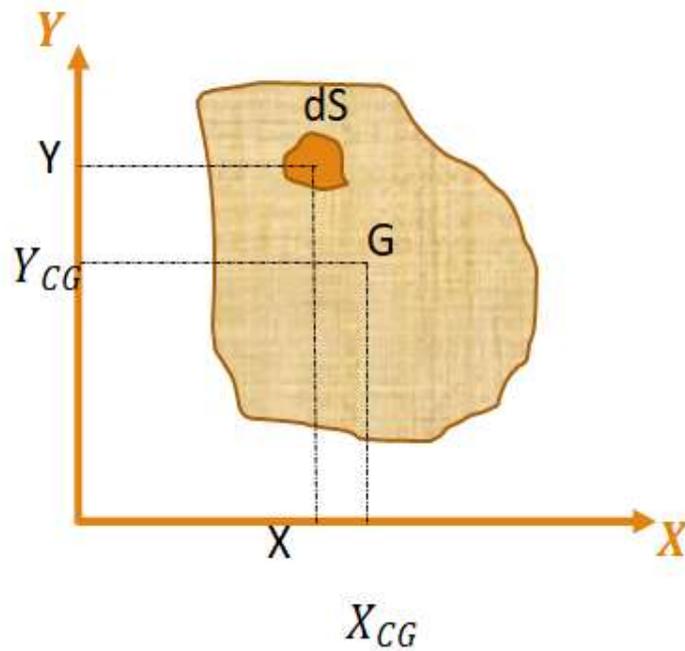


CHAPTRE 4

GEOMETRIC CHARACTERISTICS OF STRAIGHT SECTIONS



Centre of gravity



Coordinates along the x axis

$$X_{CG} = \frac{\int_s X dS}{S}$$

Coordinates along the y axis

$$Y_{CG} = \frac{\int_s Y dS}{S}$$

Centre of gravity

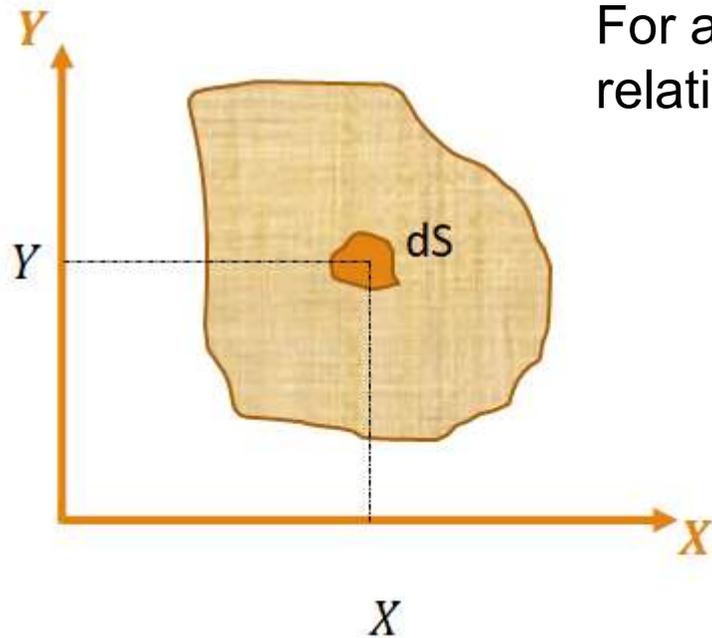
If the section S can be decomposed into n simple sub-sections of known areas S_i and known centres of gravity X_{Gi}, Y_{Gi}

