University of Tlemcen
Faculty of Sciences
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1st Year LMD-M and MI

Correction of SW N°4 of Mechanics

Relatif Motion

EXERCISE 1

$\overrightarrow{OM} = r\overrightarrow{U_x}$ in the moving (oXY) reference frame (polar coordinates).

Relative velocity : $\vec{v_r} = \frac{d\overrightarrow{O'M}}{dt}/(OXY) \; = \frac{dr}{dt}\overrightarrow{U_x}\vec{v_r} = \dot{r}\overrightarrow{U_x}$

Relative acceleration: $\vec{a_r} = \frac{d\vec{v_r}}{dt}/(OXY)$ avec$\vec{v_r} = \dot{r}\,\overrightarrow{\mathrm{U_x}}$ So $\vec{a_r} = \frac{d^2r}{dt^2}\overrightarrow{\mathrm{U_x}} = \ddot{r}\,\overrightarrow{\mathrm{U_x}}$

Training speed: $\overrightarrow{OO'} = \vec{0}$ because both markers have the same origin.

$\vec{v_e} = \frac{d\overrightarrow{OO'}}{dt} + \vec{\omega}\Lambda\overrightarrow{O'M}$ **with** $\overrightarrow{OO'} = \vec{0}$ so $\vec{v_e} = \vec{\omega}\Lambda\overrightarrow{O'M}$

$$\vec{\omega}\Lambda\overrightarrow{O'M} = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ r & 0 & 0 \end{vmatrix} = \omega r\,\overrightarrow{U_y} \text{ so } \vec{v_e} = \omega\, r\,\overrightarrow{U_y}$$

Training acceleration: $\vec{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2} + \vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) + \frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M}$

$\frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M} = \vec{0}$ because ω constant and $\frac{d^2\overrightarrow{OO'}}{dt^2} = \vec{0}$

$$\vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) = \vec{\omega}\Lambda\left(\omega\, r\,\overrightarrow{U_y}\right) = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ 0 & \omega\, r & 0 \end{vmatrix} = -\omega^2\, r\,\overrightarrow{U_x}$$

Then $\vec{a_e} = -\omega^2 r\overrightarrow{U_x}$

Coriolis acceleration : $\vec{a_c} = 2\vec{\omega}\Lambda\vec{v_r} = 2\begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ \dot{r} & 0 & 0 \end{vmatrix} = 2\omega\dot{r}\overrightarrow{U_y}$

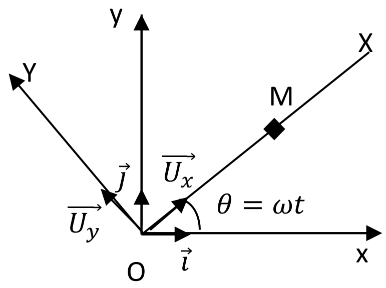
So $\vec{a_c} = 2\omega\dot{r}\overrightarrow{U_y}$

Absolute velocity: $\vec{v_a} = \vec{v_r} + \vec{v_e} = \dot{r}\overrightarrow{U_x} + \omega\, r\,\overrightarrow{U_y}$

Absolute acceleration : $\vec{a_a} = \vec{a_r} + \vec{a_c} + \vec{a_e} = (\ddot{r} - \omega^2 r)\overrightarrow{U_x} + 2\omega\dot{r}\overrightarrow{U_y}$

EXERCISE 2

The fixed frame of reference and the moving frame of reference have the same origin, so O' and O are the same.



