Continuous Mechanics Test

Exercise 1: (05 Pts)

A. The trajectory (المسار) y=f(x) of a projectile with an initial velocity($\boldsymbol{v_0}$) from a point (O) located at heigh (h) above the impact plane is given by the following formula:

$$\boldsymbol{y = \frac{g}{2v_0^2}x^2 + h}$$

If g is a gravity acceleration, show that this formula is homogeneous.

B. The expression for the period T of the simple pendulum formed of a ball (sphere) of radius R and mass m is given by the following formula: $\boldsymbol{T = k\frac{\rho R^2}{\eta}}$

With k is constant and $\boldsymbol{\rho}$ is volume mass (we give the volume of a ball is V=$\frac{4}{3}\pi R^3$).

Determine the relative uncertainty $\frac{\Delta T}{T}$ based on $\boldsymbol{\Delta\eta, \Delta R\ and\ \Delta m}$.

Exercise 2: (05 Pts)

A material point M is marked by its Cartesian coordinates (x, y) and its polar coordinates (ρ.θ).

1- Give the transition relations between these two coordinate systems.

2- Show the velocity of point M in polar coordinates.

3- What happens to this speed if : $\rho = \sqrt{t+1}$ *and* $\theta = \omega t$. (ω constant)

Exercise 3: (05 Pts)

A comet (مذنب) is moving through the solar system. His position is given by:

$$\overrightarrow{OM} = (t-1)\vec{\imath} + \frac{t^2}{2}\vec{\jmath}$$

Where O is the origin of the landmark (the sun) and t represents the time expressed in seconds. We assume that the comet remains in the plane (Oxy).

1. Write the equation of the trajectory.
2. Determine the components of the velocity vector $\vec{v}$ and the acceleration vector $\vec{a}$. Deduce the nature of the movement.
3. Give the expressions of the tangential a_T and normal a_N accelerations and deduce the radius of curvature.

Good luck

University of Tlemcen
Faculty of Science
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Duration: 01 h 30mn

Academic Year 2024/2025
1st Year LMD-M and MI

Correction of Continuous Mechanics Test

Exercise 1: (5 pts)

A. This expression is homogeneous if : $[y] = \left[\frac{g}{2v_0^2}x^2\right] = [h]$ **(0.5 pts)**

We have : $[g] = LT^{-2}, [y] = [h] = L \; and \; [v] = LT^{-1}$ **(01 pts)**

$\left[\frac{g}{2v_0^2}x^2\right] = \left[\frac{1}{2}\right]\left[\frac{g}{v_0^2}\right][x]^2 = 1\,\frac{LT^{-2}}{(LT^{-1})^2}L^2 = L,$ **(0.5 pts)**

So : $[y] = \left[\frac{g}{2v_0^2}x^2\right] = [h]$ is checked.

Hence the equation $y = \frac{g}{2v_0^2}x^2 + h$ is homogeneous. **(0.5 pts)**

B. The relative uncertainty on T= f($\Delta\eta, \Delta R, \Delta m$) ?

$$T = \frac{K\rho R^2}{\mu} \quad \text{With} \quad \rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi R^3} \quad \text{so } \boldsymbol{T} = \frac{\boldsymbol{3Km}}{\boldsymbol{4\pi R\mu}} \; \textbf{(0.5 pts)}$$

$$\Rightarrow \log T = \log\left(\frac{3mK}{4\pi R\mu}\right) = log3K + \log(m) - \log(4\pi) - \log(R) - \log(\mu) \; \textbf{(0.5 pts)}$$

$$\Rightarrow \boldsymbol{d}\log T = dlog3K + d\log(m) - \text{dlog}(4\pi) - d\log(R) - d\log(\mu) \; \textbf{(0.25 pts)}$$

$$\Rightarrow \frac{dT}{T} = \frac{dm}{m} - \frac{dR}{R} - \frac{d\mu}{\mu} \; \textbf{(0.25 pts)}$$

$$\Rightarrow \frac{\Delta T}{T} = \left|\frac{\Delta m}{m}\right| + \left|-\frac{\Delta R}{R}\right| + \left|-\frac{\Delta\mu}{\mu}\right| \; \textbf{(0.5 pts)}$$

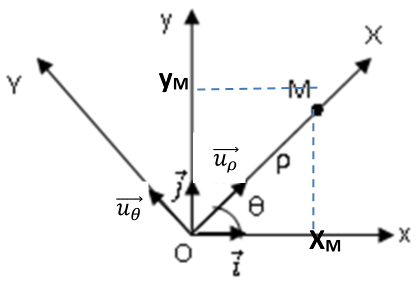
m, R, and μ are positive quantities, hence:

$$\Rightarrow \frac{\Delta T}{T} = \frac{\Delta m}{m} + \frac{\Delta R}{R} + \frac{\Delta\mu}{\mu} \; \textbf{(0.5 pts)}$$