$$X_G = \frac{\sum_{i=1}^n S_i X_{Gi}}{S}$$

$$Y_G = \frac{\sum_{i=1}^n S_i Y_{Gi}}{S}$$

if the section is symmetrical, the centre of gravity is on the axis of symmetry

Static moments

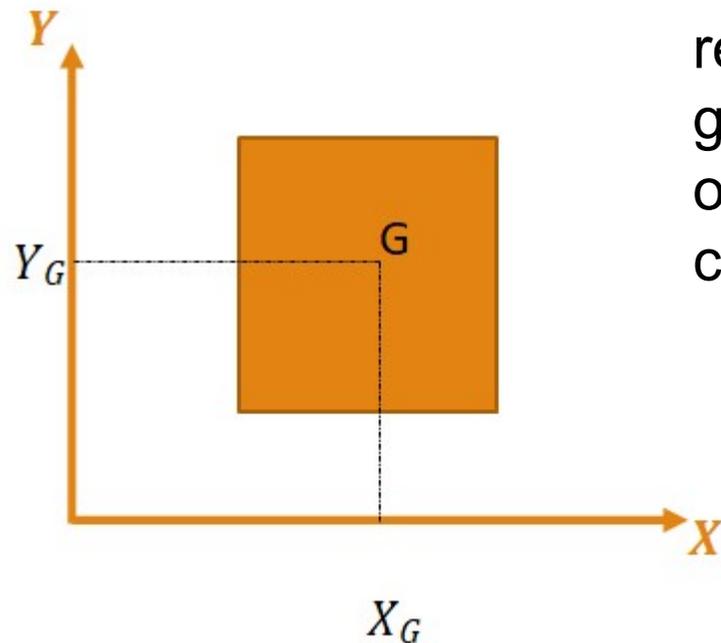


For a surface element dS the static moment relative to the axes OX and OY

$$S_X = \int_S Y dS$$

$$S_Y = \int_S X dS$$

Static moments



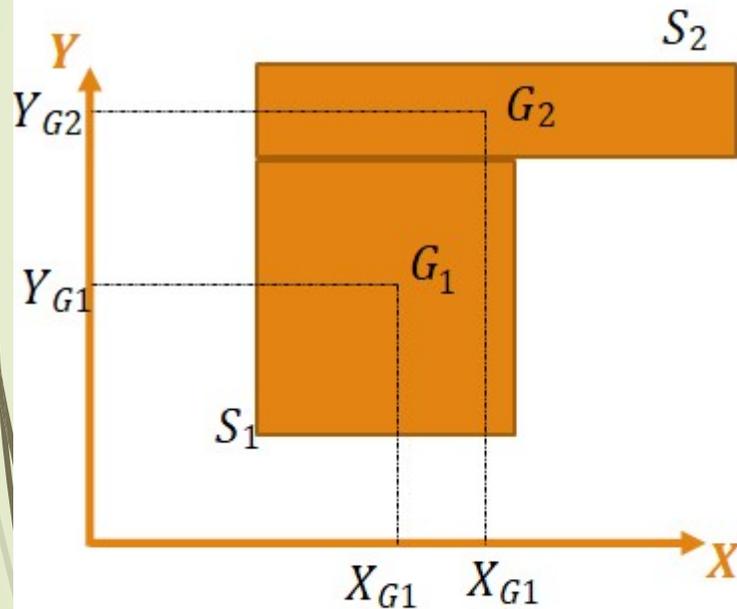
The static moment of a section relative to an axis situated in its plane is given by the product of the surface area of the section and the distance of its centre of gravity to the axis in question.

$$S_X = S \cdot Y_G$$

$$S_Y = S \cdot X_G$$

Static moments

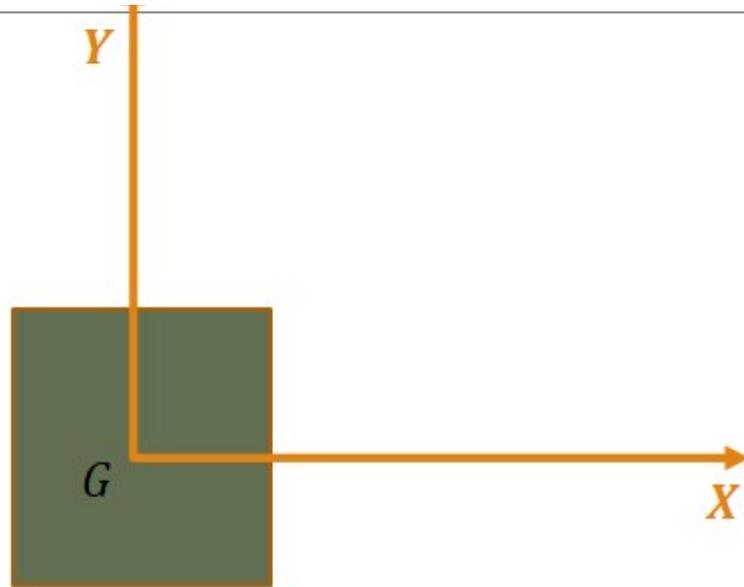
For complex surfaces composed of n simple areas



