University of Tlemcen
Faculty of Sciences
Department of Mathematics

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1ST YEAR LMD-M

ELECTRICITY COURSE

Chapter I: Electrostatic

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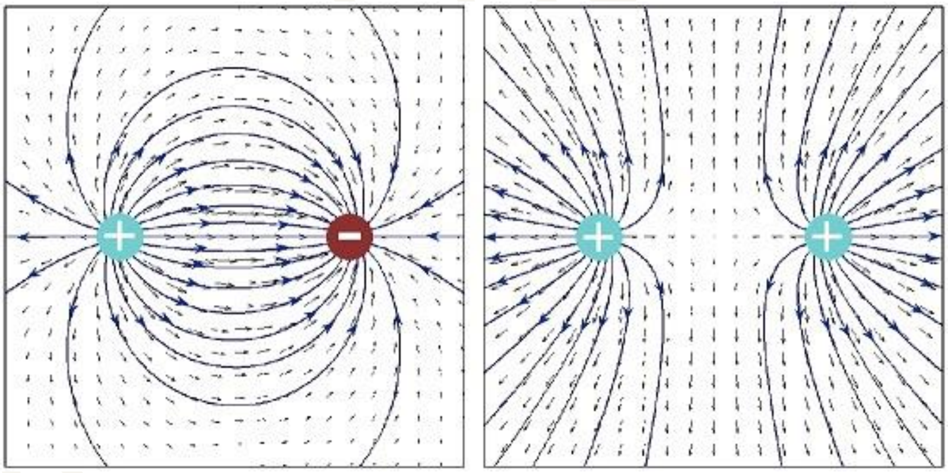

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Summary

Part 1: Electrical point charges

Part 2: Continuous charge distributions

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Part 1: Electric point charges

1. Introduction

Room lamps, electric clocks, microphones, calculators, televisions and computers only work in the presence of electricity. Cars, trains, planes (and even rockets) can't start up without electricity.

The human body needs electricity to contract its muscles (the heart, for example). The nervous system is the basis of electricity. Atoms and molecules and all chemical reactions exist because of electricity.

In 1908, Robert Andrews Millikan (1868-1953) set out to measure the elementary charge of the electron by measuring the electrical force that counteracts gravity on a drop of oil. After numerous refinements, he published the first results of his experiment in 1913. In fact, Millikan simply measured the speed of a droplet of oil he ionized by irradiating it with X-rays, using the ratio of the distance travelled to the time taken to cover it. He observed experimentally that the ionization values were all integer multiples of e = 1.592 1019 C, the constant we know today as the elementary charge.

One day, while Etienne Gray was experimenting with his leaded glass tube, which he had previously rubbed, he noticed that the cork, which closed the tube at one end, attracted a fragment of down. Yet the cork had not been rubbed. It is said that electric fluid can pass from a rubbed body to an un-rubbed one, Figure I. 1.

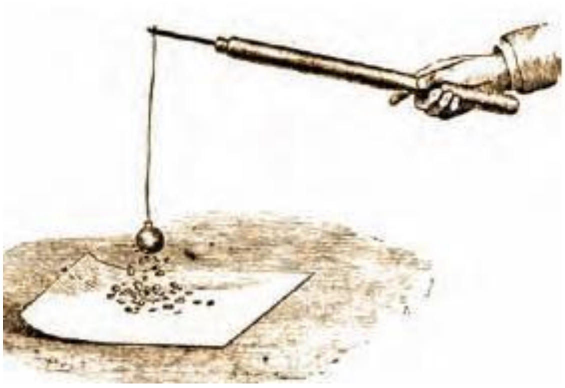

Figure I.1: Electrification experiment N°1

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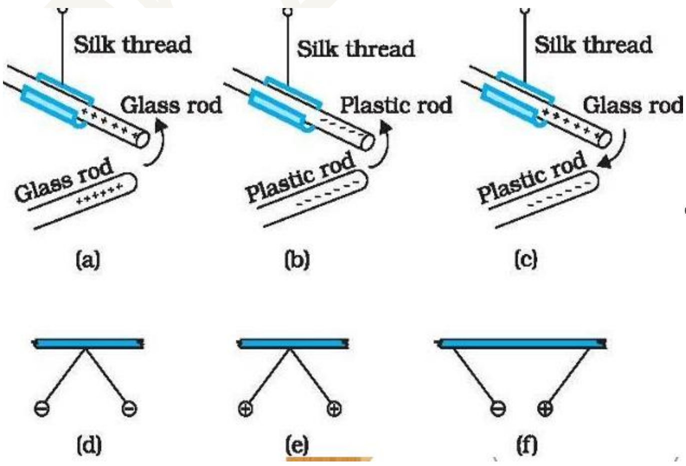
The Englishman Wheler tried the same experiment with a silk cord, observing that the electric fluid was not transmitted. Following these two experiments and another with humans, it was assumed that there were electrically conductive and insulating materials.