Then, $\overrightarrow{OO'} = \vec{0}$ and $\overrightarrow{OM} = \overrightarrow{O'M} = \frac{1}{2}\gamma t^2 \overrightarrow{U_x}$

with $\overrightarrow{U_x} = \cos\omega t\, \vec{i} + \sin\omega t\, \vec{j}$

and $\overrightarrow{U_y} = (-\sin\omega t\, \vec{i} + \cos\omega t\, \vec{j})$

we have also $\vec{\omega} = \begin{pmatrix} 0 \\ 0 \\ \omega \end{pmatrix}$ and $\frac{d\omega}{dt} = 0$

1. Absolute velocity

$\vec{v_r} = \frac{d\overrightarrow{O'M}}{dt}/(OXY)$ with $\overrightarrow{O'M} = \overrightarrow{OM} = \frac{1}{2}\gamma t^2\, \overrightarrow{U_x}$ so $\vec{v_r} = \gamma t \overrightarrow{U_x}$

Absolute acceleration :

$$\vec{a_r} = \frac{d\vec{v_r}}{dt}/(O'XY) \quad \text{with } \vec{v_r} = \gamma t\, \overrightarrow{U_x}$$

So $\vec{a_r} = \frac{d^2\overrightarrow{O'M}}{dt^2} = \gamma\, \overrightarrow{U_x}$

2. Entrainment velocity :

$\vec{v_e} = \frac{d\overrightarrow{OO'}}{dt} + \vec{\omega}\Lambda\overrightarrow{O'M}$ with $\overrightarrow{OO'} = \vec{0}$ so $\vec{v_e} = \vec{\omega}\Lambda\overrightarrow{O'M}$

$\vec{\omega}\Lambda\overrightarrow{O'M} = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ \frac{1}{2}\gamma t^2 & 0 & 0 \end{vmatrix} = \omega\frac{1}{2}\gamma t^2\, \overrightarrow{U_y}$ so $\vec{v_e} = \omega\, \frac{1}{2}\gamma t^2\, \overrightarrow{U_y}$

Entrainment acceleration

$\vec{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2} + \vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) + \frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M}$; $\frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M} = \vec{0}$ because ω constant and $\frac{d^2\overrightarrow{OO'}}{dt^2} = \vec{0}$

And $\vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) = \vec{\omega}\Lambda\left(\omega\frac{1}{2}\gamma t^2\ \overrightarrow{U_y}\right) = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ 0 & \omega\frac{1}{2}\gamma t^2 & 0 \end{vmatrix} = -\omega^2\frac{1}{2}\gamma t^2\, \overrightarrow{U_x}$

So $\vec{a_e} = -\omega^2\frac{1}{2}\gamma t^2\, \overrightarrow{U_x}$

3. Coriolis acceleration

$\vec{a_c} = 2\vec{\omega}\Lambda\vec{v_r} = 2\begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ \gamma t & 0 & 0 \end{vmatrix} = 2\omega\gamma t\overrightarrow{U_y}$ so $\vec{a_c} = 2\omega\gamma t\overrightarrow{U_y}$

4. **Absolute velocity :**

$\vec{v_a} = \vec{v_r} + \vec{v_e} = \gamma t\vec{U_x} + \ \omega\ \frac{1}{2}\gamma t^2\ \vec{U_y}$

$= \ \gamma t(\cos\omega t\,\vec{\imath} + \sin\omega t)\vec{\jmath} + \omega\ \frac{1}{2}\gamma t^2\ (-\sin\omega t\,\vec{\imath} + \cos\omega t\,\vec{\jmath})$

5. **Acceleration absolute**

$\vec{a_a} = \vec{a_r} + \vec{a_c} + \vec{a_e} = (\gamma - \omega^2\frac{1}{2}\gamma t^2)\ \vec{U_x} + 2\omega\gamma t\vec{U_y}$

$\vec{a_a} = (\gamma - \omega^2\frac{1}{2}\gamma t^2)\ (\cos\omega t\,\vec{\imath} + \sin\omega t) + 2\omega\gamma t\ (-\sin\omega t\,\vec{\imath} + \cos\omega t\,\vec{\jmath})$