$$S_X = \sum_{i=1}^n S_i Y_{G_i}$$

$$S_Y = \sum_{i=1}^n S_i X_{G_i}$$

Static moments

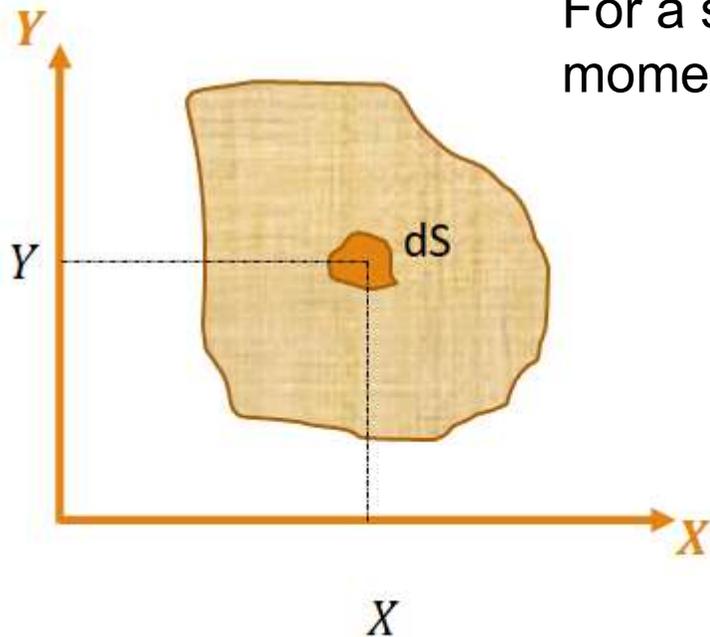


$$S_X = 0$$

$$S_Y = 0$$

The static moment of a surface relative to an axis passing through its centre of gravity is zero

Moments of inertia (Quadratic moments)



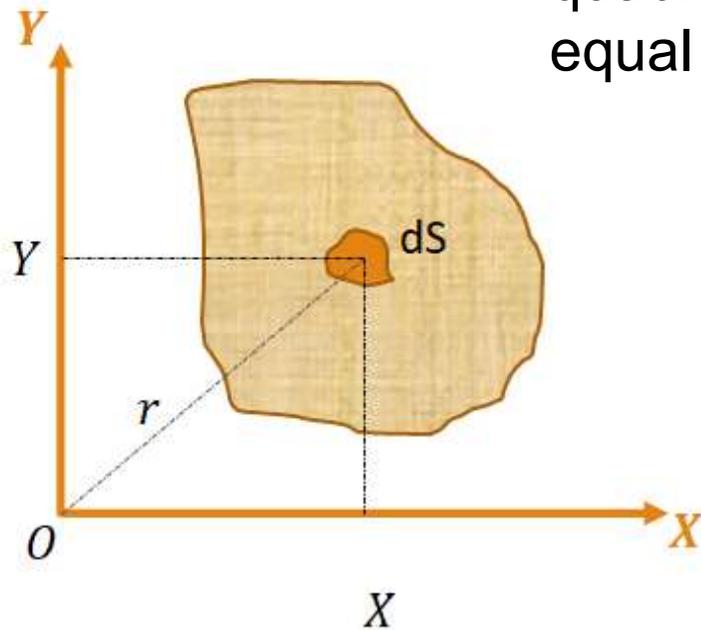
For a surface element dS , the quadratic moments relative to the axes OX and OY

