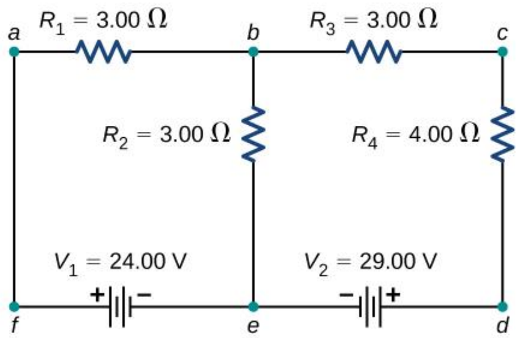
$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40} \qquad \therefore R_T = \frac{1}{0.2083} = 4.8\Omega$$



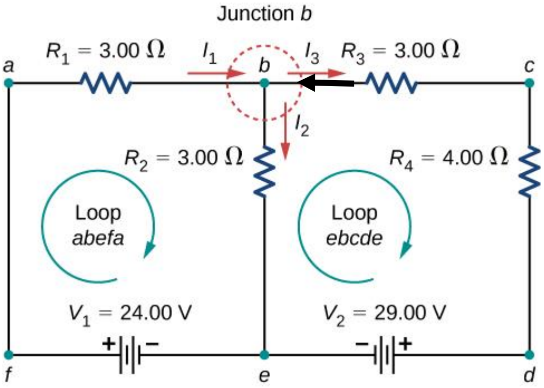

$$I_T = \frac{V_S}{R_T} = \frac{24}{4.8} = 5\text{ Amps}$$

**Example 2**

Find all currents in this circuit.



**Solution**




Junction **b** : I2=I1+I3

**I1=? I2=? I3=?**

Mesh or Loop abefa : −I1*R1 − I2*R2 + V1=0, so V1 = I1*R1 + I2*R2.

Loop ebcde : I2*R2 + I3*(R3+R4) − V2 = 0, so V2 = I2*R2 + I3*(R3+R4).

$$\begin{cases} V1 = I1*R1 + I2*R2. & eq1 \\ V2 = I2*R2 + I3*(R3+R4). & eq2 \end{cases}$$

| Node b: I2 = I1 + I3 , we replace in eq1 to obtain:<br><br>V1 = I1*R1 + (I1+I3)*R2. = (R1+R2)*I1 + R2*I3<br><br>so: 6I1 + 3I3 = 24 | Now, eq1 – eq2 to obtain:<br><br>V1 - V2 = I1*R1 - I3*(R3+R4),<br><br>so: 3I1-7I3=−5 |
|---|---|

$$\begin{cases} 6I1 + 3I3 = 24 & eq1 \\ 3I1-7I3=-5 & eq2 \end{cases}$$

Eq1 – 2*(eq2) : 6I1 + 3I3 - 6I1 + 14I3 = 24 + 10

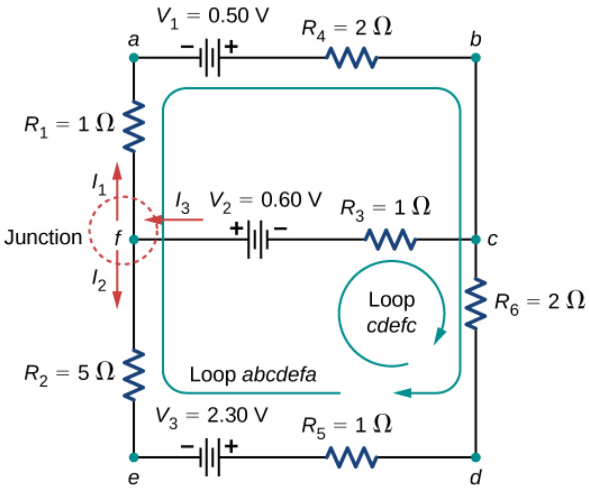
17I3 = 34 , **I3= 2A**

Eq1 : 6I1 + 6 = 24 , **I1 = 3A**

Finally: I2 = I1 + I3= 2+3, **I2 = 5A**

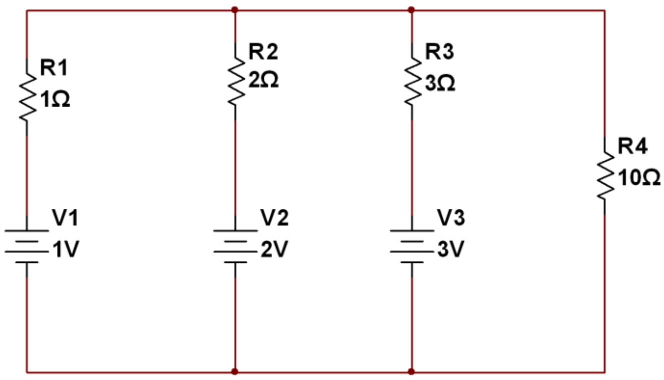
***Homework's Assignment***

Find the currents flowing in the circuit




Final Solution: ***I1=0,2A I2=0,3A I3=0,5A***

# Problem 3




1. **Apply Kirchhoff's Current Law (KCL)** at all the nodes in the circuit. Write down the KCL equations for each node.
2. **Apply Kirchhoff's Voltage Law (KVL)** to all the independent loops in the circuit. Write down the KVL equations for each loop.
3. **Using the KCL and KVL equations obtained in steps 1 and 2, set up a system of linear equations.**
4. **Solve the system of linear equations** to determine the current flowing through each resistor.
5. **Verify the power balance principle:**
    - Calculate the power supplied by each voltage source.
    - Calculate the power dissipated by each resistor.
    - Show that the sum of the powers supplied equals the sum of the powers dissipated.

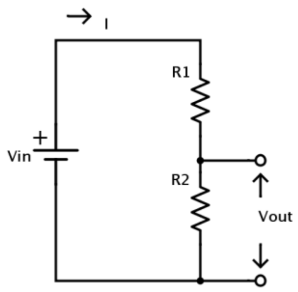
# Tutorial 4

## **Course Notes** Voltage & Current divider

### I) Voltage Divider

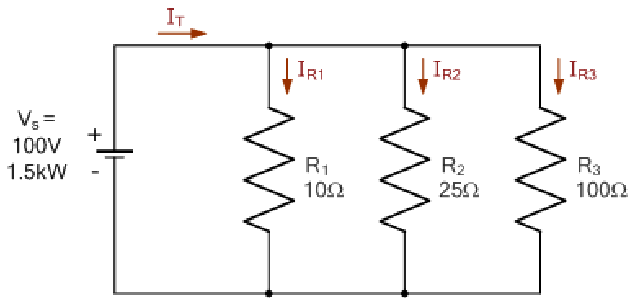
Voltage divider is a configuration of resistors connected in series that divides a voltage into smaller voltages. The voltage in each resistor is found by its resistance over the total resistance of the circuit.