EXERCISE 3

Absolute velocity : $\vec{v_r} = \frac{d\overrightarrow{O'M}}{dt}/(R')$ with $\overrightarrow{O'M} = t^2\vec{U_x}$ so $\vec{v_r} = 2t\vec{U_x}$

Training velocity: $\vec{v_e} = \frac{d\overrightarrow{OO'}}{dt} + \vec{\omega}\Lambda\overrightarrow{O'M}$

First, we look for the vector $\overrightarrow{OO'}$

Point O' moves along axis (Ox) with speed v, so $\vec{v_{O'}} = \frac{d\overrightarrow{OO'}}{dt}\vec{\imath} = v\vec{\imath}$

A t=0, x=0 So $\frac{dOO'}{dt} = v \Rightarrow OO' = vt\ \ So\ \overrightarrow{OO'} = vt\vec{\imath}$

$$\vec{\omega}\Lambda\overrightarrow{O'M} = \begin{vmatrix} \vec{U_x} & \vec{U_y} & \vec{U_z} \\ 0 & 0 & \omega \\ t^2 & 0 & 0 \end{vmatrix} = \omega t^2\vec{U_y} \quad and\ \frac{d\overrightarrow{OO'}}{dt} = v\vec{\imath}$$

So $\vec{v_e} = \omega t^2\vec{U_y} + v\vec{\imath}$

We need to write $\vec{v_e}$ in the same coordinate system, so we'll write $\vec{\imath}$ as a function of $\vec{U_x}\ and\ \vec{U_y}$

We have: $\begin{cases} \vec{U_x} = cos\ \theta\vec{\imath} + \sin\theta\,\vec{\jmath} \\ \vec{U_y}\ = -\sin\theta\vec{\imath} + cos\ \ \theta\,\vec{\jmath} \end{cases} \Rightarrow \vec{\imath} = cos\ \theta\vec{U_x} - \sin\theta\ \vec{U_y}$

so $\vec{v_e} = \omega t^2\vec{U_y} + v\left(cos\ \theta\vec{U_x} - \sin\theta\ \vec{U_y}\right) = v\ cos\ \theta\vec{U_x} + (\omega t^2 - v\ sin\ \theta)\vec{U_y}$

Absolute velocity :

$\vec{v_a} = \vec{v_r} + \vec{v_e} = 2t\vec{U_x} + \ \omega t^2\vec{U_y} + v\left(cos\ \theta\vec{U_x} - \sin\theta\ \vec{U_y}\right)$

$$\Rightarrow \overrightarrow{v_a} = (2t + v\cos\theta)\overrightarrow{U_x} + (\omega t^2 - v\sin\theta)\overrightarrow{U_y}$$

Relative acceleration : $\overrightarrow{a_r} = \frac{d\overrightarrow{v_r}}{dt}/(R')$ avec $\overrightarrow{v_r} = 2t\overrightarrow{U_x}$ so $\overrightarrow{a_r} = 2\overrightarrow{U_x}$

Training acceleration: $\overrightarrow{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2} + \vec{\omega}\Lambda(\vec{\omega}\Lambda\overrightarrow{O'M}) + \frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M}$

$$\frac{d^2\overrightarrow{OO'}}{dt^2} = \vec{0}\,, \frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M} = \vec{0} \text{ because } \omega \text{ is constant}$$

And $\vec{\omega}\Lambda(\vec{\omega}\Lambda\overrightarrow{O'M}) = \vec{\omega}\Lambda\omega t^2\overrightarrow{U_y} = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ 0 & \omega t^2 & 0 \end{vmatrix} = -\omega^2 t^2\overrightarrow{U_x}$

So $\overrightarrow{a_e} = -\omega^2 t^2\overrightarrow{U_x}$

Coriolis acceleration : $\overrightarrow{a_c} = 2\vec{\omega}\Lambda\overrightarrow{v_r} = 2\begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ 2t & 0 & 0 \end{vmatrix} = 4t\omega\,\overrightarrow{U_y}$