$$I_X = \int_S Y^2 dS$$

$$I_Y = \int_S X^2 dS$$

Polar moment of inertia

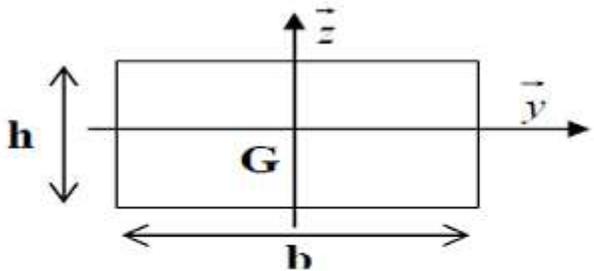
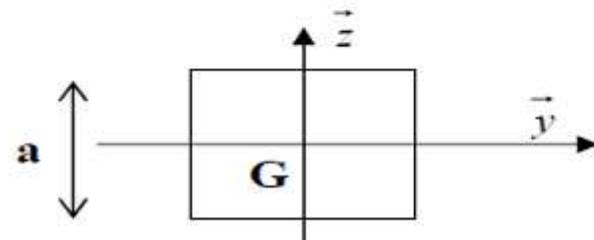
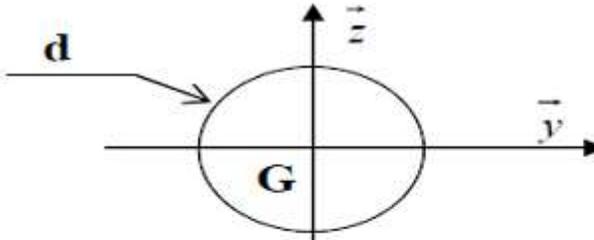
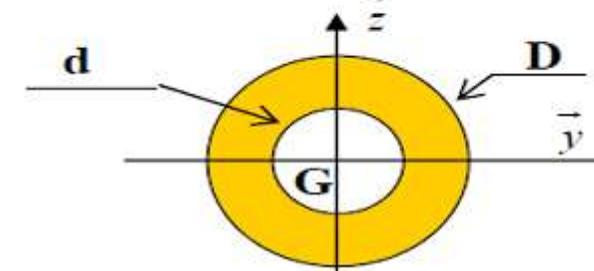
For a surface element dS , the polar quadratic moment relative to a given point is equal to



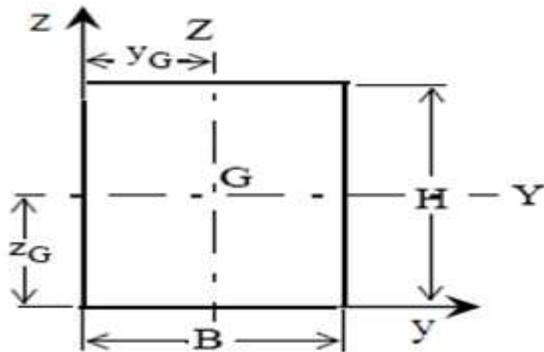
$$I_p = \int_S r^2 dS$$

$$I_p = \int_S (X^2 + Y^2) dS$$

$$I_p = I_X + I_Y$$

	I_{Gyy}	I_{Gzz}	$I_G = I_{Gyy} + I_{Gzz}$
	$\frac{b \cdot h^3}{12}$	$\frac{h \cdot b^3}{12}$	$\frac{h \cdot b}{12} (b^2 + h^2)$
	$\frac{a^4}{12}$	$\frac{a^4}{12}$	$\frac{a^4}{6}$
	$\frac{\pi \cdot d^4}{64}$	$\frac{\pi \cdot d^4}{64}$	$\frac{\pi \cdot d^4}{32}$
	$\frac{\pi}{64} (D^4 - d^4)$	$\frac{\pi}{64} (D^4 - d^4)$	$\frac{\pi}{32} (D^4 - d^4)$