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{in}$$



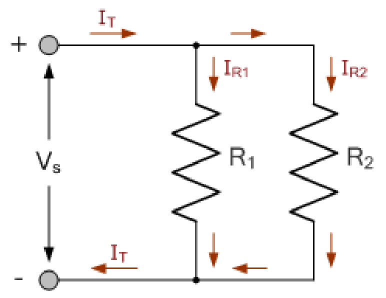

### II) Current Divider

Current divider circuits have multiple parallel branches that allow current to flow through different paths, while maintaining the same voltage across all parallel components.



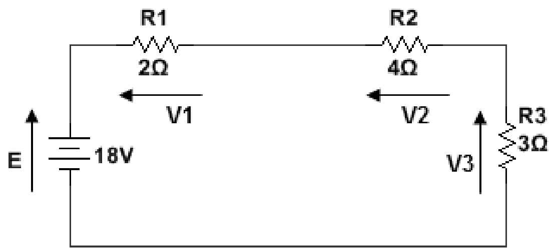

| | | |
|---|---|---|
| $P = V_S \times I_T$ <br> $I_T = \frac{P}{V} = \frac{1500}{100} = 15\,\text{Amps}$ | $R_{EQ} = \left[\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}\right]$ <br> $R_{EQ} = \left[\frac{1}{\frac{1}{10}+\frac{1}{25}+\frac{1}{100}}\right]$ <br> $R_{EQ} = \frac{1}{0.15} = 6.667\,\Omega$ | $I_{R1} = I_T\left(\frac{R_{EQ}}{R_1}\right) = 15\left(\frac{6.667}{10}\right) = 10\,\text{Amps}$ <br> $I_{R2} = I_T\left(\frac{R_{EQ}}{R_2}\right) = 15\left(\frac{6.667}{25}\right) = 4\,\text{Amps}$ <br> $I_{R3} = I_T\left(\frac{R_{EQ}}{R_3}\right) = 15\left(\frac{6.667}{100}\right) = 1\,\text{Amps}$ |

Example of the case of 2 parallel resistors:

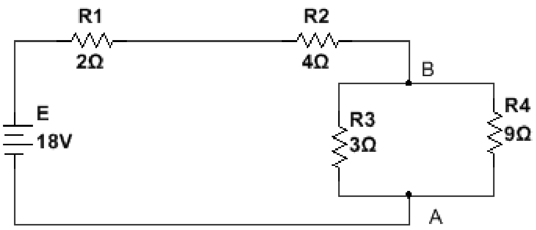
| | |
|---|---|
|  | $I_{R1} = I_T\left(\frac{R_2}{R_1 + R_2}\right)$<br><br>$I_{R2} = I_T\left(\frac{R_1}{R_1 + R_2}\right)$ |

*HomeWork's Assignment:*

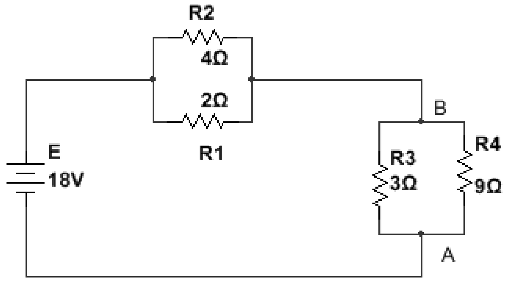
**Exercice 1** Calculate voltages across each resistors using voltage divider rule



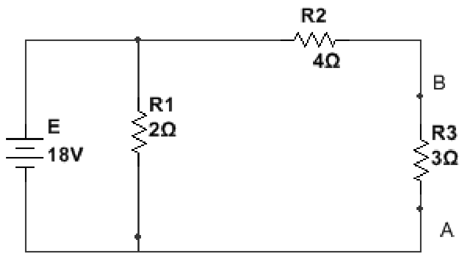

**Exercise 2** Calculate the voltage between A & B using voltage divider rule




(a)




(b)

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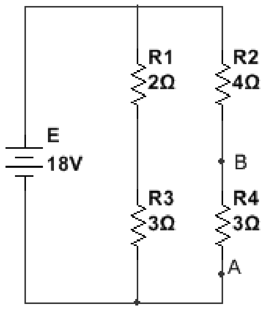


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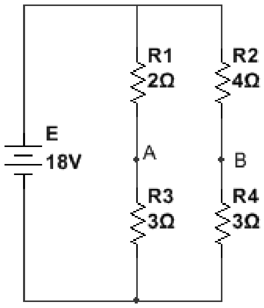


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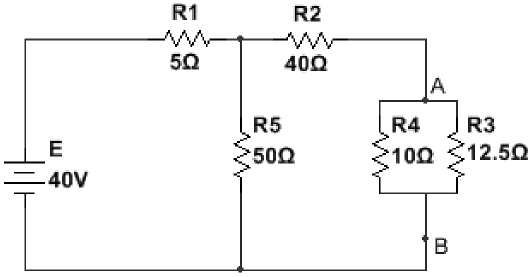
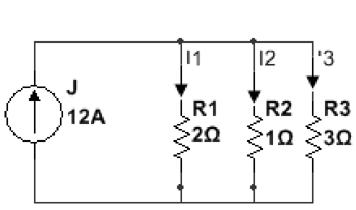
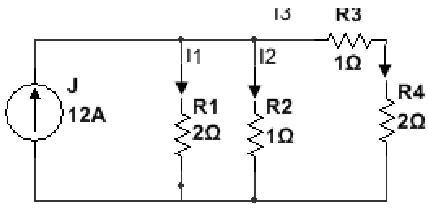



(c)

**Exercise 3** Using Current divider rule, calculate currents I1, I2, I3 & I4




(a)

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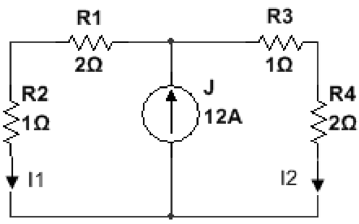


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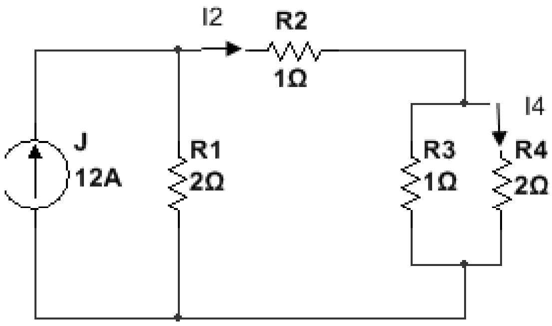
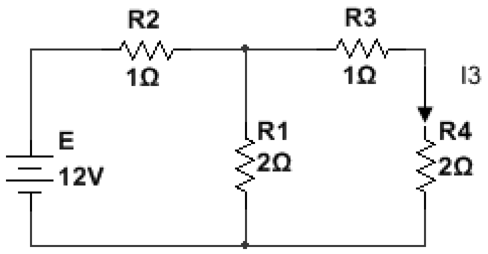


## Problem 4




1. **Calculate the total resistance (Req) of the series circuit.**
2. **Using the Voltage Divider Rule, calculate the voltage drop across each resistor (V3, V4) & (V1, V2).**
3. **Calculate the total current (I) flowing through the circuit.**
4. **Now, assume that a fourth resistor (R5 = 1 Ω) is connected in parallel with R4.**
   - Calculate the equivalent resistance of the parallel combination of R3, R4 and R5.
   - Calculate the new total resistance of the circuit.
   - Calculate the new total current (I') flowing through the circuit.

