The second semester exam of "Fundamentals of Operations Research"

Exercise N°1: (6pts)

A manufacturer produces two types of models *M* and *N*. Each *M* model requires 4 hours of grinding and 2 hours of polishing, whereas each *N* model requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers. Each grinder works for 40 hours a week and each polisher works for 60 hours a week. Profit on model *M* is 3€/Unit and model *N* is 4€/Unit. Whatever is produced in a week is sold in the market.

- ❖ How should the manufacturer allocate his production capacity to the two types of models so that he may make the maximum profit in a week? Write the LPM.

Exercise N°2: (7pts)

Solve this linear programming model using the simplex method:

$$\text{Max } Z = 3X_1 + 4X_2$$

$$\text{ST: } \begin{cases} X_1 + 2X_2 \leq 450 \text{ mn} & \text{(time constraint of machine M1)} \\ 2X_1 + X_2 \leq 600 \text{ mn} & \text{(time constraint of machine M2)} \end{cases}$$

$$X_1, X_2 \geq 0$$

Exercise N°3 : (7pts)

A company needs to transport goods from 3 warehouses to 3 retail stores. The supply at each warehouse and the demand at each store are given below. The goal is to minimize the transportation cost while meeting the demand of each store and ensuring the supply from each warehouse is fully utilized.

- Supply at Warehouses:
  - Warehouse 1: 100 uni
  - Warehouse 2: 60 uni
  - Warehouse 3: 40 uni

Demand at Retail Stores:

Store A: 90 units

Store B: 60 units

Store C: 50 units

- Transportation Costs (per unit):

	Store A	Store B	Store C
Warehouse 1	$5	$8	$6
Warehouse 2	$4	$6	$7
Warehouse 3	$3	$5	$4

Questions: 1- Arrange the data in the matrix form?

2- Formulate the balanced transportation problem as a linear programming model?

3-Using **the Northwest Corner Method** and **the Least Cost Method**, find an initial feasible solution for the transportation problem. Calculate the total costs? Explain the difference?

Model Answer and Marking Scheme for the Exam

EXERCICE N°1 (6pts)

--------------------- -----	Product M	Product N	Available hours per week
Grinding	4 hours	2 hours	$2 \times 40 = 80$ hours/ week
Polishing	2 hours	5 hours	$3 \times 60 = 180$ hours/week
Profit	3 €/ unit	4 €/ unit	----------------------------

Steps for building the Linear Programming Model:

Step N°1: Identify decision variables

Let: X_1 is the number of units of the product M produced per week; (0.5 p)

X_2 is the number of units of the product N produced per week. (0.5 p)

Step N°2: Formulate the objective function (the total profit) (0.5 p)

The total profit required per week = profit required by the product M (per week) + profit required by the product N (per week)

$$Z = 3\,X_1 + 4\,X_2$$

Step N°3 Formulate the constraints (grinding & polishing constraints)

Grinding constraint: the number of grinding hours required by the product M + the number of grinding hours required by the product N should be at most 80 hours / week(0.25 p)

$$4\,X_1 + 2\,X_2 \leq 80 \text{ hours}$$

Polishing constraint: the number of polishing hours required by the product M + the number of polishing hours required by the product N should be at most 180 hours / week (0.25 p)

$$2X_1 + 5\,X_2 \leq 180 \text{ hours}$$

Step N°3: non-negativity condition of the decision variables (0.5 p)

$$X_1 \geq 0$$

$$X_2 \geq 0$$

The linear programming model is:

$$\text{MAX } Z = Z = 3X_1 + 4X_2 \quad (1\ p)$$

$$\text{St: } \begin{cases} 4X_1 + 2X_2 \leq 80 & (1p) \\ 2X_1 + 5X_2 \leq 180 & (1\ p) \end{cases}$$

$$X_1 \geq 0 \quad (0.25\ p)$$

$$X_2 \geq 0 \quad (0.25\ p)$$

Exercise N°2: (7pts)

$$\text{MAX } Z = Z = 3X_1 + 4X_2$$

$$\text{St: } \begin{cases} 4X_1 + 2X_2 \leq 80 \\ 2X_1 + 5X_2 \leq 180 \end{cases}$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

To solve this LPM we convert it into its standard form:

$$\text{MAX } Z = Z = 3X_1 + 4X_2 + 0Y_1 + 0Y_2 \ (1\ p)$$

$$\text{St: } \begin{cases} 4X_1 + 2X_2 + Y_1 = 450 & (0.5\ p) \\ 2X_1 + 5X_2 + Y_2 = 600 & (0.5\ p) \end{cases}$$

$$X_1, X_2, Y_1, Y_2 \geq 0 \quad (0.5\ p)$$

We start by the initial simplex tableau as follows:(3p)