In 1733, Charles-François de Cisternay Dufay discovered that there were in fact two different electricities, and that two similar electricities repelled each other, while two different electricities attracted each other. He called vitreous electricity due to the electricity of glass, and resinous electricity due to the electricity of amber. Hereafter, we call positive electricity (glass electricity) and negative electricity (resin electricity) Figure I. 2.

Figure I.2: Electrification experiment N°2

Another experiment you can do yourself is to rub a glass rod with fabric (wool, cotton, fur) for 10 seconds, then approach the rod with an inflated balloon covered in Aluminium. The balloon attracts and follows the rod.



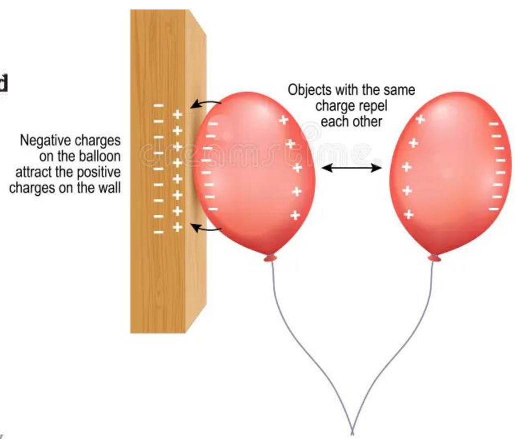


Figure I.3: Electrification experiment N°3

Explanation:

Rubbing a glass rod against a cloth is said to transfer charges. This is the phenomenon of electrifying bodies by transferring charges between two bodies (triboelectricity). This charge transfer is driven by the willingness of different materials to receive or give up electrons (a chemical unit). Thus, amber rubbed with wool or silk becomes negatively charged, whereas rubbing it with celluloid (a plastic material) makes it positively charged!

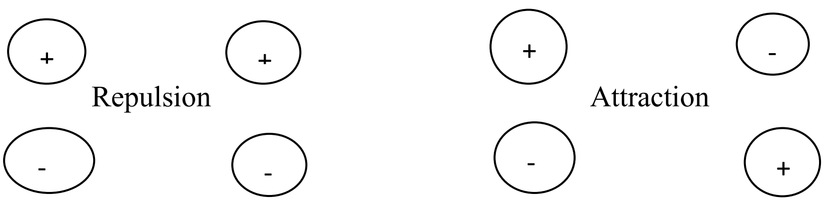
A mechanical force of attraction is therefore created between these two bodies (since the ball moves towards the rod!), one (rod) positively charged and the other (fabric) negatively charged. This force acts without contact between the two bodies (at a distance).

2. Electric charge

Matter is formed by chemical species linked together by bonds to maintain their cohesion. These chemical species are atoms, identified in Mendeleev's table by their atomic numbers. These atoms, which are electrically neutral, are made up of charged particles, namely electrons and protons. Ionizing an atom means removing negative charges from it, making it positively charged; adding negative charges makes it negatively charged.

Charging a material means ionizing its atoms, which means that the charge is a multiple of the elementary charge and is worth 1.610^{-19} ***Coulomb***. If the volume of each charged body is small in front of all other dimensions, an approximation is to identify each of these bodies with a point. This mathematical abstraction is known as the point charge approximation. Electrostatics is the study of phenomena produced by the presence of electric charges at rest.

There are two types of electric charge: positive and negative. Two charges of the same sign repel each other, while two charges of opposite signs attract each other.



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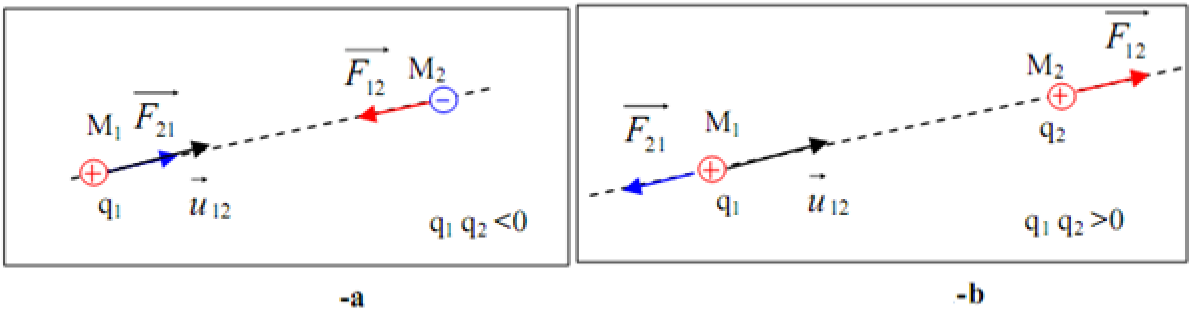


Figure I.4: The two types of electrostatic forces: a- attractive and b- repulsive

Notes :

The electric charge carried by a body can only take on quantified values. This quantum was first measured in 1909 by Robert Millikan: "$e=1.6.10^{-19}$C".