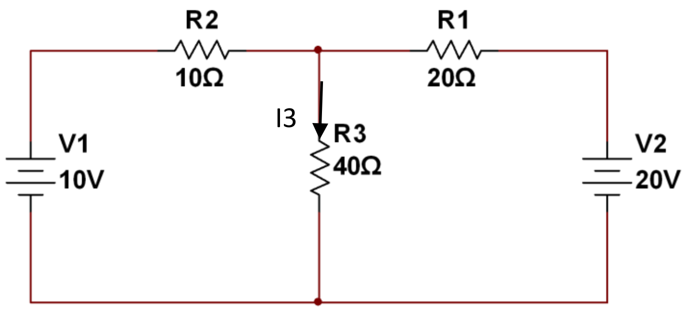
5. **Using the Current Divider Rule, calculate the current flowing through R4 and R5 in the modified circuit.**

# Tutorial 5

## **Course Notes** Superposition theorem

Superposition Theorem states that any the total response (current or voltage) at a point in a circuit can be found by the calculation of the sum of the responses caused by each source acting individually while the others are considered inactive [3]

**Example:** find current I3 flowing via R3 using superposition theorem




Superposition theorem:

$$I_{R3} = I_{R3(1)} + I_{R3(2)}$$

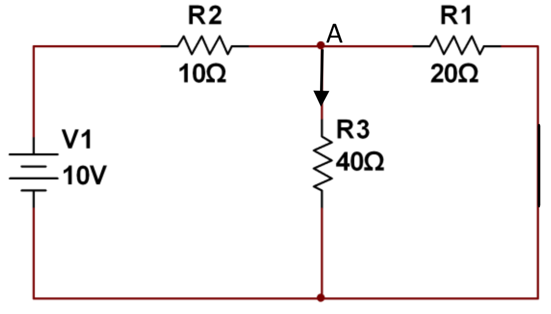
$I_{R3(1)}$ : the current flowing through $R_3$ as a result of $V_1$ acting alone in the circuit

$I_{R3(2)}$ : the current flowing through $R_3$ as a result of $V_2$ acting alone in the circuit

**Solution**

**Current $I_{R3(1)}$**

First, we change V2 by short-circuit so that V1 will acting alone on the circuit




20Ω Resistor in Parallel with the 40Ω Resistor

$$R_{EQ1} = \frac{R_2 \times R_3}{R_2 + R_3} = \frac{20 \times 40}{20 + 40} = 13.33\Omega$$

Using voltage divider, we calculate $V_A$ at the node A

$$V_A = V_1\left(\frac{R_{EQ1}}{R_1 + R_{EQ1}}\right) = 10\left(\frac{13.33}{10+13.33}\right) = 5.71\,\text{volts}$$

Then ,

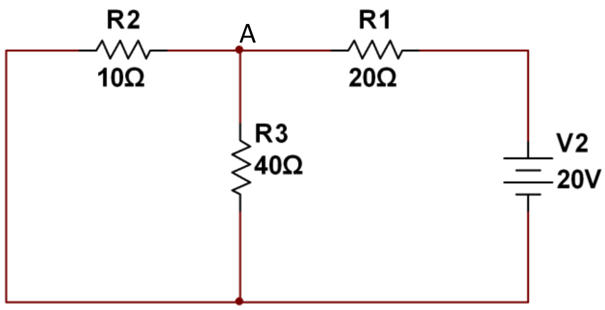
$I_{R3(1)} = V_A/R_3 = 5.71 / 40 = 0.143$ A = 143 mA

So, the current flowing through $R_3$ as a result of $V_1$ acting alone in the circuit is **143mA**.

## Current $I_{R3(2)}$

We Use the same method as before to calculate the current flowing through $R_3$ being driven by only voltage $V_2$.

$V_1$ is replaced by a short circuit. $V_2$ is acting alone in the circuit.




10Ω Resistor in Parallel with the 40Ω Resistor

$$R_{EQ2} = \frac{R_1 \times R_3}{R_1 + R_3} = \frac{10 \times 40}{10 + 40} = 8\Omega$$

Using voltage divider, we find voltage at node A as a result of $V_2$.

$$V_A = V_2\left(\frac{R_{EQ2}}{R_2 + R_{EQ2}}\right) = 20\left(\frac{8}{20+8}\right) = 5.71\,\text{volts}$$

Then,

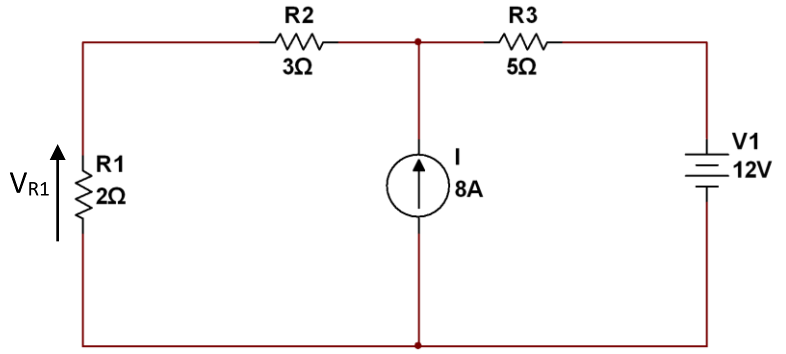
$I_{R3(2)} = V_A / R_3 = 5.71 / 40 = 0.143$ A = 143 mA

So, the current flowing through $R_3$ as a result of $V_2$ acting alone in the circuit is **143mA**.

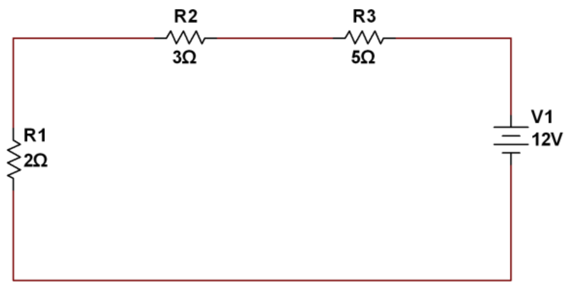
Now to find the final value and direction of $I_{R3}$ we calculate the sum of $I_{R3(1)}$ and $I_{R3(2)}$ :

$I_{R3} = I_{R3(1)} + I_{R3(2)} = 0.143 + 0.143 = 0.286 =$ **286mA**

Example2: Using superposition theorem, find $V_{R1}$



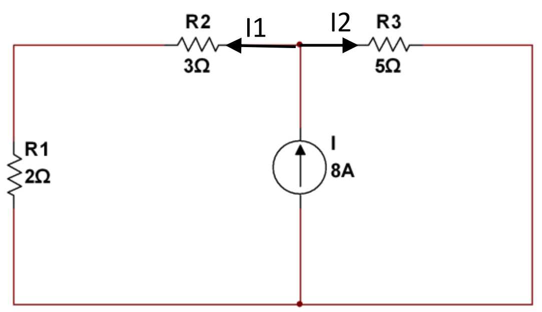

Superposition theorem: $V_{R1} = V_{R1(1)} + V_{R1(2)}$




I1 = V1/(R1+R2+R3) =20/10 = 2A

The individual voltage drop across resistor $R_1$ is calculated as being:

$V_{R1(1)} = I_1 \times R_1 = 2 \times 2 =$ **4 volts**




**I1 = ( R3 / R3+R1+R2 ) * I**
**= 4 A**

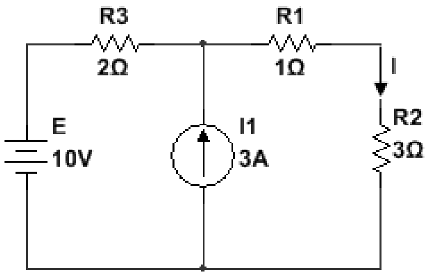
Voltage across $R_1$ :

$V_{R1(2)} = I_1 \times R_1 = 4 \times 2 =$ **8 volts**

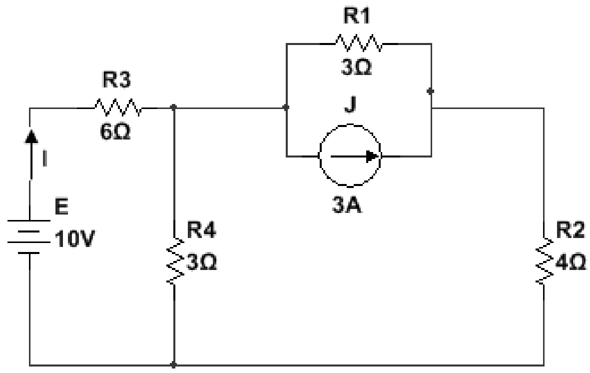
Finally, $V_{R1} = V_{R1(1)} + V_{R1(2)} = 4V + 8V =$ **12 Volts**

**Exercises (Homework's Assignment)**

**Exercise 1** Calculate I using superposition theorem in each of the following circuits.




(a)

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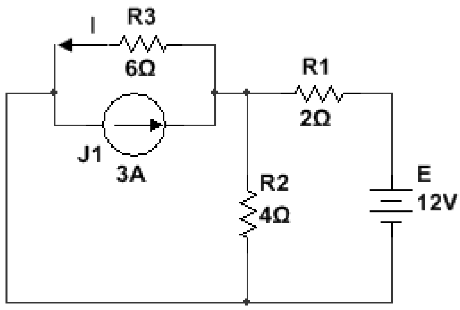


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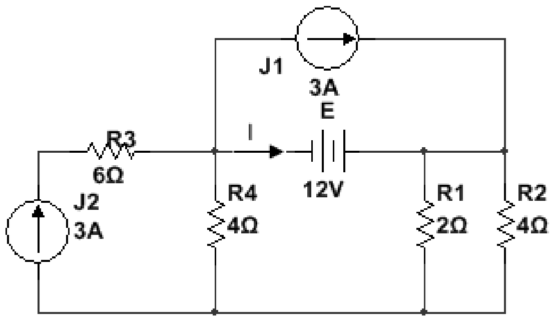
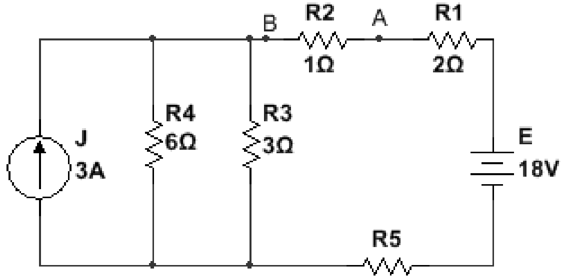


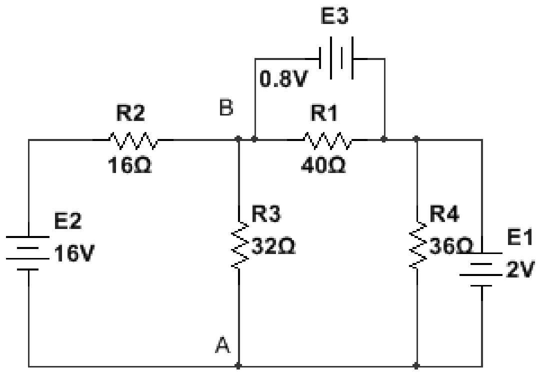

(d)

## Problem 5

Calculate voltage between points A & B using superposition theorem in each of the following circuits




(a)

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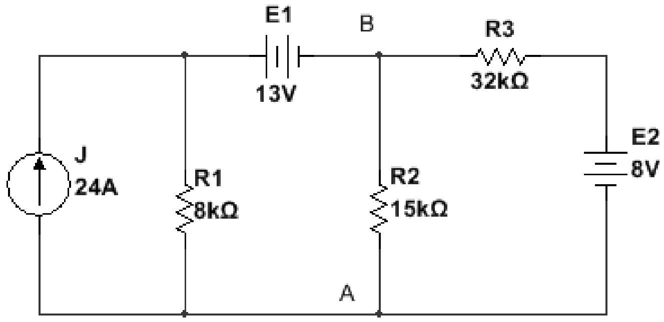


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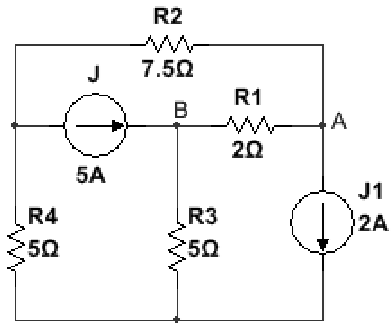


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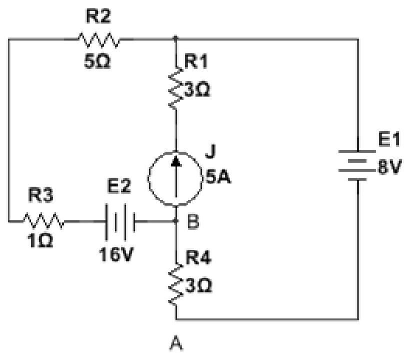
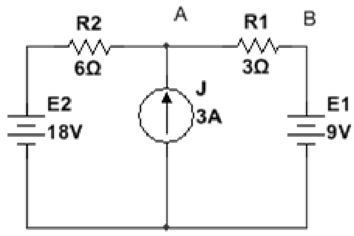
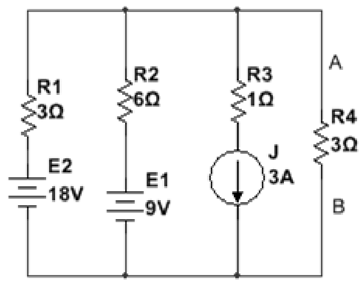



**e**




**f**



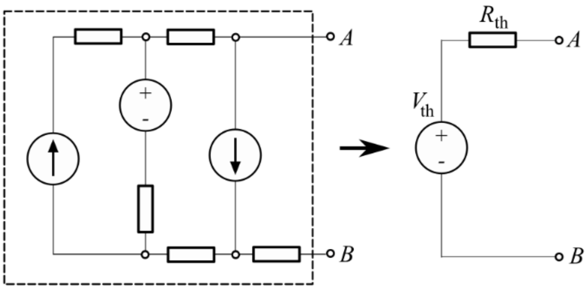

**G**

# Tutorial 6

## **Course Notes** Thevenin theorem

Thevenin Theorem states that a complex circuit can be simplified into a circuit with a single voltage source and a single resistor connected in series. This equivalent circuit will produce the same voltage and current at its terminals as the original circuit [1].

