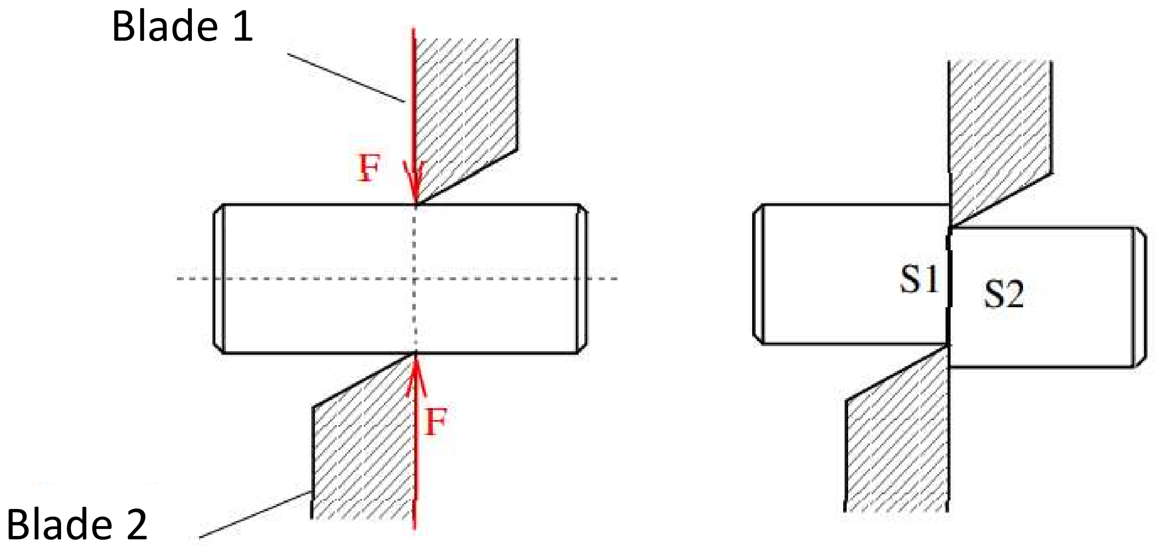
Chapter 3

Shear

Example 1: Sheared workpiece



the shearing of the piece results in the sliding of the straight section S1 against the straight section S2 which is in direct contact with it.

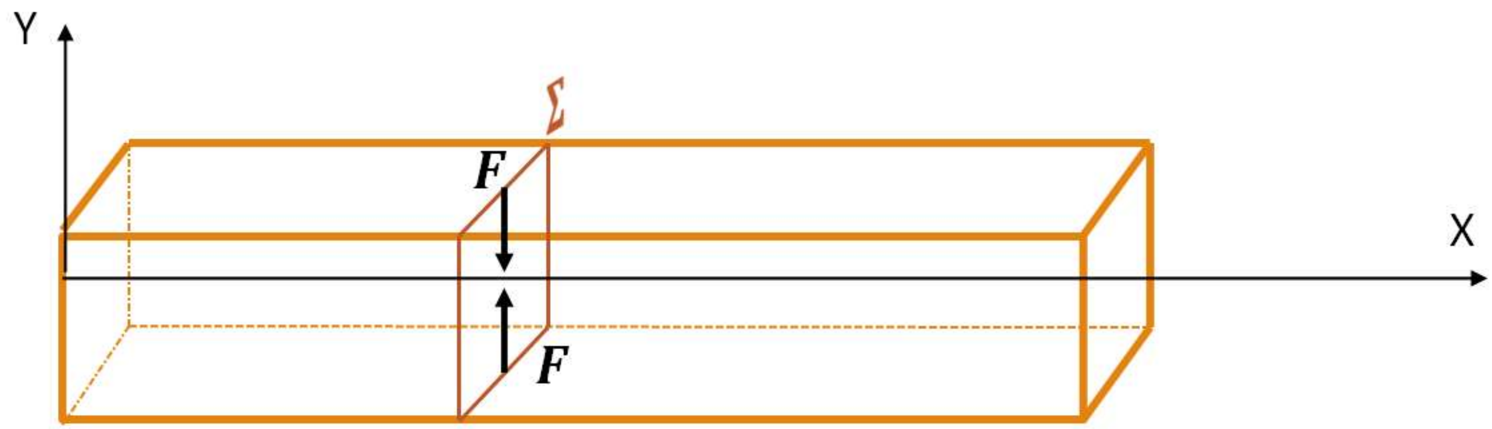
1.Definition:

A beam is subjected to pure shear stress when the cohesive torsor at the centre of gravity of a section has the form

$$\tau_{coh} = \begin{Bmatrix} 0 & 0 \\ T_y & 0 \\ T_z & 0 \end{Bmatrix}$$

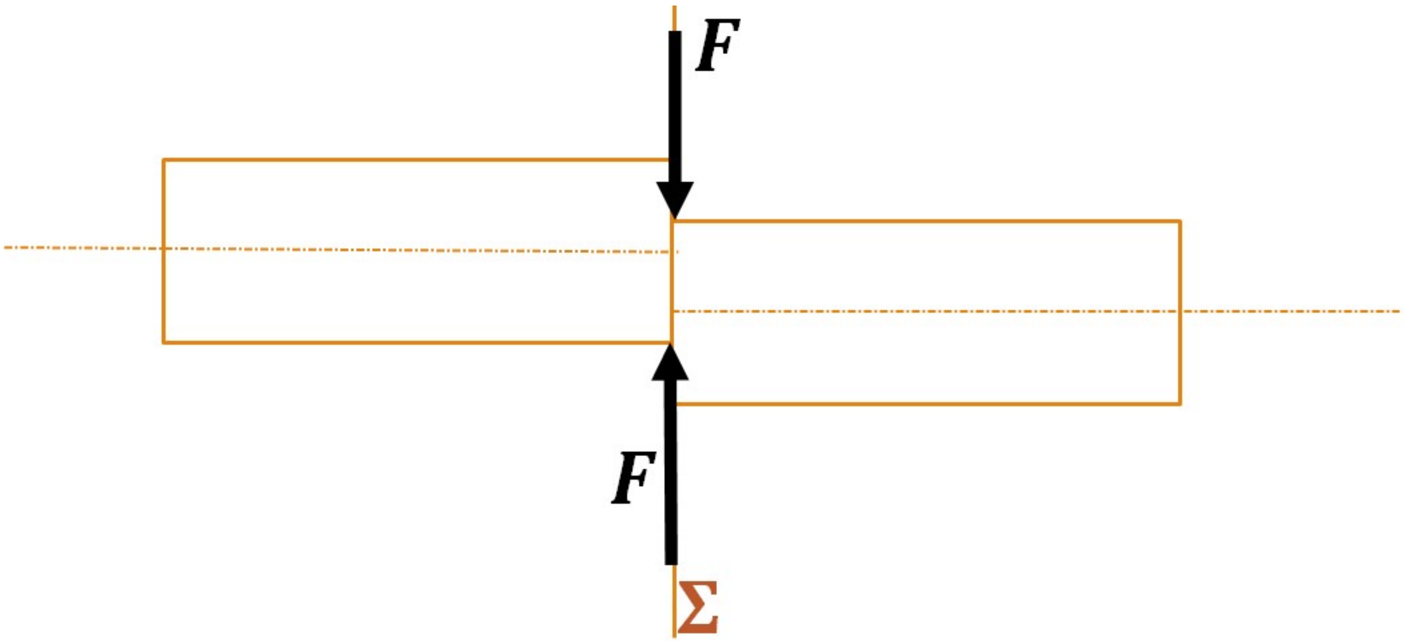
1.Definition :

Shear stress occurs when a beam is subjected to two equal and directly opposed forces whose support is in a plane perpendicular to the mean line.



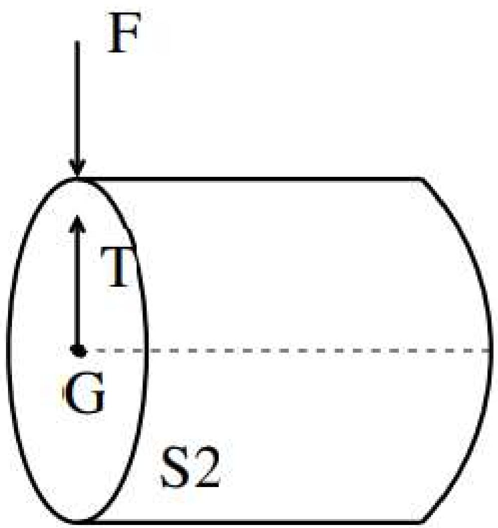
1.Definition :

Under the action of these two forces, the beam tends to separate into two parts sliding relative to each other in the plane of the straight section (Σ)



2.Shear force T

The only internal force is T (N = Mf = 0).According to the equilibrium T=F.