We refer to the quantity of electric charge carried by a body as "q" (any charge must be equal to an integer multiple of "e").

The charge of an electron is : $q_e= -e=-1.6\ 10^{-19}$ C and $m_e=9.1\ 10^{-31}$kg

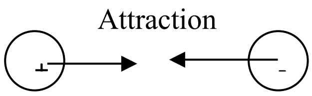
The charge of the proton : $q_p=1.6\ 10^{-19}$ C and $m_p=e=1.67\ 10^{-31}$kg

3. Electric point charges الشحنة الكهربائية النقطية

3.1. Electrostatic force القوة الكهربائية (Coulomb's law)

All objects with an electrical charge can cause an electrical force (charge plays the same role in electricity as mass does in mechanics).

Two charges in contact behave differently, either attracting each other or pulling apart, which explains the presence of an electric force.



Analogy:

In his work on the interaction between two immobile (static) electric charges, Coulomb, using a device known as a torsion balance and drawing on the law of gravitation previously

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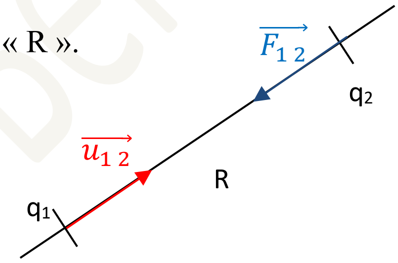
established by Isaac Newton (1643/1727), was able to make a number of pertinent observations on the phenomenon under study, namely:

1. The force exerted by one of the charges on the other is radial, i.e. carried by the straight line connecting the two charges.
2. The force is proportional to the product of the loads.
3. The force varies as the inverse of the square of the distance between the two charges.
4. The force is attractive between two charges of opposite sign (different polarity) and repulsive between two charges of the same sign (same polarity).

This force is similar to the gravitational force $\vec{F} = -G\frac{MM\prime}{R^2}\vec{u}$ by replacing mass by charge. It is given by Coulomb's law in the following form:

$$\overrightarrow{F_{1\,2}} = k\frac{q_1 q_2}{R^2}\overrightarrow{u_{1\,2}} \qquad \text{(I.1)}$$



Where : q_1 and q_2 two electric charges separated by a distance « R ».

$\overrightarrow{F_{1\,2}}$ is the electrostatic force exerted by q_1 on q_2.

$\overrightarrow{F_{21}}$ is the electrostatic force exerted by q_2 on q_1.

$\overrightarrow{u_{1\,2}}$ is a unit vector, carried by the support carrying the two charges, directed from q_1 to q_2

K is a constant: $k = \frac{1}{4\pi\varepsilon_0} \approx 9.10^9\ S.I$

ε_0 is the dielectric permittivity in vacuum (سماحية الكهربائية في الفراغ) $\varepsilon_0 \approx 8{,}85.10^{-12}$SI

Note : In the drawing, $q_1\ q_2<0$ is assumed. (i.e. the two charges have opposite signs).

Indeed, there are two types of forces:

a. Repulsive force:

We have a repulsive force if the two charges are of the same sign (either both positive or both negative). The two charges of the same sign will move away from each other.

$$\overrightarrow{F_{1\,2}} = k\frac{q_1 q_2}{R^2}\overrightarrow{u_{1\,2}} \quad and \quad \overrightarrow{F_{21}} = k\frac{q_2 q_1}{R^2}\overrightarrow{u_{21}} \qquad \text{(I.2)}$$

So the product $q_1 q_2 > 0$ then $\overrightarrow{F_{1\,2}}\ and\ \overrightarrow{u_{1\,2}}$ are in the same direction and $q_2 q_1 > 0$ so $\overrightarrow{F_{21}}\ and\ \overrightarrow{u_{21}}$ are in the same direction.

q2

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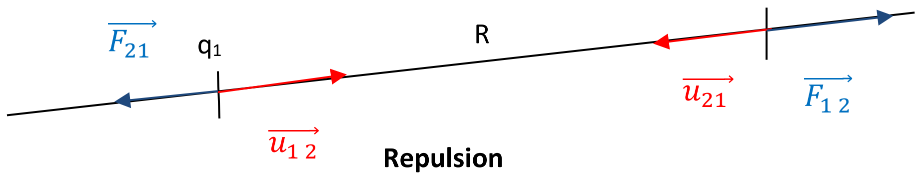
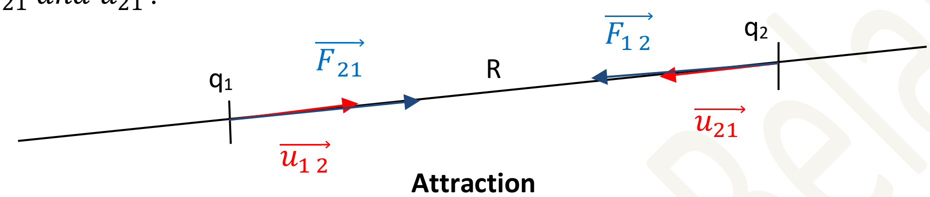