***Example***

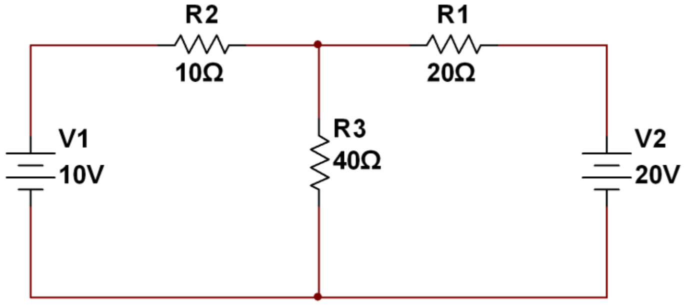



A complex network of resistors and voltage sources can be simplified into only one resistor (Rth) and one voltage source (Vth), with :

- $R_{th}$ is the equivalent resistance between A and B.
- $V_{th}$ is the voltage measured between A and B when there's no load connected.

**Example**

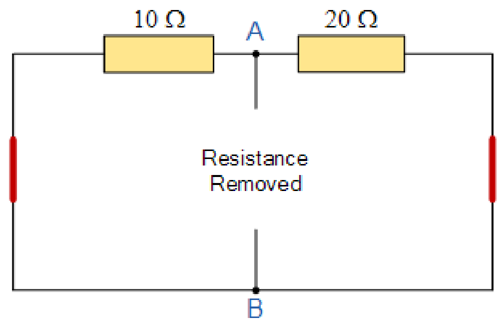
Find current flowing through the 40 ohm resistor using thevenin theorem.



**Solution**

- First, we start by excluding the middle load resistor between points A and B, and eliminate the power supply sources.

- Then, Calculate the equivalent resistance Rth between A and B.



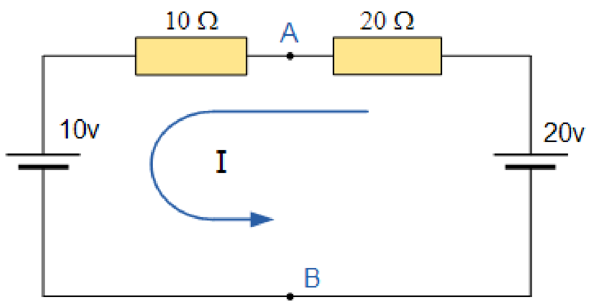

To find the thevenin resistor Rth:

10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$

- The voltage Vth is calculated by measuring the voltage between terminals A and B when the load resistor RL is disconnected.

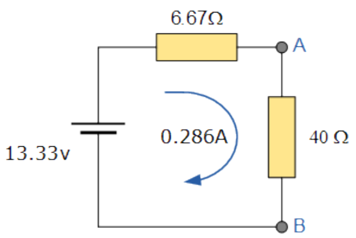
$$I = \frac{V}{R} = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\ amps$$




Mesh law :

$V_{th} = V_{AB} =$ V2 – R1 * I = 20 – 20 x 0.33 = 13.33 volts.

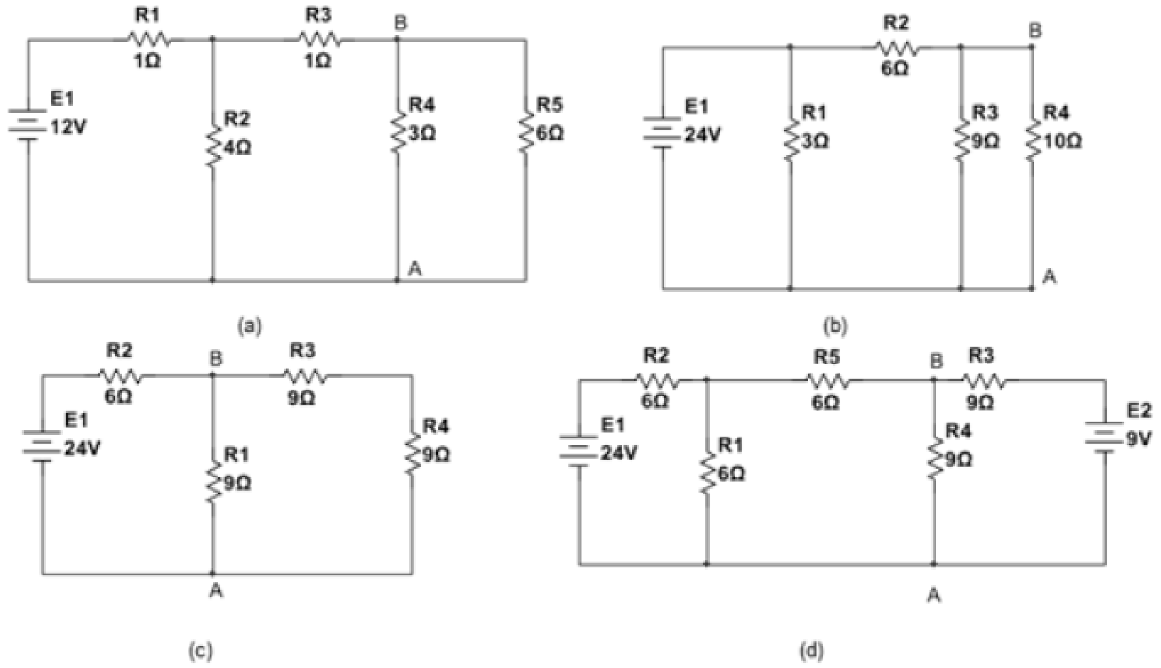
The Thevenin equivalent circuit comprises a 6.67 ohms resistor connected in series with a 13.33-volt voltage source. If we reattach the 40-ohm resistor to this equivalent circuit, we have:



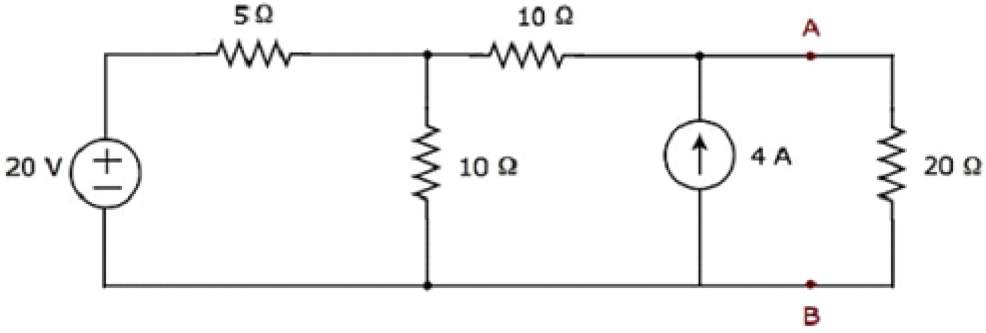

$$I = \frac{V}{R} = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286\ amps$$

## Exercises Homework's Assignment

**Exercise 1** Calculate voltage across R4 using Thevenin theorem in each of the following circuits.



## Problem 6




**Find the Thevenin equivalent voltage (Vth) across terminals A and B.**

- Remove the load resistor (RL) from the circuit.
- Calculate the open-circuit voltage between terminals A and B using appropriate circuit analysis techniques (e.g., nodal analysis, mesh analysis).

**Find the Thevenin equivalent resistance (Rth) across terminals A and B.**

- Replace all voltage sources with short circuits.
- Replace all current sources with open circuits.
- Calculate the equivalent resistance between terminals A and B.

**Draw the Thevenin equivalent circuit.**

- Represent the original circuit with a single voltage source (Vth) in series with a single resistor (Rth).

**Reconnect the load resistor (RL) to the Thevenin equivalent circuit.**

- Calculate the current flowing through the load resistor (RL) using Ohm's Law.

**Compare the current through RL calculated in step 4 with the current through RL in the original circuit.**

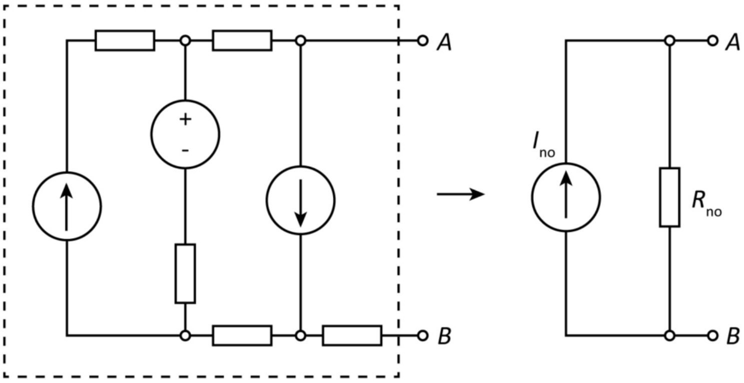
- Verify that both methods yield the same result.

# Tutorial 7

## **Course Notes:** Norton theorem

Norton's Theorem is a method for simplifying complex circuits.

A circuit with multiple sources will be replaced by only one current source and one parallel resistor [2].

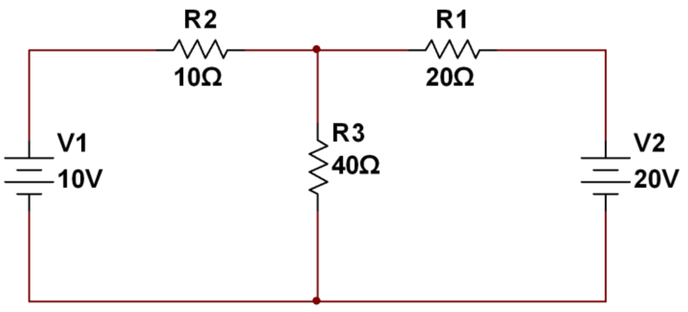



$I_{no}$ : current flowing between terminals A and B if they are shorted.

$R_N$ the Norton resistance is the resistance measured looking into the circuit from its terminals A and B (similar to Thevenin).

**Example**

Calculate current flowing through the 40 ohm resistor using thevenin theorem.

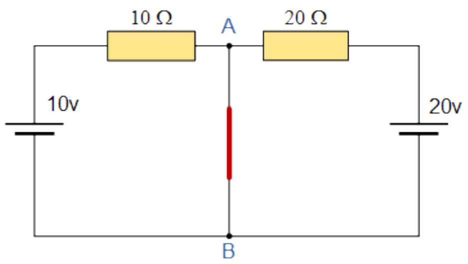


*Calculation of* ***I****N*

Remove the 40 ohm load resistor and connect terminals A and B directly together.

$$I_1 = \frac{10v}{10\Omega} = 1amp, \quad I_2 = \frac{20v}{20\Omega} = 1amp$$

$$\text{therefore,} \quad I_{\text{short-circuit}} = I_1 + I_2 = 2amps$$

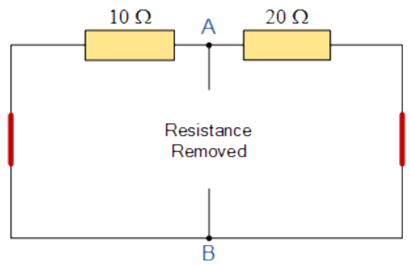



*Calculation of* ***R****N*

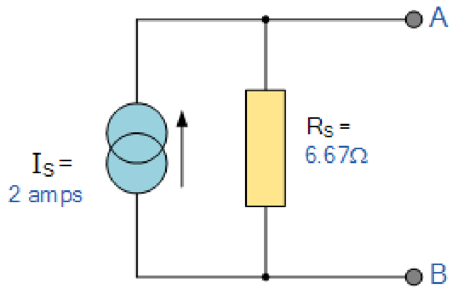
Short circuit the power sources & remove the 40 ohm resistor to open-circuit terminals A and B

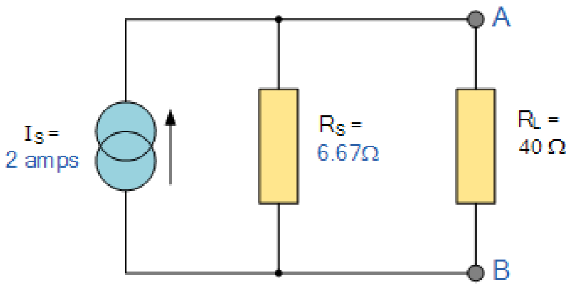
10Ω Resistor in Parallel with the 20Ω Resistor

$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{20 \times 10}{20 + 10} = 6.67\Omega$$




Norton Theorem Equivalent Circuit :




$$R_T = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{6.67 \times 40}{6.67 + 40} = 5.72\Omega$$