Exercise 2 : (05 pts)

1- The transition relationships between Polar and Cartesian coordinates are.

$$\begin{cases} cos\theta = \frac{x_M}{\rho} \\ Sin\theta = \frac{y_M}{\rho} \end{cases} \Rightarrow \begin{cases} x_M = \rho cos\theta \\ y_M = \rho sin\theta \end{cases} \; \textbf{(01 pts)}$$



So the vector $\overrightarrow{OM}$ in coordinates cartesian

is written $\overrightarrow{OM} = x_M\vec{i} + y_M\vec{j}$ **(0.25 pts)**

We had : $\overrightarrow{OM} = \rho\overrightarrow{u_\rho}$ (in polar coordinates) **(0.25 pts)**

Then $\overrightarrow{OM} = \rho(cos\theta\vec{i} + sin\theta\vec{j})$ **(0.5 pts)**

By identification $\overrightarrow{u_\rho} = cos\theta\vec{i} + sin\theta\vec{j}$ **(0.5 pts)**

and $\overrightarrow{u_\theta} = \frac{d\,\overrightarrow{u_\rho}}{d\theta} = -sin\theta\vec{i} + cos\theta\vec{j}$ **(0.5 pts)**

2- The velocity vector $\vec{v}$ of point M in polar coordinates will be:

$$\Rightarrow \vec{v} = \frac{\mathrm{d}\overrightarrow{OM}}{\mathrm{dt}} = \frac{\mathrm{d}\rho}{\mathrm{dt}}\overrightarrow{u_\rho} + \rho\,\frac{d\overrightarrow{u_\rho}}{dt} = \rho'\overrightarrow{u_\rho} + \rho\,\frac{d\theta}{dt}\frac{d\overrightarrow{U_\rho}}{d\theta} = \rho'\overrightarrow{u_\rho} + \rho\theta'\overrightarrow{U_\theta} \; \textbf{(0.5 pts)}$$

$$\vec{v} = \rho'\overrightarrow{u_\rho} + \rho\theta'\overrightarrow{U_\theta} \; \textbf{(0.5 pts)}$$

3- If: $\rho = \sqrt{t+1}$ and $\theta = \omega t.$

$$\Longrightarrow \vec{v} = \frac{1}{2\sqrt{t+1}}\overrightarrow{U_\rho} + \sqrt{t+1}.\omega.\overrightarrow{U_\theta}$$ **(01 pts)**

<u>Exercise 3</u> : (05 pts)

$$\overrightarrow{OM} = (t-1)\vec{\imath} + \frac{t^2}{2}\vec{\jmath}$$

1- The equation of the trajectory

$$\overrightarrow{OM} = (t-1)\vec{\imath} + \frac{t^2}{2}\vec{\jmath} \Rightarrow \begin{cases} x = t-1 \\ y = \frac{t^2}{2} \end{cases}$$

$t=x+1 \Rightarrow y = \frac{(x+1)^2}{2}$ **(0.5 pts)**

2- The components of velocity and acceleration, and their magnitudes :

- Velocity

$$\begin{cases} v_x = \frac{dx}{dt} \\ v_y = \frac{dy}{dt} \end{cases} \Rightarrow \begin{cases} v_x = 1 \\ v_y = t \end{cases}$$ **(01 pts)** $\vec{v} = v = \vec{\imath} + t\vec{\jmath} \Rightarrow |\vec{v}| = \sqrt{1+t^2}$ **(0.25 pts)**

- Acceleration

$$\begin{cases} a_x = \frac{dv_x}{dt} \\ a_y = \frac{dv_y}{dt} \end{cases} \Rightarrow \begin{cases} a_x = 0 \\ a_y = 1 \end{cases}$$ **(01 pts)** $\vec{a} = 1\vec{\jmath} \Rightarrow |\vec{a}| = a = 1$ **(0.25 pts)**

- The nature of the movement

  $\vec{a}.\vec{v} = t > 0$ so The movement in this case is uniformly accelerated. **(0.5 pts)**

3- Normal and tangential accelerations

- Tangential acceleration

$$a_T = \frac{d|\vec{v}|}{dt} = \frac{d\left(\sqrt{1+t^2}\right)}{dt} = \frac{t}{\sqrt{t^2+1}}$$ **(0.5 pts)**

- Normal acceleration

  We have $a^2 = a_T^2 + a_N^2$ so $a_N^2 = a^2 - a_T^2$

$a_N^2 = 1 - \frac{t^2}{t^2+1} \Rightarrow a_N^2 = \frac{1}{v^2} \Rightarrow a_N = \frac{1}{v}$ **(0.5 pts)**

- The radius of curvature

$$a_N = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_N} = \frac{v^3}{1} = v^3$$ **(0.5 pts)**