Repulsion

b. Attractive force:

We have an attractive force if the two charges are of opposite signs (one positive and the other negative, or vice versa). The two charges of different natures will attract each other.

so the product $q_1 q_2 < 0$ then $\overrightarrow{F_{12}}$ et $\overrightarrow{u_{12}}$ have two opposite meanings, and the same applies to $\overrightarrow{F_{21}}$ and $\overrightarrow{u_{21}}$.



Attraction

Notes :

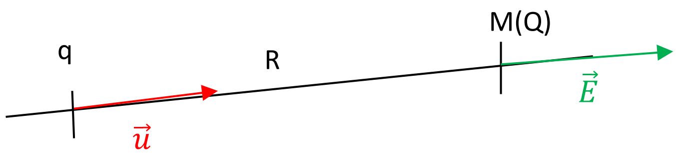
1. The unit of electrical charge is the Coulomb, which is a very large unit! This is why we work with sub-units of the Coulomb: Micro Coulomb (μC), Nano Coulomb (nC), etc....
2. Coulomb's law is only valid for immobile (static) charges. This is why the branch of physics that deals with this situation is called electrostatics.

We have also : $\overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$ and $\|\overrightarrow{F_{21}}\| = \|\overrightarrow{F_{12}}\|$

3.2. Electric field الحقل الكهربائي

a. Definition :

An electric field is said to exist at a given point in space, if an electrostatic force $\overrightarrow{F_e}$ acts on a point electric charge q placed at that point.



It is given by: $$\vec{E}_M = k\frac{q}{R^2}\vec{u} \qquad (I.3)$$

(In the drawing, q>0 is assumed, so the field is outward).

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$\vec{u}$ is a unit vector carried by the support carrying the charge q and the point M.

Notes:

- The relationship between the electrostatic force and the electric field is :

$$\vec{F}_M = Q\vec{E}_M = k\frac{qQ}{R^2}\vec{u} \quad \text{(I.4)}$$

This is the force of the charge q on the charge Q.

- The unit of electric field is (V/m).

b. Representation of field lines:

The lines of an electrostatic field created by a point charge are radial, i.e. they are like the radii of a circle. But we also deduce that the field lines follow the nature of the charge:

- If $q > 0$ the electric field $\vec{E}$ is in the same direction as $\vec{u}$, the field is centrifugal. The field lines run from the positive point charge away from it.

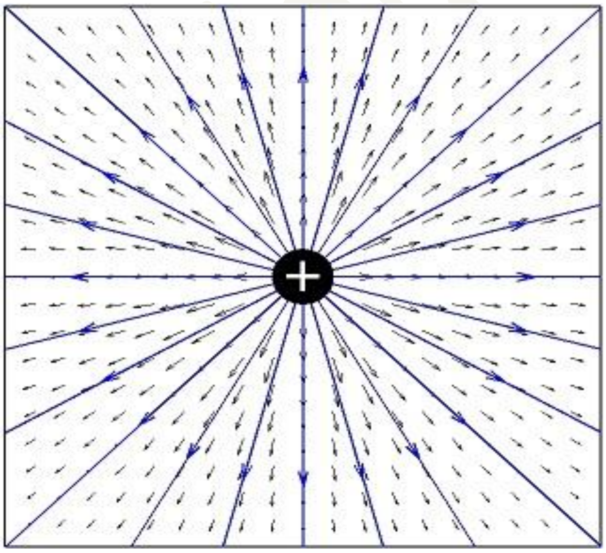


Figure I. 5: Field lines for a positive point charge

- If $q < 0$ the electric field $\vec{E}$ is in the opposite direction to $\vec{u}$; the field is centripetal. The field lines go towards the negative charge.

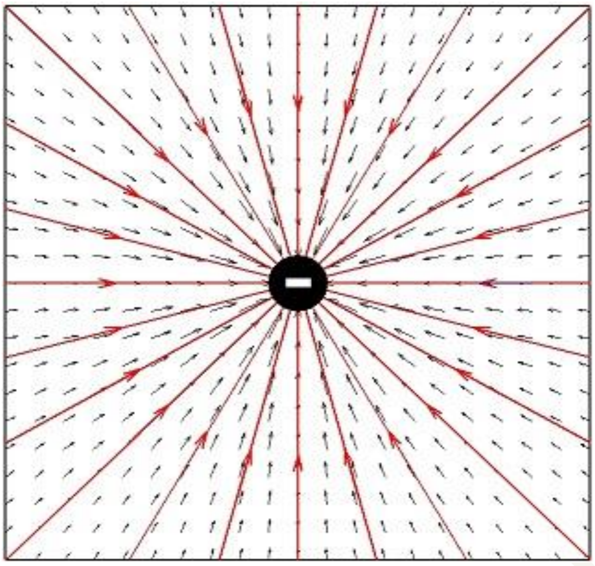


Figure I.6: Field lines for a negative point charge

In the presence of two point charges, there are two types of electrostatic force: repulsive (if the two charges have the same sign) and attractive (if the two charges have two different signs), as shown in figure 7:

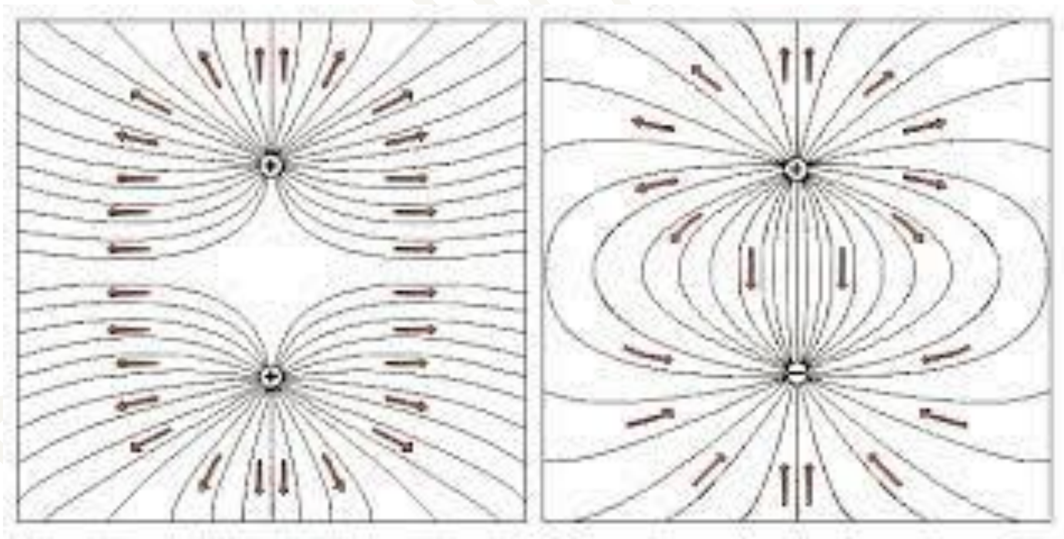


Figure I.7: Field lines in the presence of two point charges

3.3. Electrical potential: الكمون الكهربائي

The electric potential is a scalar given by the following relation :

$$V = k\frac{q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} \qquad (I.5)$$

Notes :

- The relation between V and $\vec{E}$ is : $\vec{E} = -\overrightarrow{grad}V = -\frac{dV}{dr}\vec{u}$ (I.6)

- As r tends towards infinity, the potential V becomes zero.

- The unit of electric potential is volt (V).

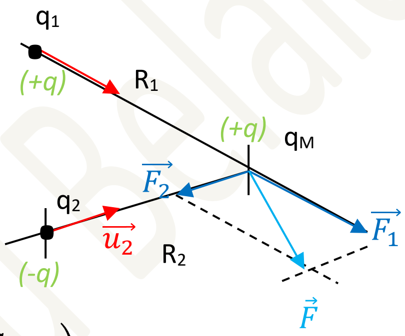
3.4. Superposition principle

3.4.1. Electrostatic force

$\vec{F_1}$ Force exerted by q_1 on q with: $\vec{F_1} = k\frac{q_1 q}{R_1^2}\vec{u_1}$

$\vec{F_2}$ Force exerted by q_2 on q with: $\vec{F_2} = k\frac{q_2 q}{R_2^2}\vec{u_2}$

$\vec{F}$ Force exerted on q is: $\vec{F_M} = \vec{F_1} + \vec{F_2}$



$$\vec{F_M} = kq\left(\frac{q_1}{R_1^2}\vec{u_1} + \frac{q_2}{R_2^2}\vec{u_2}\right)$$

For several charges we have:

$$\vec{F_M} = \sum_{i=1}^{N}\vec{F_i} = kq\left(\sum_{i=1}^{N}\frac{q_i}{R_i^2}\vec{u_i}\right) \qquad (I.6)$$

3.4.2. Electrostatic field

Using the relationship between the electrostatic force and the electrostatic field:

$$\vec{F_M} = q\vec{E} \Rightarrow \vec{F_i} = q\vec{E_i}$$

$$\Rightarrow \vec{E_i} = \frac{\vec{F_i}}{q} = k\left(\sum_{i=1}^{N}\frac{q_i}{R_i^2}\vec{u_i}\right) \qquad (I.7)$$

