University of Tlemcen
Faculty of Sciences
Duration : 01 h 30mn

Academic Year: 2024/2025

Final Exam of Mechanics

(For students M and MI with debt)

Exercise 1 : (6pts)

Let the simple pendulum formed of a ball (sphere) of **radius R** (نصف قطر) and **mass m.** The study of the effect of the air on this pendulum shows that its **period T (الدور)** depends on a **constant k**, the coefficient of the air η, the radius of the ball R and its **density ρ (الكتلة الحجمية)**.

1- Find the expression of the period assuming the form:

$$T = K\eta^x R^y \rho^z \ \ with \ [\eta] = ML^{-1}T^{-1}$$

2- The average velocity of the molecules in a gas is given by the following formula: $v = \sqrt{\frac{PV}{m}}$

Determine the relative uncertainty on v based on $\Delta p, \Delta V \ and \ \Delta m$.

Exercise 2 : (6pts)

A particle is launched with an initial horizontal speed v_0 according to the time-dependent equations:

$$\begin{cases} x = v_0 t \\ y = \frac{1}{2} g t^2 \end{cases}$$

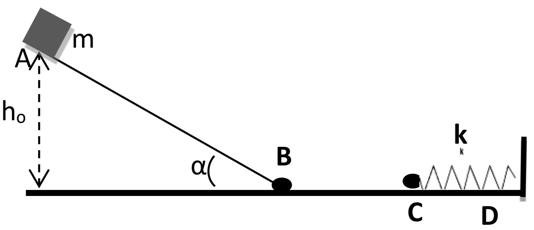
Determine:

1. The trajectory equation.
2. The components of speed and its module.
3. The components of acceleration and its module.
4. Tangential and normal accelerations.

Exercise 3 : (8pts)

Consider a small block of mass m =5kg dropped without initial velocity at point A of an inclined plane at an angle α=30° to the horizontal. Point A is at a height h_0=5m from the horizontal.

1- Knowing that the coefficient of dynamic friction on plane AB is μ_d=0.2, applying the fundamental principle of dynamics:



- What is the nature of the motion on plane AB?
- Calculate the speed of the block when it reaches point B.

2- After passing through point B at speed V_B, the mass arrives at point C. Knowing that the coefficient of friction is negligible on plane BC :

- Deduce the speed at point C?
- Calculate the maximum compression of the spring, given a stiffness constant equal to k=100N/m? (g =10 m/s^2).

Bon courage

The correction of final exam

Exercise 1: (6 pts)

1- The period of a pendulum is written :

$T = K\eta^x R^y \rho^z$ such $[\eta] = ML^{-1}T^{-1}$

Suppose the relationship is homogeneous so $[T] = [k][\eta]^x[R]^y[\rho]^z$ **(0.5 pts)**

With $\begin{cases} [\eta] = ML^{-1}T^{-1} \\ [R] = L \; and \; [k] = 1 \\ [\rho] = \left[\frac{m}{V}\right] = \frac{M}{L^3} = ML^{-3} \\ [T] = T \end{cases}$ **(01 pts)**

So $[T] = (ML^{-1}T^{-1})^x L^y (ML^{-3})^z = T$

$$\Rightarrow T = M^x L^{-x} T^{-x} L^y \quad M^z L^{-3z} \Rightarrow M^0 L^0 T^1 = M^{x+z} L^{-x+y-3z} \quad T^{-x}$$

$$by\ identification: \begin{cases} x + z = 0 \\ -x + y - 3z = 0 \\ -x = 1 \end{cases}$$

$\Rightarrow \begin{cases} x = -1 \\ y = x + 3z = 2 \\ z = -x = 1 \end{cases}$ **(0.75 pts)** $\Rightarrow T = K\eta^{-1}R^2\rho^1$

So $\boldsymbol{T = k\frac{\rho R^2}{\eta}}$ **(0.75 pts)**

2- The relative uncertainty on $\boldsymbol{\vartheta} = \sqrt{\frac{PV}{m}}$

$$\Rightarrow \boldsymbol{\vartheta^2} = \frac{PV}{m} \Rightarrow \log(\boldsymbol{\vartheta^2}) = \log\frac{PV}{m}$$ **(01 pts)**

$$\Rightarrow 2\log\vartheta = \log P + \log V - \log m$$

$$\Rightarrow 2\frac{d\vartheta}{\vartheta} = \frac{dP}{P} + \frac{dV}{V} + \frac{dm}{m}$$ **(01 pts)**

$$\Rightarrow 2\frac{\Delta\vartheta}{\vartheta} = \frac{\Delta P}{P} + \frac{\Delta V}{V} + \frac{\Delta m}{m}$$

$$\Rightarrow \frac{\Delta\vartheta}{\vartheta} = \frac{1}{2}\left(\frac{\Delta P}{P} + \frac{\Delta V}{V} + \frac{\Delta m}{m}\right)$$ **(01 pts)**

Exercise 2 : (6 pts)

The x and y coordinates of a mobile point M in the (xy) plane vary wit

h time t according to the following relationships: $\begin{cases} x = v_0 t \\ y = \frac{1}{2} g t^2 \end{cases}$

1- The equation of the trajectory is then written as follows:

Here, we will express t as a function of x: $t = \frac{x}{v_0}$ So $y = \frac{1}{2} g \left(\frac{x}{v_0}\right)^2 = \frac{g}{2v_0^2} \boldsymbol{x^2}$

The equation of the trajectory is: $\mathbf{y(x)} = \frac{g}{2v_0^2} \boldsymbol{x^2}$ **(0.5 pts)**