3.State of constraint

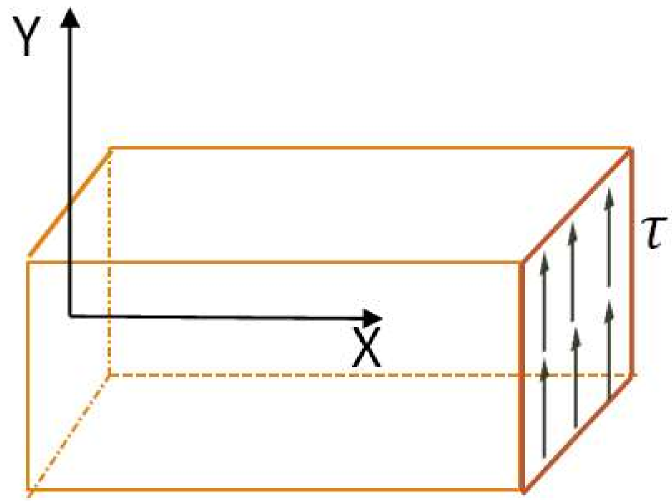
Each surface element $\boldsymbol{dS}$ supports a shear stress $\boldsymbol{\tau}$ contained in the sheared section $\boldsymbol{S}$.

$$T(x) = \int_S \tau . dS$$

$$T(x) = \tau . \int_S dS$$

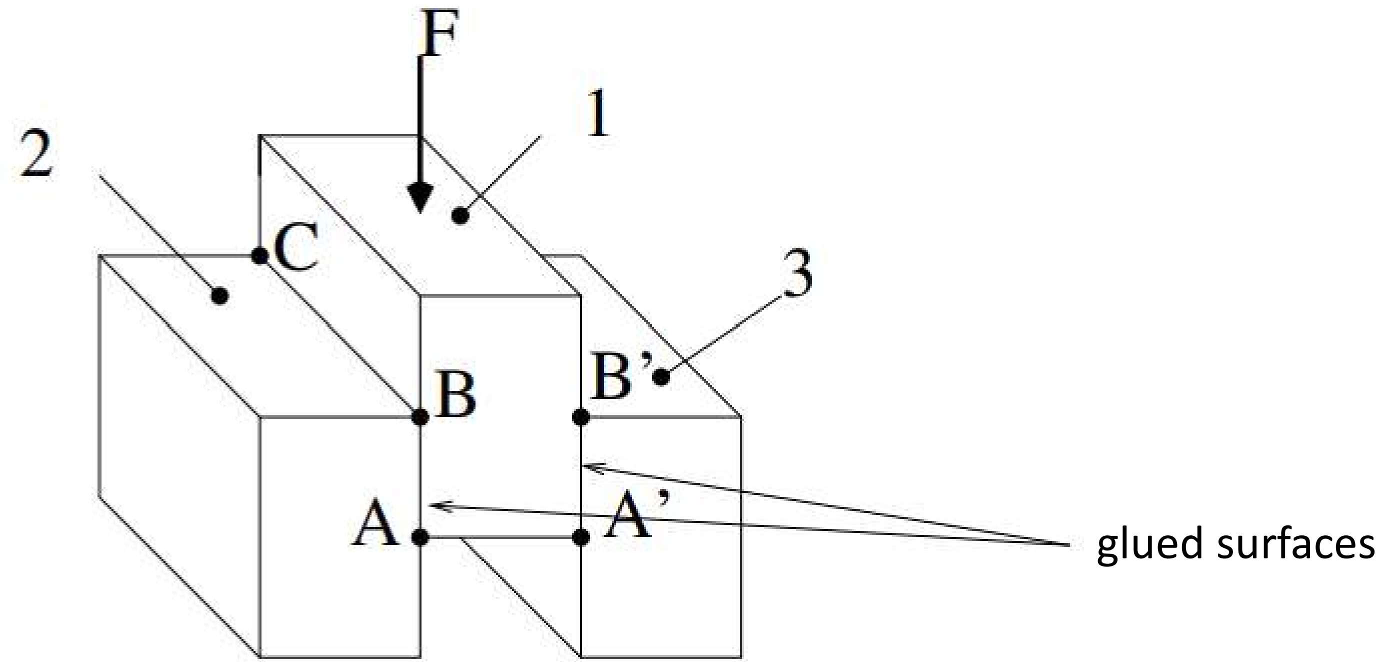
$$T(x) = F$$

$$\tau = \frac{F}{S}$$

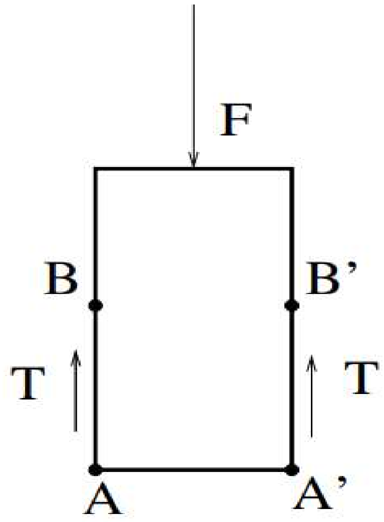


Example 2

Three identical blocks of wood (1), (2) and (3) are glued together. The assembly supports a load F along its axis of symmetry. The glued surfaces are ABCD and A'B'C'D'.



To calculate T, we isolate block (1) and study its equilibrium.



$$T = \frac{F}{2}$$

Note

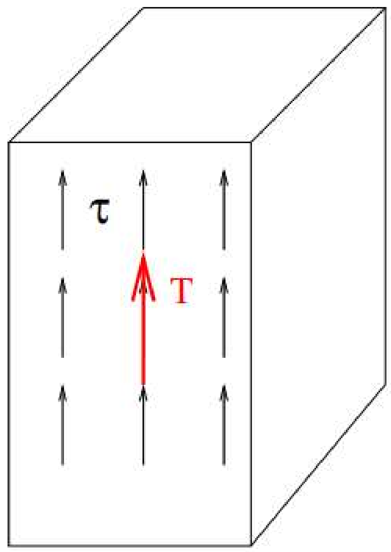
A simple relationship can be deduced between the applied force F and T :

$$T = \frac{F}{n}$$

Where n is the number of sheared surfaces.