3.4.3. Electrostatic potential

For several charges we have: $V = V_1 + V_2 + \ldots\ldots + V_n$

$$V = \sum_{i=1}^{N} V_i = k\left(\sum_{i=1}^{N}\frac{q_i}{R_i}\right) \qquad (I.8)$$

Notes :

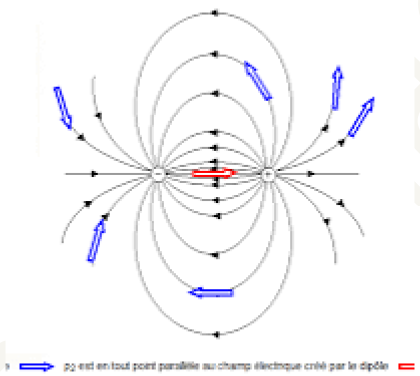
- The circulation of the electric field along a trajectory connecting two points A and B is equal to the difference between these two points. $C_{\vec{E}} = V_A - V_B$.
- The electric field is always oriented in the direction of decreasing potentials.
- Field circulation on a closed contour is zero.
- The work of an electric force W=q.$(V_A - V_B)$ =q.U

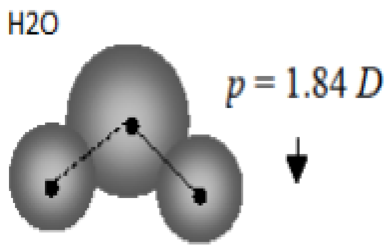
4. Electric Dipole

What is Electric Dipole?

An electric dipole is defined as a couple of opposite charges "q" and "–q" separated by a distance "d". By default, the direction of electric dipoles in space is always from negative charge "-q" to positive charge "q". The midpoint "q" and "–q" is called the centre of the dipole.

Example: H_2O molecule (H_3O^+, OH^-).



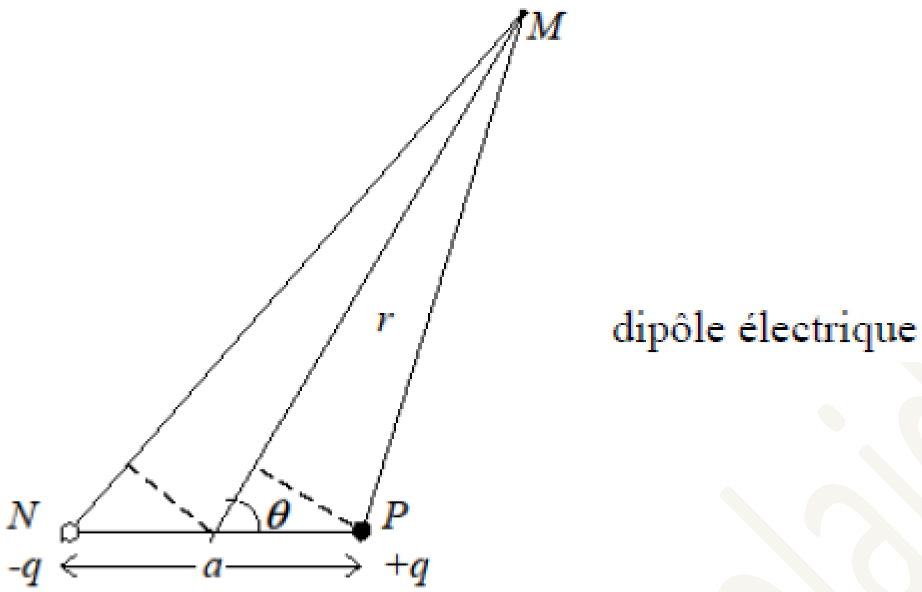


Potential of electric dipole

We use polar coordinates to calculate the potential created by the electric dipole. ... We take the origin O as the center of NP, and the line NP as the origin of the angles. In the case of a dipole OM=r >>a.

By definition:

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{PM} - \frac{1}{NM}\right)$$



$PM \approx r - \frac{a}{2}\cos\theta \; PM \approx r\left(1 - \frac{a}{2r}\cos\theta\right)$

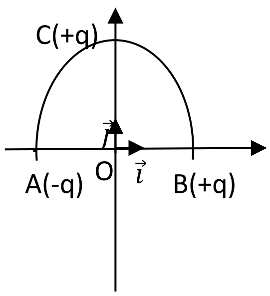
The potential of electric dipole is given by:

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{a\cos\theta}{r^2}\right)$$

5. Application exercise :

Consider three point charges q_A, q_B and q_C placed at three points A, B and C such that:

$q_A = -q$, $q_B = q_C = +q$ and OA=OB=OC=R.



1. Calculate the potential at point O.
2. Calculate the electric field at point O.
3. Place a charge q' = (+q) at point O. Deduce the resultant of the electrostatic forces acting on this charge.