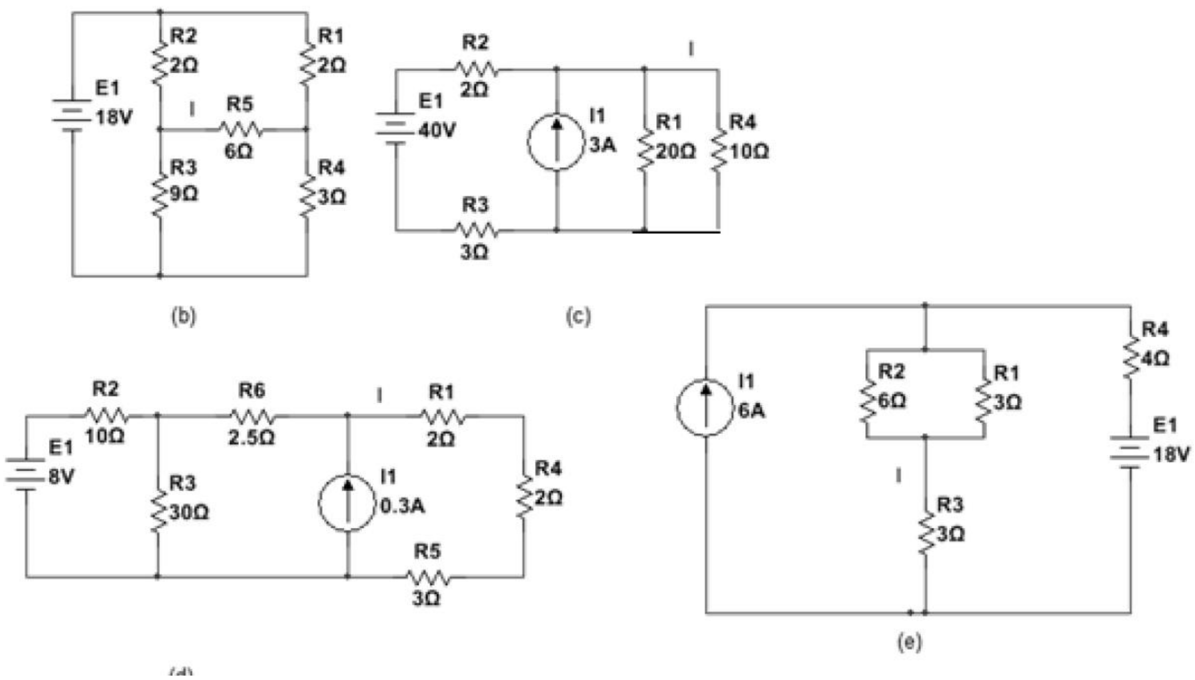
$$V_{A\text{-}B} = I \times R = 2 \times 5.72 = 11.44v$$

So,

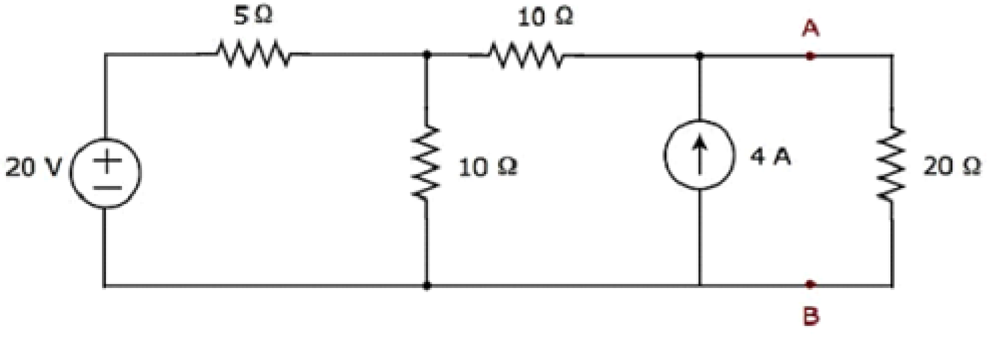
$$I = \frac{V}{R} = \frac{11.44}{40} = 0.286 \text{ amps}$$

**Exercises (Homework's Assignment)**

1) Calculate current I using Norton equivalent circuit in each of the following circuits



## Problem 7




1. **Find the Norton equivalent current (IN) across terminals A and B.**
   - Remove the load resistor (RL) from the circuit.
   - Calculate the short-circuit current flowing between terminals A and B.
2. **Find the Norton equivalent resistance (RN) across terminals A and B.**
   - Replace all voltage sources with short circuits and current sources with open circuits.
   - Calculate the equivalent resistance between terminals A and B.
3. **Draw the Norton equivalent circuit.**
   - Represent the original circuit with a single current source (IN) in parallel with a single resistor (RN).
4. **Reconnect the load resistor (RL) to the Norton equivalent circuit.**
   - Calculate the current flowing through the load resistor (RL) using current divider rule.
5. **Compare the current through RL calculated in step 4 with the current through RL in the original circuit.**
   - Verify that both methods yield the same result.

# Tutorial 8

## **Course Notes:** Millman Theorem

Millman's Theorem states that in a circuit with many parallel branches containing voltage sources, the equivalent voltage across the terminals is the sum of each branch's voltage divided by the sum of the reciprocals of the branch impedances [3].

This method can only be used for circuits that can be rearranged into this parallel branch structure.

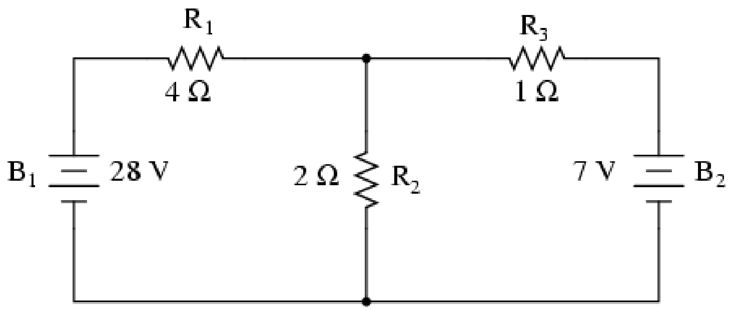
### Equivalent Voltage (Veq):

$V_{eq} = (\Sigma (V_i / R_i)) / (\Sigma (1 / R_i))$

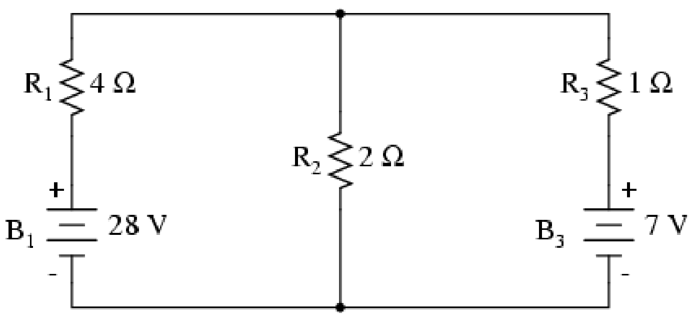
where:

- $V_i$ is the voltage of the ith branch
- $R_i$ is the resistance of the ith branch