4.Shear stress τ

In shear, normal stresses are null. It is supposed that all tangential stresses are the same. In other words, there is a uniform stress distribution in the sheared section.



$$T = \int_S \tau dS = \tau \int_S dS = \tau S \Rightarrow \quad \tau = \frac{T}{S}$$

Example 2

Take example 2 with F=200 daN, AB=CD=3 cm, AD=BC=10 cm. Calculate τ in the glued joint.

Solution

$$S = 30 \times 100 = 3000mm^2, \ T = \frac{F}{2} = 1000N$$
$$\tau = \frac{T}{S} = \frac{1000}{3000} = 0.333Nmm^{-2} = 0.333MPa$$

5.Resistance condition

$$\tau = \frac{T}{S} \leq R_{ps} \text{ avec : } R_{ps} = \frac{R_{es}}{s}$$

R_{ps}: practical resistance to sliding or shearing (Admissible stress)
R_{es}: elastic limit in shear.
s: safety coefficient.

Example 3

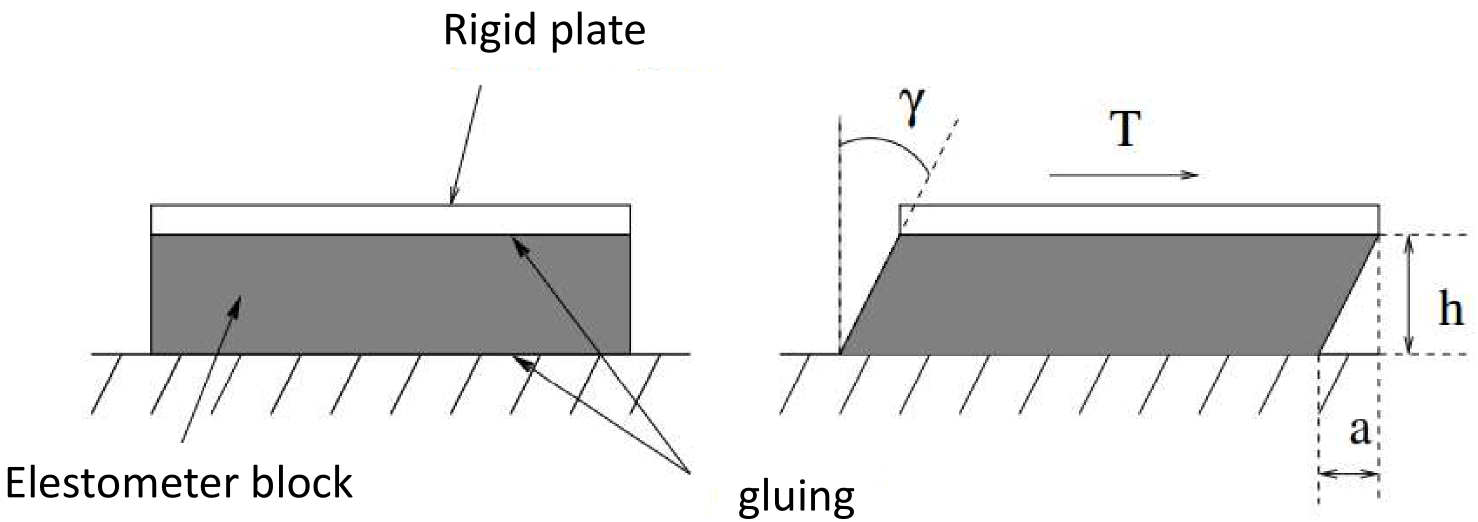
Let's return to example 2.