Correction of application exercise

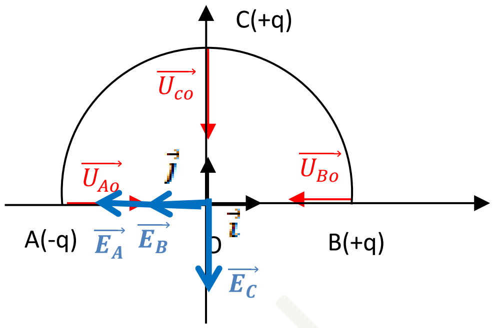
- Potential at the point O :

$$V_O = V_A + V_B + V_C = k\frac{q_A}{OA} + k\frac{q_B}{OB} + k\frac{q_C}{OC}$$

OA=OB=OC=R

$$V_O = k\frac{(-q)}{R} + k\frac{(+q)}{R} + k\frac{(+q)}{R} \Rightarrow V_O = k\frac{q}{R}$$

- The field at the point O



$\overrightarrow{E_O} = \overrightarrow{E_A} + \overrightarrow{E_B} + \overrightarrow{E_C}$

with

$\overrightarrow{E_A} = k\frac{q_A}{(OA)^2}\overrightarrow{u_{AO}}$, $\overrightarrow{E_B} = k\frac{q_B}{(OB)^2}\overrightarrow{u_{BO}}$

$\overrightarrow{E_C} = k\frac{q_C}{(OC)^2}\overrightarrow{u_{CO}}$

$$\overrightarrow{u_{AO}} = \vec{\imath},\ \ \overrightarrow{u_{BO}} = -\vec{\imath},\qquad \overrightarrow{u_{CO}} = -\vec{\jmath}$$

so $\overrightarrow{E_A} = k\frac{-q}{(R)^2}\vec{\imath}$, $\overrightarrow{E_B} = k\frac{q}{(R)^2}\overrightarrow{(-\imath)}$

$\overrightarrow{E_C} = k\frac{q}{(R)^2}\overrightarrow{(-\jmath)}$

Then : $\overrightarrow{E_O} = k\frac{(-q)}{R^2}\vec{\imath} + k\frac{q}{R^2}(\overrightarrow{-\imath}) + k\frac{q}{R^2}(\overrightarrow{-\jmath}) \Rightarrow \overrightarrow{E_0} = -k\frac{q}{R^2}(2\vec{\imath} + \vec{\jmath})$

3. Place a charge q' = (+q) at point O. Deduce the electrostatic force at point O:
   we have: $\overrightarrow{F_0} = q'\,\overrightarrow{E_O} = q\overrightarrow{E_O} = -k\frac{q^2}{R^2}(2\vec{\imath} + \vec{\jmath})$

Part 2: Continuous electric charge distributions

توزيع مستمر للشحن

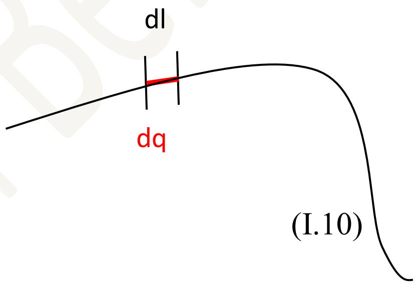
In the case of a very large number of point charges, we can distinguish between three types of charge distribution, which may be uniformly distributed along a straight line (linear), on a surface or in a volume.

1. Linear charge distribution

For a linear charge distribution, the elementary charge is written as: $dq = \lambda dl$

Where λ is the unit linear density (C/m).

The charge in this case is calculated by: $Q = \int \lambda . dl$ (II.9)



The field can be written as: $\vec{E} = k \int \frac{\lambda dl}{r^2} \vec{u}$ (I.10)

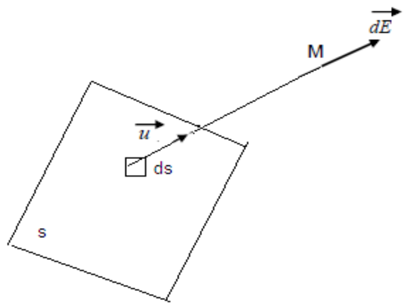
The potential written by : $V = k \int \frac{\lambda . dl}{r}$ (I.11)

2. Surface distribution of charges

For a surface charge distribution, the elementary charge dq is written as:

$$dq = \sigma dS \Rightarrow Total\ charge\ is\ Q = \iint \sigma . dS \quad (I.12)$$

Where σ is the surface charge density.