Absolue acceleration :

$$\overrightarrow{a_a} = \overrightarrow{a_r} + \overrightarrow{a_c} + \overrightarrow{a_e} = 2\overrightarrow{U_x} - \omega^2 t^2\overrightarrow{U_x} + 4t\omega\,\overrightarrow{U_y}$$

Then $\overrightarrow{a_a}$=$(2 - \omega^2 t^2)\overrightarrow{U_x} + 4t\,\omega\overrightarrow{U_y}$

EXERCISE 4

The coordinates of point M in the moving reference frame M(t^2, t)/(R'). So $\overrightarrow{O'M}$ is written :

$\overrightarrow{O'M} = t^2\overrightarrow{U_x} + t\overrightarrow{U_y}$

O' moves on the axis (Oy) with a constant acceleration γ. At instant t=0, the axis (O'X) is confused with (Ox). So v_0=0 and y_0=0 then:

$\textit{the acceleration of } O' is: \gamma = \frac{dv}{dt} \Rightarrow dv = \gamma dt$

After integration ***v=γ.t***

and $\frac{dy}{dt} = \gamma t \Rightarrow dy = \gamma t dt$ *so* $y = \frac{1}{2}\gamma t^2$ and $\overrightarrow{OO'} = \frac{1}{2}\gamma t^2\vec{j}$

Relative velocity: $\overrightarrow{v_r} = \frac{d\overrightarrow{O'M}}{dt}/(R')$ with $\overrightarrow{O'M} = t^2\overrightarrow{U_x} + t\overrightarrow{U_y}$ so $\overrightarrow{v_r} = 2t\overrightarrow{U_x} + \overrightarrow{U_y}$

Training velocity : $\vec{v_e} = \frac{d\overrightarrow{OO'}}{dt} + \vec{\omega}\Lambda\overrightarrow{O'M}$ with $\overrightarrow{OO'} = \frac{1}{2}\gamma t^2\vec{j} \Rightarrow \frac{d\overrightarrow{OO'}}{dt} = \gamma t\vec{j}$

$$\vec{\omega}\Lambda\overrightarrow{O'M} = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ t^2 & t & 0 \end{vmatrix} = -\omega t\overrightarrow{U_x} + \omega t^2\overrightarrow{U_y} \quad so \quad \vec{v_e} = \gamma t\vec{j} - \omega t\overrightarrow{U_x} + \omega t^2\overrightarrow{U_y}$$

We need to write $\vec{v_e}$ in the same coordinate system, so we'll write $\vec{j}$ as a function of $\overrightarrow{U_x}$and $\overrightarrow{U_y}$

We have : $\begin{cases} \overrightarrow{U_x} = \cos\theta\vec{i} + \sin\theta\,\vec{j} \\ \overrightarrow{U_y} = -\sin\theta\vec{i} + \cos\,\theta\,\vec{j} \end{cases} \Rightarrow \vec{j} = \sin\theta\overrightarrow{U_x} + \cos\theta\,\overrightarrow{U_y}$

So $\vec{v_e} = \omega t^2\overrightarrow{U_y} - \omega t\overrightarrow{U_x} + \gamma t\left(\sin\theta\overrightarrow{U_x} + \cos\theta\,\overrightarrow{U_y}\right)$

$$\Rightarrow \vec{v_e} = (\gamma t\sin\theta - \omega t)\overrightarrow{U_x} + (\omega t^2 + \gamma t\cos\theta)\overrightarrow{U_y}$$

Absolue velocity : $\vec{v_a} = \vec{v_r} + \vec{v_e} = 2t\overrightarrow{U_x} + \overrightarrow{U_y} + (\gamma t\sin\theta - \omega t)\overrightarrow{U_x} + (\omega t^2 + \gamma t\cos\theta)\overrightarrow{U_y}$