If the admissible stress in the glued joint is 900 kPa, determine the maximum load F that can be supported

Solution

$$\tau = \frac{T}{S} = \frac{F}{2S} = \frac{F}{2\times 30\times 100} \leq 0.9 Nmm^{-2}$$
$$F \leq 0.9 \times 2 \times 3000 = 5400 N.$$

6. Deformation - Sliding angle γ



Deformation can be characterized by the angle γ, known as the angle of sliding angle.

$$\tan\gamma = \frac{a}{h}$$ **if γ is small** $$\tan\gamma \approx \gamma = \frac{a}{h}$$

7.Relation between τ and γ

As in the case of tension, in the elastic domain, the relationship between τ and γ is linear.

$\tau = G.\gamma$

γ: sliding angle (radian)

G: transverse modulus of elasticity (MPa)

Coulomb Modulus G

$$G = \frac{E}{2.(1+\nu)}$$

Where ν is the Poisson coefficient

Example 4

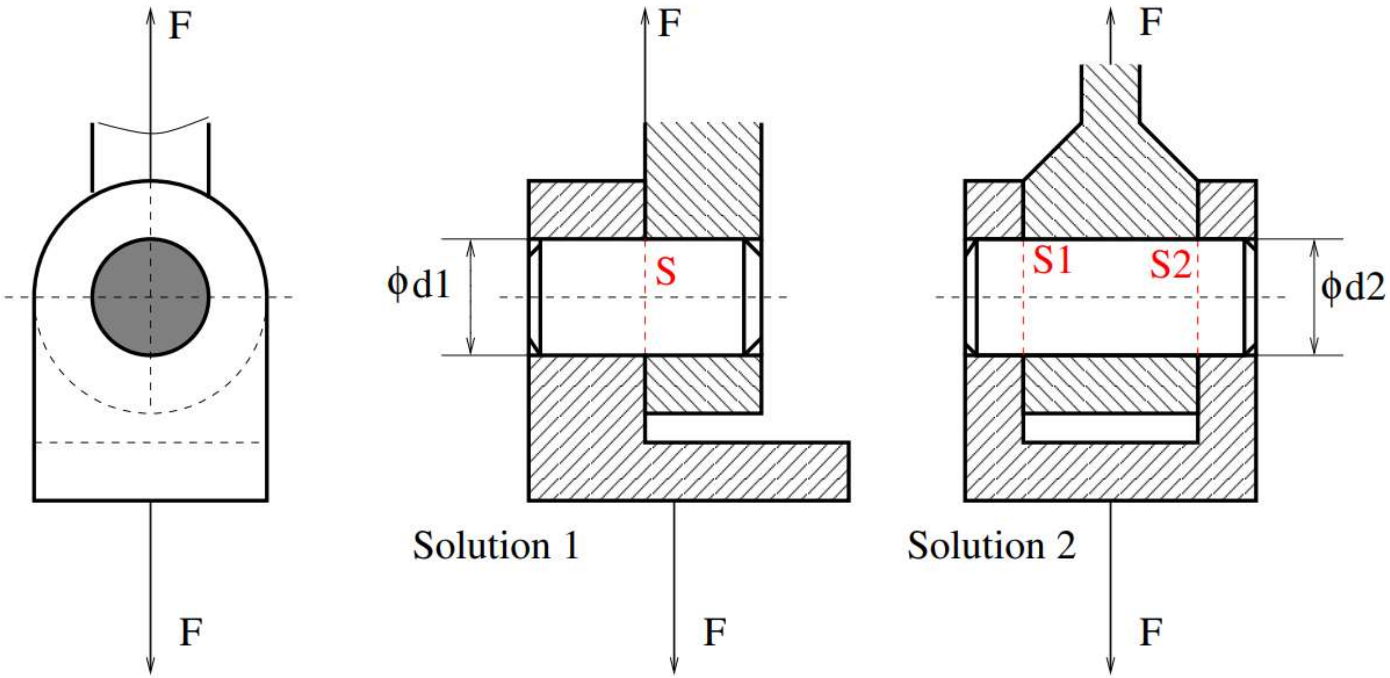
Let's take the example of the elastomer block ($c \times b \times h$) with c=50mm, b=100 mm and G=800 kPa. Determine γ if T=100 daN and h=25 mm. calculate a.