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The field is written by : $$\vec{E} = k \iint \frac{\sigma.dS}{r^2} \vec{u}$$ (I.13)

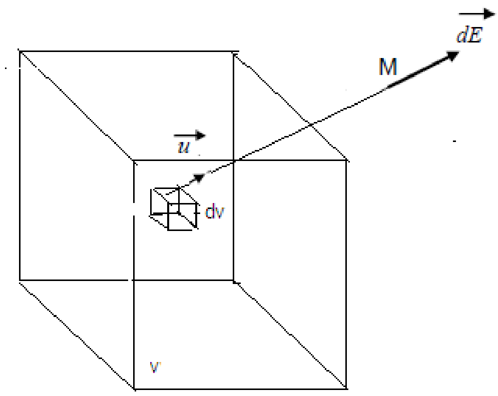
The potential is written by : $$V = k \int \int \frac{\sigma.dS}{r}$$ (I.14)

3. Volumetric charge distribution

For a volumetric charge distribution, the elementary charge dq is written as :

$$dq = \rho dv \Rightarrow Q = \iiint \rho.dV$$ (I.15)

Where ρ is the volume charge density.



The field is written by : $$\vec{E} = k \iint \int \frac{\rho.dV}{r^2} \vec{u}$$ (I.16)

The potential is written by: $$V = k \int \int \int \frac{\rho.dV}{r}$$ (I.17)

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4. Summary of the three cases

- Linear Distribution:

The elementary electric field $d\vec{E}$ created by an element of charge dq present in an element of length dl is written by:

$$d\vec{E} = k\frac{dq}{R^2}\vec{u} \ \ with \ \ dq = \lambda dl \quad so \qquad \vec{E} = k\int\frac{\lambda dl}{R^2}\vec{u} \qquad (I.18)$$

- Surface Distribution :

The elementary electric field $d\vec{E}$ created by an element of charge dq present in an element of surface dS is written by :

$$d\vec{E} = k\frac{dq}{R^2}\vec{u} \ \ with \ \ dq = \sigma ds \ \ so \qquad \vec{E} = k\iint\frac{\sigma ds}{R^2}\vec{u} \qquad (I.19)$$

- Volumique Distribution :

The elementary electric field $d\vec{E}$ created by an element of charge dq present in an element of volume dV is written by:

$$d\vec{E} = k\frac{dq}{R^2}\vec{u} \ \ with \ \ dq = \rho dV \quad so \qquad \vec{E} = k\iiint\frac{\rho dV}{R^2}\vec{u} \qquad (I.20)$$

- The relationship between the electrostatic field and potential is:

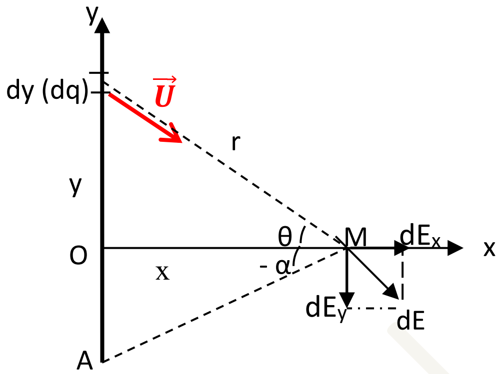
$$\vec{E} = -\overrightarrow{grad}V \qquad (I.21)$$

5. Application exercise

Let a wire (Ay) carry a linear charge distribution of uniform positive linear density λ.

1- Determine the electric field E_M produced at a point M by this continuous distribution. We give OM=x.

Correction of application exercise :



The electric field E on point M.

$$\begin{cases} \overrightarrow{dE} = k\dfrac{dq}{r^2}\vec{U} \\ dq = \lambda dy \\ \vec{U} = cos\theta\,\vec{\imath} - sin\theta\,\vec{\jmath} \end{cases}$$

$$\overrightarrow{dE} = k\frac{dq}{r^2}\vec{U} \quad = k\frac{\lambda dy}{r^2}(cos\theta\,\vec{\imath} - sin\theta\,\vec{\jmath})$$

or: $dE_x = dE\cos\theta = k\frac{\lambda dy}{r^2}\,cos\theta$

$$dE_y = -dE\sin\theta = -k\frac{\lambda dy}{r^2}\,sin\theta$$

On the other hand : $tg\theta = \frac{y}{x} \Rightarrow y = xtg\theta \Rightarrow dy = \frac{x}{\cos^2\theta}d\theta$

with $\cos\theta = \frac{x}{r} \Rightarrow r = \frac{x}{cos\theta}$

so $dE_x = \frac{k\lambda}{x}\,cos\theta d\theta$

And $dE_y = \frac{-k\lambda}{x}\,sin\theta d\theta$

Where : $E_x = \int dE_x = \frac{k\lambda}{x}\int_{-\alpha}^{\pi/2}\cos\theta d\theta$

and $E_y = \int dE_y = \frac{k\lambda}{x}\int_{-\alpha}^{\pi/2} -\sin\theta d\theta$

The final expressions of field's components are :

$\Rightarrow\ E_x = \frac{k\lambda}{x}(1 + sin\alpha)$

and $E_y = \frac{-k\lambda}{x}cos\alpha$

The modulus of electric field will be: $E = \sqrt{E_x^2 + E_y^2} = \frac{k\lambda}{x}\sqrt{2 + 2sin\alpha}$

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