The components of the velocity

$$\begin{cases} v_x(t) = \frac{dx(t)}{dt} = v_0 \\ v_y(t) = \frac{dy(t)}{dt} = gt \end{cases}$$ **(01 pts)**

The velocity is expressed as: $\overrightarrow{\mathbf{v(t)}} = v_0 \vec{\mathbf{i}} + gt\, \vec{\mathbf{j}}$

The magnitude of the velocity: $|\vec{v}(t)| = \sqrt{v_0^2 + (gt)^2} = \sqrt{v_0^2 + g^2 t^2}$ **(0.5 pts)**

The components of the acceleration

$$\begin{cases} a_x(t) = \frac{dv_x(t)}{dt} = 0 \\ a_y(t) = \frac{dv_y(t)}{dt} = g \end{cases}$$ **(01 pts)** The acceleration is expressed as: $\overrightarrow{\mathbf{a(t)}} = \mathbf{g}\vec{\mathbf{j}}$

The magnitude of the acceleration $|\vec{a}(t)| = g$ **(0.5 pts)**

2- The nature of the movement

$\overrightarrow{a(t)}.\overrightarrow{v(t)} = v_0(0) + gt(g) = g^2 t > 0$ **(0.25 pts)**

The movement in this case is uniformly accelerated. **(0.25 pts)**

3- Normal and tangential accelerations.

- Tangential acceleration:

$a_T = \frac{d|\overrightarrow{v(t)}|}{dt}$ avec $|\overrightarrow{v(t)}| = \sqrt{v_0^2 + g^2 t^2}$ so $a_T = \frac{d(\sqrt{v_0^2+g^2t^2})}{dt} = \frac{2g^2 t}{2\sqrt{v_0^2+g^2t^2}}$

$a_T = \frac{g^2 t}{\sqrt{v_0^2+g^2t^2}} = \frac{g^2 t}{v}$ **(01 pts)**

- Normal acceleration

The accelerations a_N and a_T are the normal and tangential components of the acceleration vector $\vec{a}$.

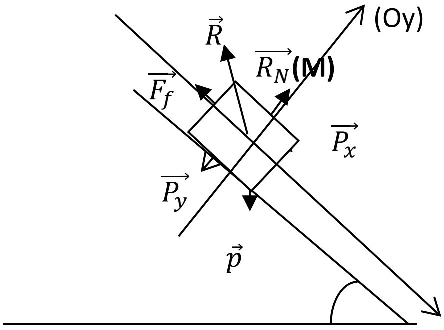
$(\vec{a} = a_T \overrightarrow{U_T} + a_N \overrightarrow{U_N}) \Rightarrow a^2 = a_T^2 + a_N^2$ so $a_N^2 = a^2 - a_T^2$

$$a_N^2 = g^2 - \left(\frac{g^2 t}{\sqrt{v_0^2 + g^2t^2}}\right)^2 = g^2 - \frac{g^4t^2}{v_0^2 + g^2t^2} \Rightarrow a_N^2 = \frac{g^2v_0^2 + g^4t^2 - g^4t^2}{v_0^2 + g^2t^2}$$

So $a_N = \sqrt{\frac{g^2v_0^2}{v_0^2+g^2t^2}} = \frac{gv_0}{v}$ **(01 pts)**

Exercise 3:

(0.5 pts)



1. Knowing that the coefficient of dynamic friction on plane AB is μ_d=0.2, apply the fundamental principle of dynamics:

- What is the nature of the motion on AB? **a= ?**

$\Sigma\vec{F} = m\vec{a} = \vec{p} + \overrightarrow{R_N} + \overrightarrow{F_f}$ **(0.5 pts)**

Following (Ox): $-F_f + p_x = -F_f + m\,g\sin\alpha = ma$ **(0.5 pts)**

Following (Oy): $R_N - p_y = 0 \Rightarrow R_N = m\,g\cos\alpha$ **(0.5 pts)**

$\mu_d = tg\varphi = F_f/R_N \Rightarrow F_f = R_N\, tg\,\varphi = \mu_d\, m\, g\cos\alpha$ **(0.5 pts)**

$-\mu_d\, m\, g\cos\alpha + m\, g\sin\alpha = m.a$

$\Rightarrow a = g.(\sin\alpha - \mu_d\cos\alpha)$

So: **a = 3.26 m/s²**. **(0.5 pts)**

- Calculate the speed of the block when it reaches point B.

$v_B^2 - v_A^2 = 2al \Rightarrow v_B^2 = 2al = 2a(\frac{h}{sin\alpha})$ **(0.5 pts)**

$v_B = \sqrt{2a(\frac{h}{sin\alpha})} = 8.074\, m/s$ **(0.5 pts**

2. $V_c = V_B$ because we have an MRU (principle of inertia or Newton's 1st law)

We calculate the compression distance of the spring:

$\Delta Ec = \Sigma W_{f_{ext}} \Rightarrow Ec_D - Ec_C = W_p + W_{Fr} + W_{RN}$ **(0.5 pts)**

$-\frac{1}{2}kx^2 = -\frac{1}{2}mv_c^2$ so: $x = \sqrt{\frac{mv_c^2}{k}} = 1.8\, m$ **(0.5 pts)**

2nd Method: Between points C and D

$E_{M_C} = E_{M_D} \Rightarrow E_{C_C} + E_{P_C} = E_{C_D} + E_{P_D} \Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv_c^2$ **(0.5 pts)**

So; $x = \sqrt{\frac{mv_c^2}{k}} = 1.8\, m$ **(0.5 pts)**