Chapter three : The Graphical Method for solving Linear Programming Models

3-1- Introduction

The graphical method is a visual approach for solving linear programming (LP) problems with **two decision variables**.

3-2- Steps for Solving Linear Programming Problems Graphically

Here are the steps for solving a linear programming model (LPM) graphically:

1. Formulate the Linear Programming Problem: Define the decision variables, objective function, and constraints clearly.

2. Construct a graph: Set up a coordinate system (typically the x_1-x_2 plane) to represent the decision variables.

3. Plot the Constraint Lines: For each constraint, convert the inequalities into equations and plot the corresponding lines on the graph.

4. Determine the Feasible Region: Identify the area that satisfies all the constraints. This region is where all the inequalities overlap.

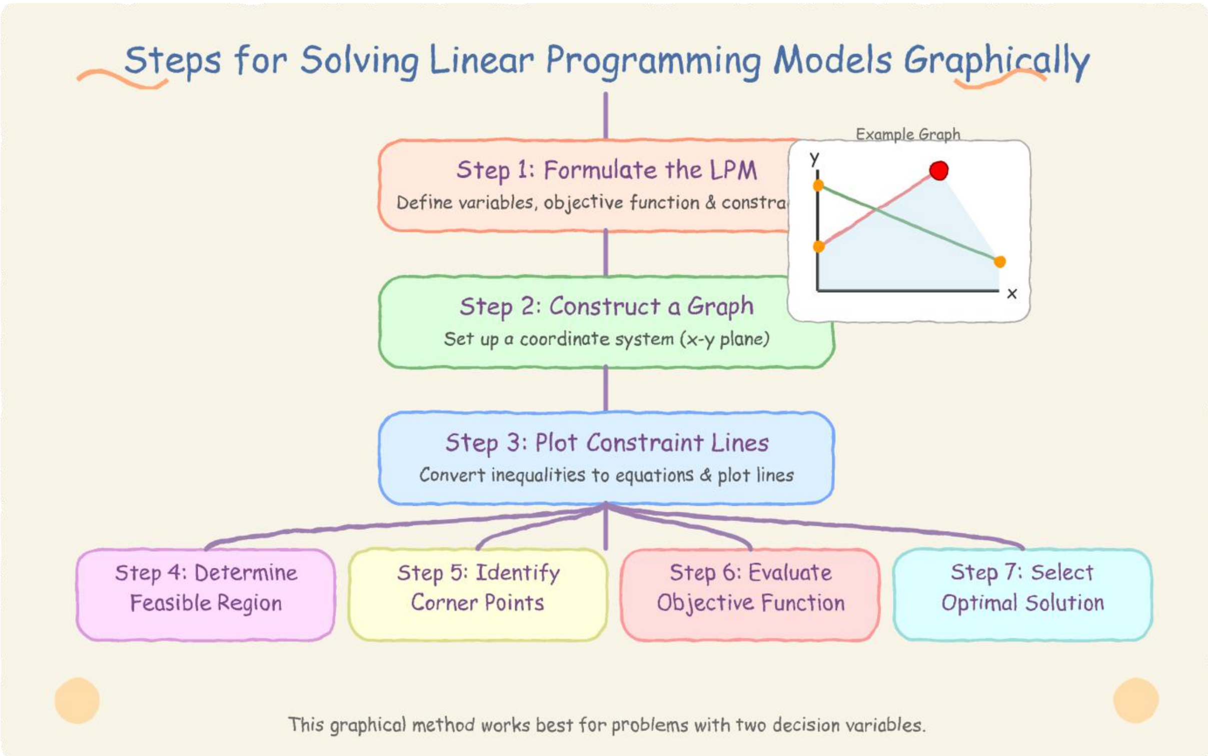
5. Identify the Corner Points: Locate the vertices (corner points) of the feasible region. These points are potential candidates for the optimal solution.

6. Evaluate the Objective Function: Calculate the value of the objective function at each corner point to determine which point yields the maximum or minimum value.