Rectangle



$$A = B.H$$

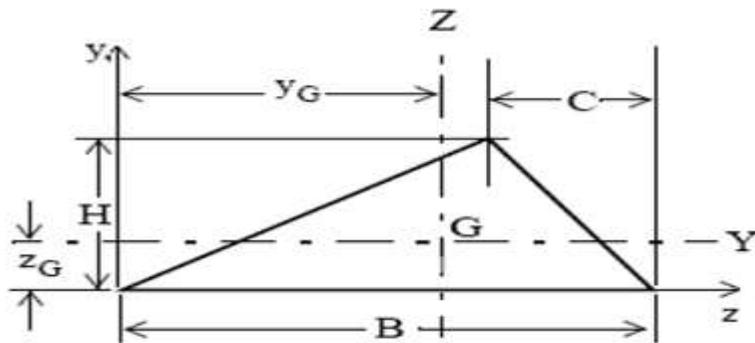
$$y_G = \frac{B}{2}$$

$$z_G = \frac{H}{2}$$

$$I_Y = \frac{BH^3}{12}$$

$$I_Z = \frac{HB^3}{12}$$

Triangle quelconque



$$A = B.H / 2$$

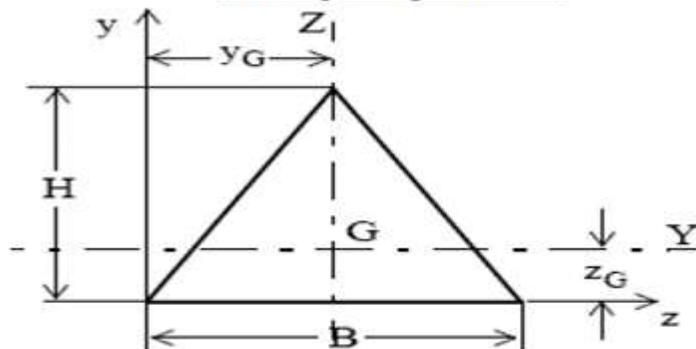
$$y_G = \frac{2B + C}{3}$$

$$z_G = \frac{H}{3}$$

$$I_Y = \frac{BH^3}{36}$$

$$I_Z = \frac{BH}{36} \cdot (B^2 + C^2 - BC)$$

Triangle équilatéral



$$A = B.H / 2$$

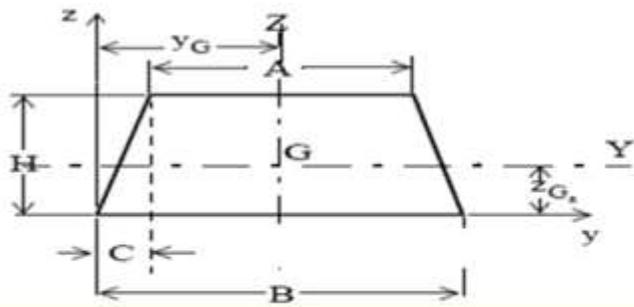
$$y_G = \frac{B}{2}$$

$$z_G = \frac{H}{3}$$

$$I_Y = \frac{BH^3}{36}$$

$$I_Z = \frac{B^3 H}{48}$$

Trapèze isocèle



$$A = (A + B) \cdot H / 2$$

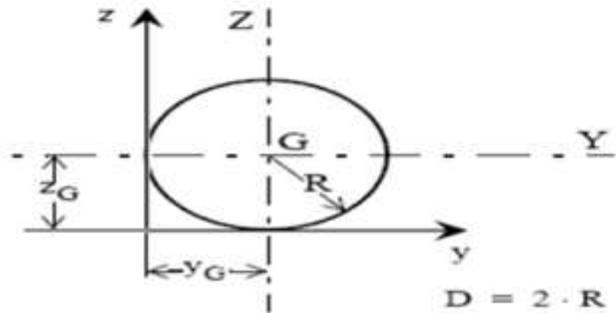
$$y_G = \frac{B}{2}$$

$$z_{G_a} = \frac{H \cdot (B + 2A)}{3 \cdot (B + A)}$$

$$I_Y = \frac{H^3 \cdot (A^2 + 4AB + B^2)}{36 \cdot (A + B)}$$

$$I_Z = \frac{H \cdot (A + B) \cdot (A^2 + B^2)}{48}$$

Cercle



$$A = \pi R^2$$

$$y_G = R$$

$$z_G = R$$

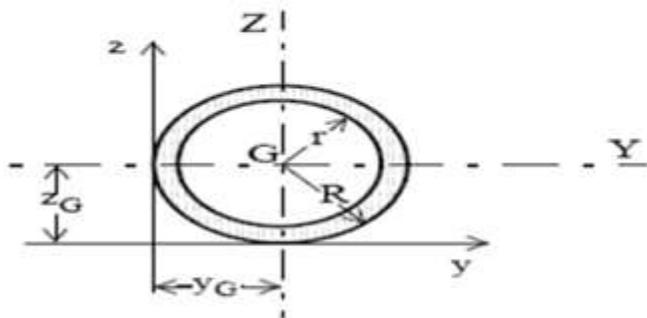
$$I_Y = \frac{\pi \cdot R^4}{4}$$

$$= 0.0491 \cdot D^4$$

$$I_Z = \frac{\pi \cdot R^4}{4}$$

$$= 0.0491 \cdot D^4$$

Cercle creux (tube)