$\Rightarrow \vec{v_a} = (\gamma t\sin\theta - \omega t + 2t)\overrightarrow{U_x} + (\omega t^2 + \gamma t\cos\theta + 1)\overrightarrow{U_y}$

Relative acceleration : $\vec{a_r} = \frac{d\vec{v_r}}{dt}/(R')$ avec$\vec{v_r} = 2t\overrightarrow{U_x} + \overrightarrow{U_y}$ so $\vec{a_r} = 2\overrightarrow{U_x}$

Training acceleration : $\vec{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2} + \vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) + \frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M}$

$\frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M} = \vec{0}$ because ω constant and $\frac{d^2\overrightarrow{OO'}}{dt^2} = \gamma\vec{j}$

$$and \quad \vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) = \vec{\omega}\Lambda\left(-\omega t\overrightarrow{U_x} + \omega t^2\overrightarrow{U_y}\right) = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ -\omega t & \omega t^2 & 0 \end{vmatrix} = -\omega^2 t^2\overrightarrow{U_x} - \omega^2 t\overrightarrow{U_y}$$

Then $\vec{a_e} = \vec{j} - \omega^2 t^2\overrightarrow{U_x} - \omega^2 t\overrightarrow{U_y} = \gamma\left(\sin\theta\overrightarrow{U_x} + \cos\theta\,\overrightarrow{U_y}\right) - \omega^2 t^2\overrightarrow{U_x} - \omega^2 t\overrightarrow{U_y}$

$\vec{a_e} = (\gamma\sin\theta - \omega^2 t^2)\overrightarrow{U_x} + (\gamma\cos\theta \; - \omega^2 t)\overrightarrow{U_y}$

Coriolis acceleration : $\vec{a_c} = 2\vec{\omega}\Lambda\vec{v_r} = 2\begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ 2t & 1 & 0 \end{vmatrix} = 4t\omega\overrightarrow{U_y} - 2\omega\overrightarrow{U_x}$

Absolue acceleration : $\overrightarrow{a_a} = \overrightarrow{a_r} + \overrightarrow{a_c} + \overrightarrow{a_e}$

$\Rightarrow \overrightarrow{a_a} = 2\overrightarrow{U_x} + (\gamma \sin\theta - \omega^2 t^2)\overrightarrow{U_x} + (\gamma \cos\theta \ - \omega^2 t)\overrightarrow{U_y} + 4t\,\omega\overrightarrow{U_y} - 2\omega\overrightarrow{U_x}$

So $\overrightarrow{a_a}$=$(2 - 2\omega + \gamma \sin\theta - \omega^2 t^2)\overrightarrow{U_x} + (\gamma \cos\theta \ - \omega^2 t + 4t\omega\,)\overrightarrow{U_y}$

Supplementary exercise:

$$r = r_0(\cos\ \omega t + \sin\omega t)\,\overrightarrow{U_x}$$

Relative velocity : $\overrightarrow{v_r} = \frac{d\overrightarrow{O'M}}{dt}/(R')$

O' is confused with O, then : $\overrightarrow{O'M} = \overrightarrow{O'M} \Rightarrow \overrightarrow{v_r} = \frac{d\overrightarrow{OM}}{dt}/(R')$

$$\overrightarrow{v_r} = r_0\omega(-sin\omega t + \cos\omega t)\,\overrightarrow{U_x}$$

Training velocity : $\overrightarrow{v_e} = \frac{d\overrightarrow{OO'}}{dt} + \vec{\omega}\Lambda\overrightarrow{O'M}$ with $\overrightarrow{OO'} = \vec{0} \Rightarrow \frac{d\overrightarrow{OO'}}{dt} = \vec{0}$

$$\vec{\omega}\Lambda\overrightarrow{O'M} = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ r & 0 & 0 \end{vmatrix} = \omega\, r\,\overrightarrow{U_y} \quad so\ \overrightarrow{v_e} = \omega\, r\,\overrightarrow{U_y} = \omega\, r_0(\cos\ \omega t + \sin\omega t)\,\overrightarrow{U_y}$$