7. Select the Optimal Solution

The corner point that provides the best value for the objective function (maximum or minimum) is the optimal solution.

This graphical method is particularly effective for problems involving two decision variables, allowing for a visual representation of constraints and the feasible region.



Example:(The graphical method for solving LPM)

Solve this linear programming model using graphical method;

Max $Z= 6x_1 + 9x_2$

$$\text{ST:} \begin{cases} x_1 + 3x_2 \leq 9 \\ x_1 + x_2 \leq 5 \end{cases}$$

$X_1 \geq 0$, $X_2 \geq 0$

Solution:

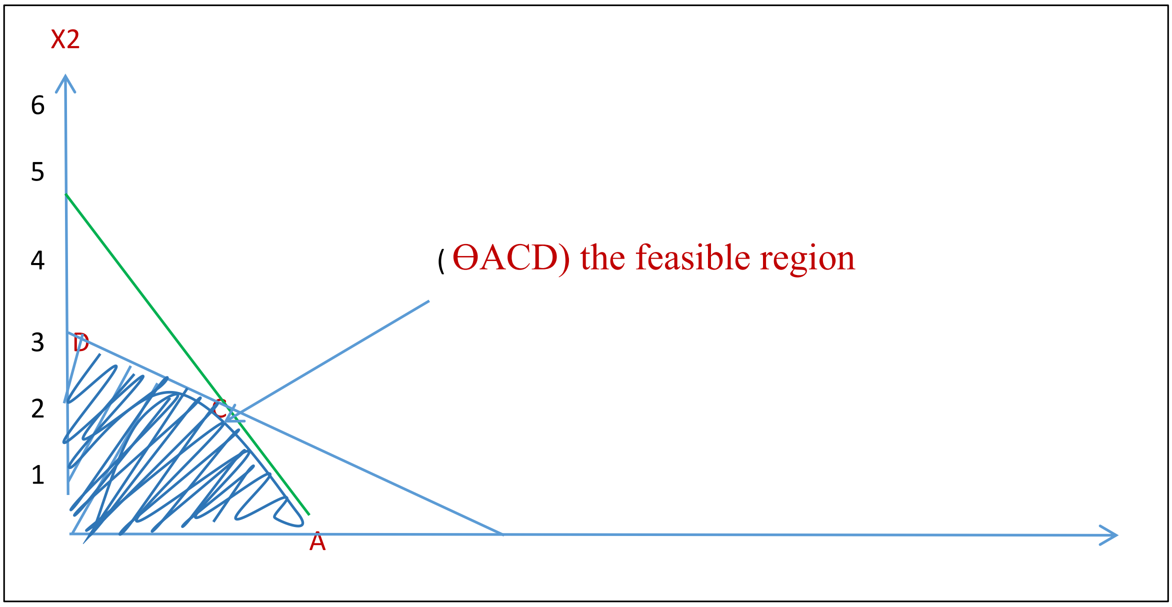
For each constraint, convert the inequalities into equations and plot the corresponding lines on the graph:

(Δ_1)......... $x_1 + 3x_2 = 9$

X1	0	9
X2	3	0

(Δ_2) ....$x_1 + x_2 = 5$

X1	0	5
X2	5	0



The region shaded limited by the polygon (ӨACD) is the area that satisfies all the constraints. This region is where all the inequalities overlap.

The vertices (corner points) (ӨACD) of the feasible region are potential candidates for the optimal solution.

Evaluate the Objective Function: Calculate the value of the objective function at each corner point to determine which point yields the maximum or minimum value:

The vertices	The coordinates		The objective function
	X1	X2	$Z = 6x_1 + 9x_2$
O	0	0	$Z_O=6(0)+9(0)=0$
A	5	0	$Z_A=6(5)+9(0)= 30$
C	**3**	**2**	$Z_C=6(3)+9(2)=36$
D	0	3	$Z_D=6(0)+9(3)=27$

The optimal solution is: $X_1=3$, $X_2=2$, Max $Z= 36$