$$A = \pi (R^2 - r^2)$$

$$y_G = R$$

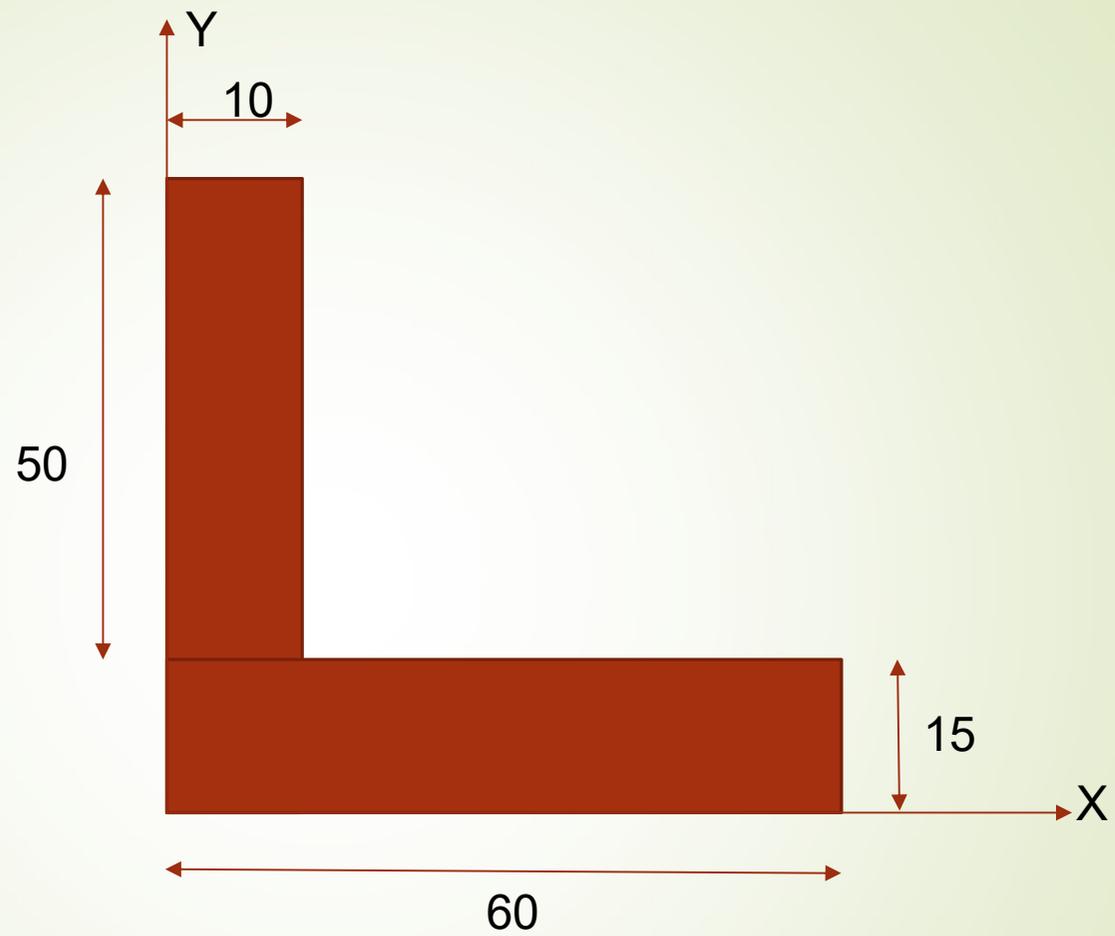
$$z_G = R$$

$$I_Y = \frac{\pi \cdot (R^4 - r^4)}{4}$$

$$I_Z = \frac{\pi \cdot (R^4 - r^4)}{4}$$

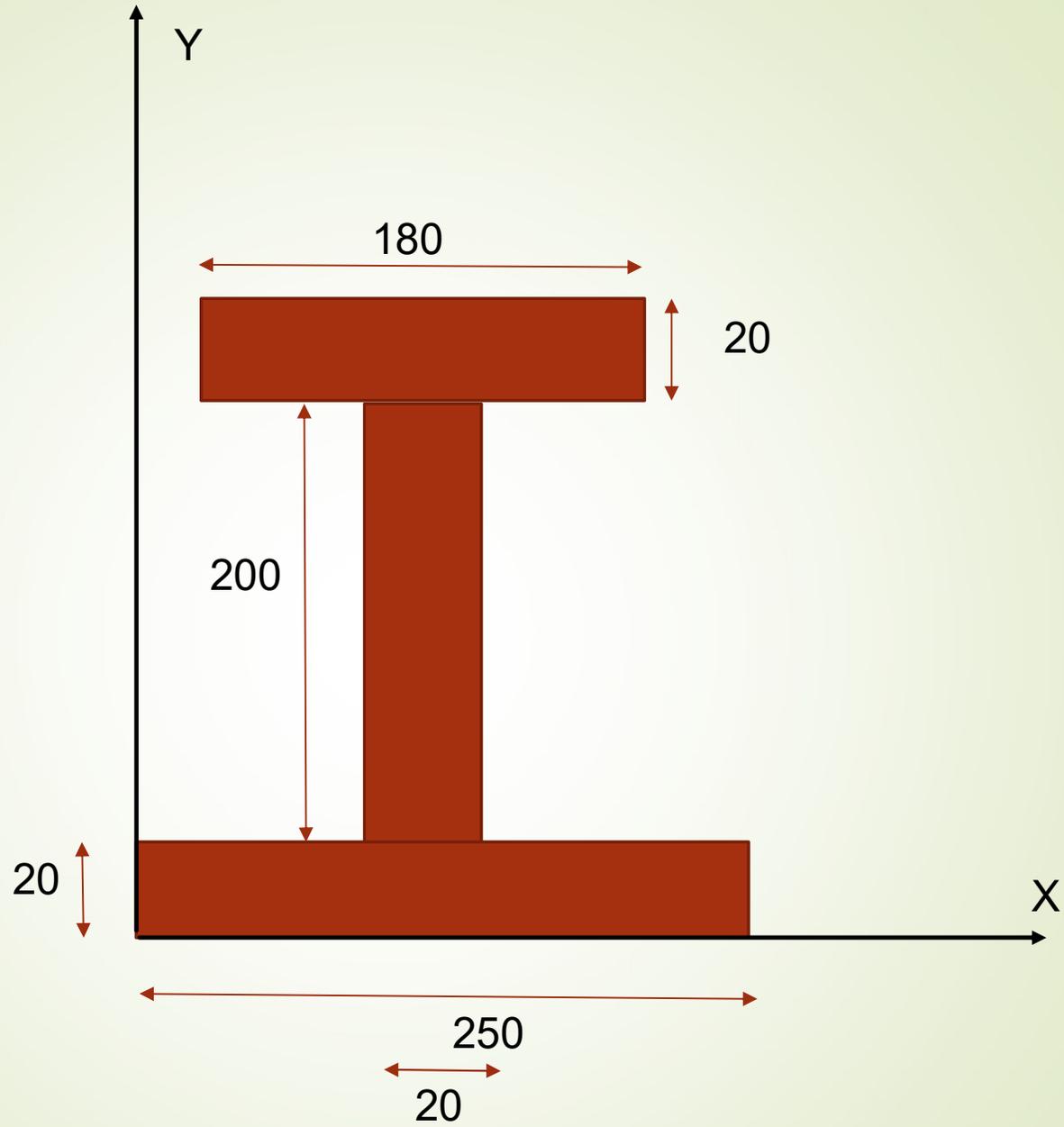
Exemple 1

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Exemple 2

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Exemple 3

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