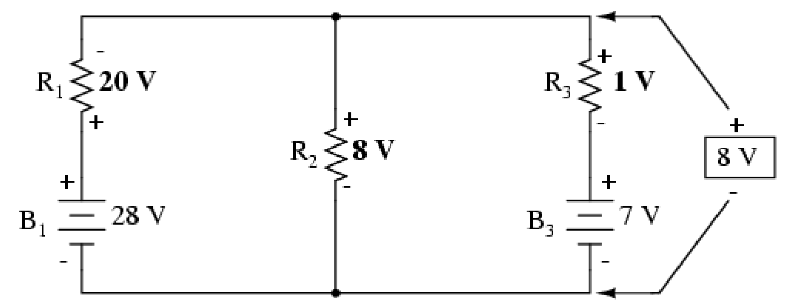
**Example**



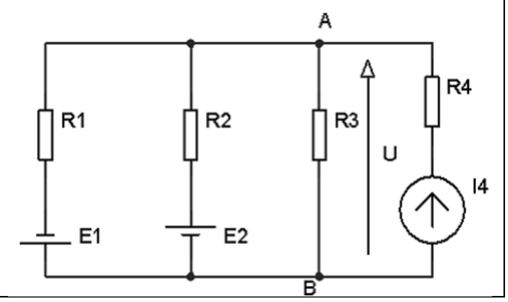

**Solution**

| | |
|---|---|
| Let's redraw the circuit in a way that shows the parallel branches more clearly. |   |

| | |
|---|---|
| *Millman's Theorem Equation* <br> $\frac{\frac{E_{B1}}{R_1}+\frac{E_{B2}}{R_2}+\frac{E_{B3}}{R_3}}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$ = Voltage across all branches | $\frac{\frac{28\ V}{4\ \Omega}+\frac{0\ V}{2\ \Omega}+\frac{7\ V}{1\ \Omega}}{\frac{1}{4\ \Omega}+\frac{1}{2\ \Omega}+\frac{1}{1\ \Omega}} = 8\ V$ |

| | |
|---|---|
| The value "8 volts" is the voltage seen across all parallel branches, like this: |   |

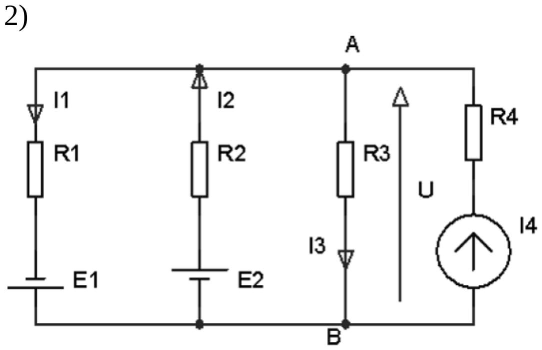
## Example 2

| | |
|---|---|
| E1 = 5V, E2 = 12V, I4 = 3mA , R1 = 1kΩ , R2 = R3 = 2kΩ <br> 1) Calculate voltage U between A and B <br> 2) Calculate & indicate direction of current flow through each resistor <br> 3) Confirm the results with the KCL rule |   |

## Solution

1) $U = \frac{\frac{-E_1}{R_1}+\frac{E_2}{R_2}+I_4}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{\frac{-5}{1}+\frac{12}{2}+3}{\frac{1}{1}+\frac{1}{2}+\frac{1}{2}} = -2V$

2)




$U + E_1 = R_1 I_1 \rightarrow I_1 = (2V + 5V) / 1k\Omega \rightarrow I_1 = 7mA$

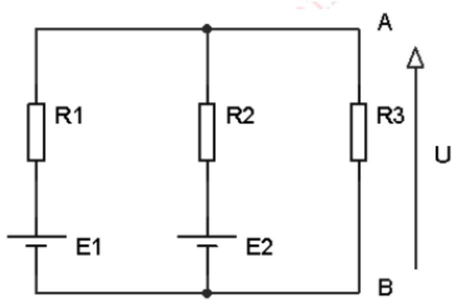
$U - E_2 = -R_2 I_2 \rightarrow I_2 = -(2V - 12V) / 2k\Omega \rightarrow I_2 =$ 5mA

$U = R_3 I_3 \rightarrow I_3 = 2V / 2k\Omega \rightarrow I_3 = 1mA$

3) $I_1 - I_2 + I_3 - I4 = 7 - 5 + 1 - 3 = 0$

## Exercises: (Homework Assignment)

**Exercise1:** Calculate U using Millman's theorem.



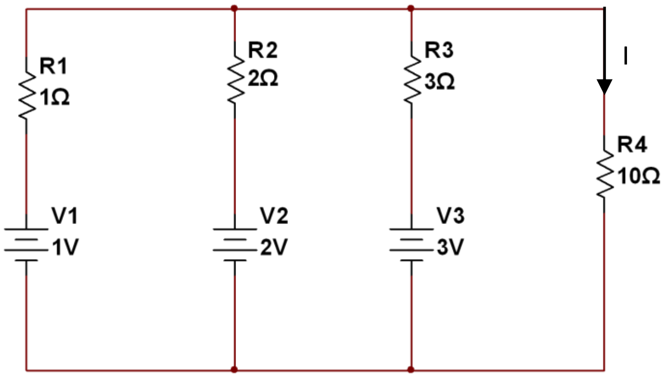

E1 = 5volts

E2 = 12 volts

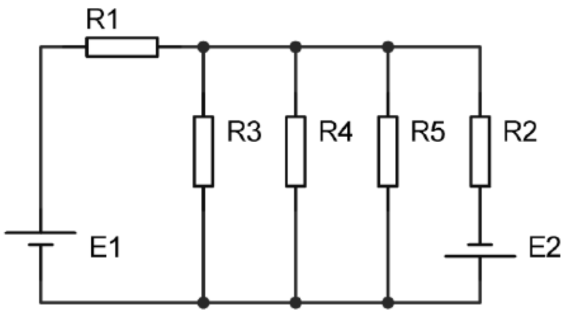
R1 = 1 kOhms

R2 = R3 = 2 kOhms

**Exercise 2** Find the current I using Millman's theorem.



## Problem 8




E1=10V E2=5V R1=R2=1k R3= R4=R5=1,5k

1) **Apply Millman's Theorem** to determine the equivalent voltage (Vm) across the parallel branches.
2) Calculate and indicate the direction of the current flowing through each resistor
3) Verify the KCL node rule.

# Acronym List

DC = Direct Current

AC = Alternating Current

I = current

V : Voltage (“U” is also used for voltage)

R : Resistance

E :Energy

P : Power

A = Ampere

v : volts

Ω : Ohm

# Bibliography

Here are some references on electrical circuits or basics of electricity with exercises and solutions

[1] Dorf, Richard C., and James A. Svoboda. ***Introduction to Electric Circuits***. Hoboken, NJ: Wiley, 2015. ISBN: 978-1119560983.

[2] Seymour, Charles D., and Herman W. Cooke. ***Basic Electricity***. Mineola, NY: Dover Publications, 2013. ISBN: 978-0486495708.

[3] Johnson, David E., and Johnny R. Johnson. ***Electric Circuit Analysis***. Hoboken, NJ: Wiley, 2016. ISBN: 978-1119235385.

[4] Patrick, Dale R., and Stephen W. Fardo. ***Electricity and Electronics Fundamentals***. Clifton Park, NY: Delmar Cengage Learning, 2012. ISBN: 978-1111128538.