Basic variables	X1	X2	Y1	Y2	B	B/X2
Y1	1	2	1	0	450	450/2 =225
Y2	2	1	0	1	600	600/1 = 600
Z	3	4	0	0	0	B/X1
X2	½	1	½	0	225	225/1/2=450
Y2	3/2	0	-2	1	375	375/3/2= 250
Z	1	0	-2	0	900	
X2	0	1	2/3	-1/3	100	
X1	1	0	-1/3	2/3	250	
Z	0	0	-5/6	-2/3	1150	

The optimal solution is:

X1= 250 units of the product M should be produced per week(0.5 p)

X2=100 units of the product N should be produced per week (0.5 p)

Max Profit Required (Per Week) = 1150 € (0.5 p)

EXERCISE N° 3: (7pts)

1° Formulate the transportation table: (1p)

	Store1	Store2	Store3	Σ Supply
Warehouse 1	5 X11	8 X12	6 X13	100
Warehouse 2	4 X21	6 X22	7 X23	60
Warehouse 3	3 X31	5 X32	4 X33	40
Σ Demand	90	60	50	200=200

2°/ Formulate the balanced transportation problem as a LPM:

Let X_{ij} the number of units shipped from the warehouse number i to the store number j

Where: i= 1,2,3 and j= 1,2,3 (0.5)

The linear programming model is :

$$\text{Min } Ct = 5X_{11} + 8X_{12} + 6X_{13} + 4X_{21} + 6X_{22} + 7X_{23} + 3X_{31} + 5X_{32} + 4X_{33} \quad (0.25p)$$

ST :

$$\begin{cases} X_{11} + X_{12} + X_{13} = 100 \ (0.25p) \\ X_{21} + X_{22} + X_{23} = 60 \ (0.25p) \\ X_{31} + X_{32} + X_{33} = 40 (0.25p) \end{cases} \quad \text{Supply constraints}$$

$$\begin{cases} X_{1} + X_{21} + X_{31} = 90 (0.25p) \\ X_{12} + X_{22} + X_{32} = 60 \ (0.25p) \\ X_{13} + X_{23} + X_{33} = 50 \ (0.25p) \end{cases} \quad \text{Demand constraints}$$

$X_{ij} \geq 0 \quad i=1,2,3 \qquad j= 1,2,3$ (0.25p)

2°/ find the initial solution:

- The initial feasible solution using **the North West Corner Method** is as follows:(1p)

	Store 1		Store 2		Store 3		*Σ Supply*
Warehouse 1	5	90	8	10	6	-----	100
Warehouse 2	4	-----	6	50	7	10	60
Warehouse 3	3	-----	5	-----	4	40	40
Σ Demand	90		60		50		200=200

We check the condition : the number of occupied cells = (m+n) – 1

m is the number of warehouses and n is the number of stores

so: according to the NWCMethod we have the number of occupied cells = 5

(M+N) = (3+3) – 1= 6-1= 5 ,We accept this initial solution (0.25p)

The total cost = 90(5) + 10(8) + 50(6) + 10(7) + 40(4) = 1060 € (0.25p)

- The initial feasible solution using the **Least Cost Method** is as follows:

	Store 1	Store 2	Store 3	Σ *Supply*
Warehouse 1	5 -----	8 50	6 50	100
Warehouse 2	4 50	6 10	7 -----	60
Warehouse 3	3 40	5 -----	4 -----	40
Σ *Demand*	90	60	50	200=200

We check the condition : the number of occupied cells = (m+n) – 1

m is the number of warehouses and n is the number of stores

so:

according to the Least Cost Method we have the number of occupied cells = 5

(M+N) = (3+3) – 1= 6-1= 5 ,We accept this initial solution (0.25p)

THE total cost = 50(8)+ 50(6) + 50(4) + 10(6) + 40(3) =1080 € (0.25p)

In transportation problems, **the Northwest Corner Method (NWC)** is a basic feasible solution method that does **not consider costs** during allocation—it simply follows a fixed rule: start from the top-left (northwest) cell and move right or down based on supply and demand.

The Least Cost Method (LCM), on the other hand, **tries to minimize cost** by allocating as much as possible to the cells with the lowest transportation cost first.

The Northwest Corner Method **can sometimes** lead to a lower total cost than the Least Cost Method **by chance**, due to the structure of the cost matrix and the specific supply/demand values. (0.5P)