

TD2: Numerical series

Exercise 1:

Calculate the partial sums of the following series, deduce their sums if they exist:

$$1) \sum_{n \geq 1} \frac{2^n}{3^{n-1}}, \quad 2) \sum_{n \geq 1} \ln \left(\frac{n+1}{n} \right), \quad 3) \sum_{n \geq 0} U_n = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \dots$$

Homework: Practice and summing telescoping series (to do by student)

$$\sum_{n \geq 1} \frac{1}{n} - \frac{1}{n+2}, \quad \sum_{n \geq 1} \frac{\sin(\frac{1}{n} - \frac{1}{n+1})}{\cos(\frac{1}{n}) \cos(\frac{1}{n+1})}.$$

Exercise 2:

Study the nature of the following numerical series

$$\begin{aligned} & \sum_{n \geq 0} \frac{e^n}{e^n + 1}, \quad \sum_{n \geq 1} \frac{\ln(1 + \frac{1}{n})}{\sin(\frac{1}{n})}, \quad \sum_{n \geq 0} \frac{5^n}{e^{n-2}}, \quad \sum_{n \geq 0} \frac{(n!)^2}{(2n)!}, \\ & \sum_{n \geq 1} \frac{n^n}{n!}, \quad \sum_{n \geq 1} \left(\sqrt{n^2 + n} - \sqrt{n} \right)^n, \quad \sum_{n \geq 2} \frac{n}{\sqrt{n^3 - 1}}, \\ & \sum_{n \geq 1} \ln \left(1 + \frac{2}{n(n+3)} \right), \quad \sum_{n \geq 1} n \left(\cos\left(\frac{1}{n}\right) - 1 \right), \quad \sum_{n \geq 3} \frac{1}{n \ln n}, \end{aligned}$$

Exercise 3:

Study the nature of the following numerical series

$$\begin{aligned} 1) & \sum_{n \geq 2} \sin \left(\frac{n+1}{n-1} \right), \quad 2) \sum_{n \geq 0} (-1)^n \frac{2^n}{n!}, \quad 3) \sum_{n \geq 0} \frac{\sin(2n)}{n^2 - n + 1}, \\ 4) & \sum_{n \geq 0} (-1)^n (\sqrt{n+1} - \sqrt{n}), \quad 5) \sum_{n \geq 3} (-1)^n \frac{\ln(n)}{n}. \end{aligned}$$

Additional exercises:

Study the nature of the following numerical series

$$\begin{aligned} & \sum_{n \geq 1} \frac{1}{n(n + \ln n)}, \quad \sum_{n \geq 1} \frac{1}{2^n} \left(1 + \frac{1}{n} \right)^{n^2}, \quad \sum_{n \geq 0} \frac{2n+1}{(\sqrt{2})^n}, \quad \sum_{n \geq 0} \frac{n+2}{n^3+1}, \\ & \sum_{n \geq 0} \frac{\sqrt{3^n}}{3n+1}, \quad \sum_{n \geq 1} \left(\frac{n-1}{3n} \right)^n, \quad \sum_{n \geq 0} \left(\frac{3n+1}{2n+3} \right)^{\frac{n}{2}}, \quad \sum_{n \geq 0} \frac{a^n}{n+b^n}, \quad a > 0, b > 0, \\ & \sum_{n \geq 1} \frac{(-1)^n}{n} \arctan\left(\frac{1}{n}\right), \quad \sum_{n \geq 1} \frac{(-1)^n}{n + (-1)^n}, \quad \sum_{n \geq 1} \frac{(-1)^n}{n + 2 \sin n}, \quad \sum_{n \geq 0} \frac{\sin n}{\sqrt{n} + \cos n}. \end{aligned}$$