Absolue velocity : $\overrightarrow{v_a} = \overrightarrow{v_r} + \overrightarrow{v_e} = r_0\omega[(-sin\omega t + \cos\omega t)\overrightarrow{U_x} + (\cos\ \omega t + \sin\omega t)\overrightarrow{U_y}]$

$\Rightarrow \lceil\overrightarrow{v_a}\rceil = r_0\omega\,\sqrt{(-sin\omega t + \cos\omega t)^2 + (\cos\ \omega t + \sin\omega t)^2}$

Then $\lceil\overrightarrow{v_a}\rceil = r_0\omega\sqrt{2}$ so $\lceil\overrightarrow{v_a}\rceil$ is constant

Relative acceleration : $\overrightarrow{a_r} = \frac{d\overrightarrow{v_r}}{dt}/(R')$ $\quad \overrightarrow{v_r} = r_0\omega(-sin\omega t + \cos\omega t)\,\overrightarrow{U_x}$

so $\overrightarrow{a_r} = r_0\omega^2(-\cos\omega t - sin\omega t)\,\overrightarrow{U_x}$

Training acceleration : $\overrightarrow{a_e} = \frac{d^2\overrightarrow{OO'}}{dt^2} + \vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) + \frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M}$

avec $\frac{d\vec{\omega}}{dt}\Lambda\overrightarrow{O'M} \ = \vec{0}$ because ω constant and $\frac{d^2\overrightarrow{OO'}}{dt^2} = \vec{0}$

and $\vec{\omega}\Lambda\left(\vec{\omega}\Lambda\overrightarrow{O'M}\right) = \vec{\omega}\Lambda\left(\omega\, r\,\overrightarrow{U_y}\right) = \begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ 0 & \omega\, r & 0 \end{vmatrix} = -\omega^2\, r\overrightarrow{U_x}$

then $\quad \overrightarrow{a_e} = -\omega^2 r_0(\cos\ \omega t + \sin\omega t)\overrightarrow{U_x}$

Coriolis acceleration: $\overrightarrow{a_c} = \ 2\vec{\omega}\Lambda\overrightarrow{v_r} = 2\begin{vmatrix} \overrightarrow{U_x} & \overrightarrow{U_y} & \overrightarrow{U_z} \\ 0 & 0 & \omega \\ v_r & 0 & 0 \end{vmatrix} = 2\omega v_r\overrightarrow{U_y}$

so $\overrightarrow{a_c} = 2\omega v_r\overrightarrow{U_y} = 2r_0\omega^2(-sin\omega t + \cos\omega t)\overrightarrow{U_y}$

Absolue acceleration : $\overrightarrow{a_a} = \overrightarrow{a_r} + \overrightarrow{a_c} + \overrightarrow{a_e}$

$\overrightarrow{a_a} = -r_0\omega^2(\cos\omega t + sin\omega t)\,\overrightarrow{\mathrm{U_x}} - \omega^2 r_0(\cos\ \omega t + \sin\omega t)\overrightarrow{U_x} + 2r_0\omega^2(-sin\omega t + \cos\omega t)\overrightarrow{U_y}$

$\Rightarrow \overrightarrow{a_a} = -2r_0\omega^2(\cos\ \omega t + \sin\omega t)\overrightarrow{U_x} + 2r_0\omega^2(-sin\omega t + \cos\omega t)\overrightarrow{U_y}$

$\lceil\overrightarrow{a_a}\rceil = 2r_0\omega^2\ \sqrt{(-(\cos\ \omega t + \sin\omega t))^2 + (-sin\omega t + \cos\omega t)^2}$

Then $\lceil\overrightarrow{a_a}\rceil = 2\, r_0\omega^2\sqrt{2}$ donc $\lceil\overrightarrow{a_a}\rceil$ is constant.