Solution

$$\tau = \frac{T}{c \times b} = \frac{1000}{50 \times 100} = 0.2 Nmm^{-2}$$

$$\gamma = \frac{\tau}{G} = \frac{0.2}{0.8} = 0.25 rad = 14.3^\circ \text{ et } a = h \tan \gamma = 6.4 mm.$$

Example 5



F=10000 daN, the axes are made from the same steel with an admissible shear stress of 5 daN/mm2.

Solution 1

one sheared section $\Rightarrow T = F$

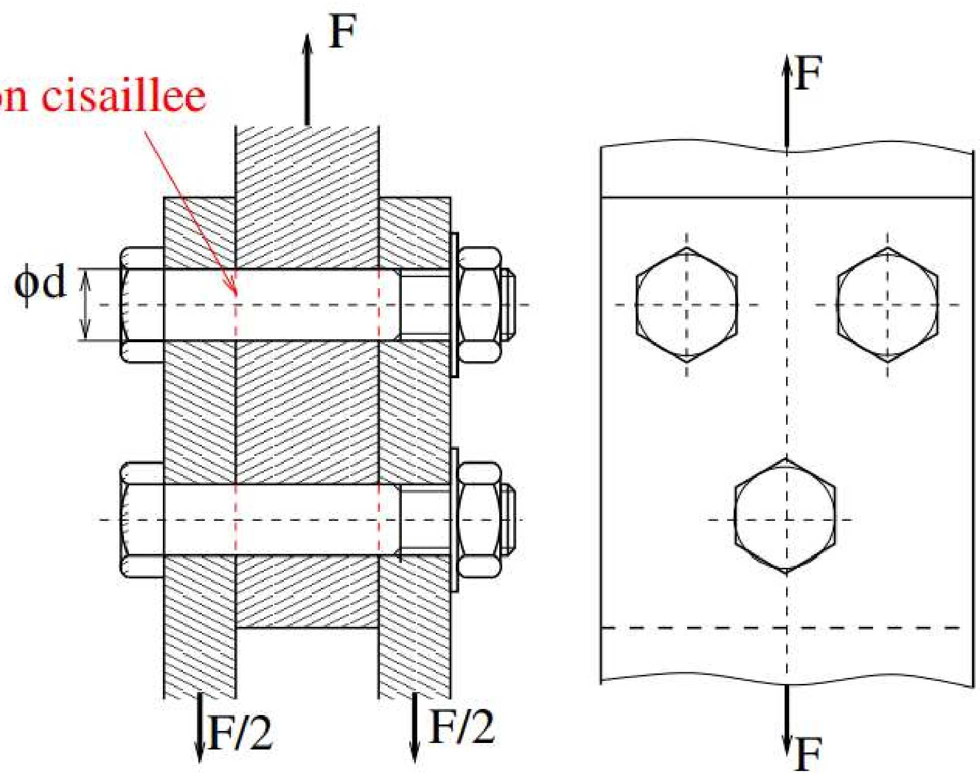
$$\tau = \frac{F}{S} = \frac{4F}{\pi d_1^2} \leq R_{pg} \Rightarrow d_1 \geq \sqrt{\frac{4F}{\pi R_{pg}}} = 50,5mm$$

Solution 2

Two sheared section S_1 et $S_2 \Rightarrow T = \frac{F}{2}$

$$\tau = \frac{F}{2S} = \frac{2F}{\pi d_2^2} \leq R_{pg} \Rightarrow d_2 \geq \sqrt{\frac{2F}{\pi R_{pg}}} = 35,7mm$$

Example 6



For the above connection with three steel bolts, d= 12mm, the permissible shear stress is R_{ps} = 30 daN/mm2
Determining the admissible force

$$T = \frac{F}{6} \quad \tau = \frac{F}{6S} \leq R_{pg} \Rightarrow F \leq 6SR_{pg} = 6\frac{\pi d^2}{4}R_{pg} = 20358 \ daN$$