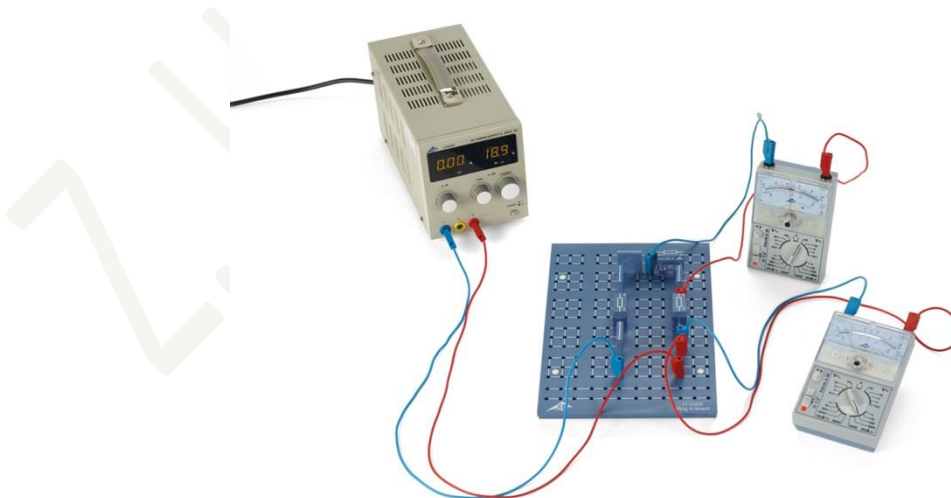


**1ST YEAR LMD-M AND MI
ELECTRICITY COURSE**

Chapter IV : Electro-kinetics

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1. What is an electric current?

1.1. Introduction

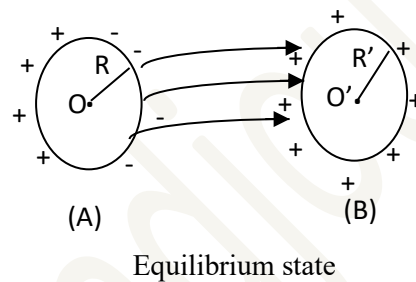
Electrokinetics is the study of the movement of charges. In other words, it's the study of electrical circuits and, above all, the movement of electricity in conductive material media, as opposed to electrostatics, which studies the phenomena and laws relating to immobile electricity.

As a conductor is a medium in which charges are able to move freely, we can cite metals, ionic solutions and semiconductors. In metals, only electrons move freely. In other media, there are different charge carriers in motion, as in the case of electrolytes with several positive and negative ions, and ionized gases.

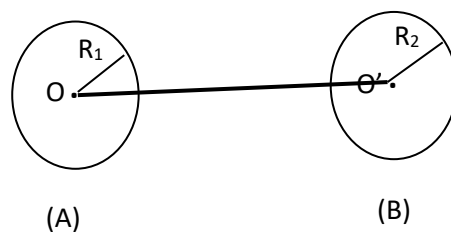
An electric current is a movement of electric charges in a conducting medium.

1.2. Origin of electric current

Consider two conductors A and B in electrostatic equilibrium under partial influence.



If we connect the two conductors by a conducting wire, we'll have a single conductor under the action of an electric field; we'll have a movement of charges passing from one conductor to the other through the conducting wire, this circulation of charges constitutes an "electric current".



Note: This current is temporary; it stops when equilibrium is established. To obtain a direct current, we use a generator.

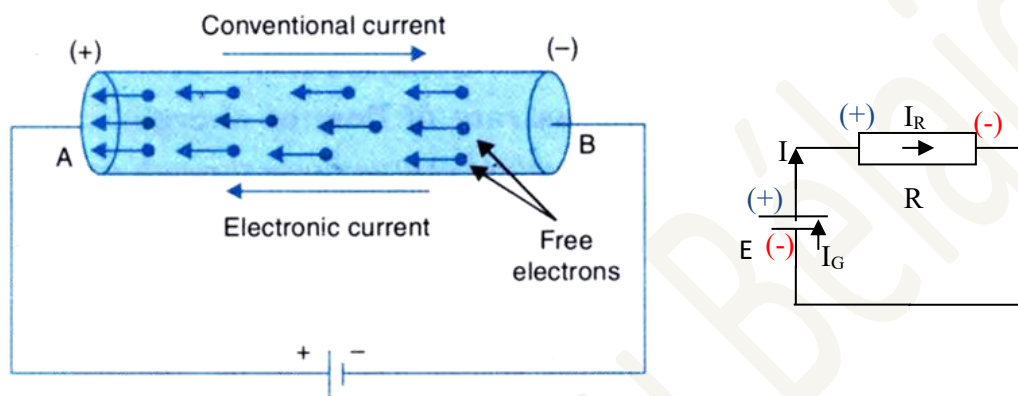
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Definition: An electric current is an overall movement of electric charge carriers. In metals, these carriers are electrons (negative charge), and in liquids and gases, the carriers are electrons or ions, positive or negative.

The elementary charge is that of the electron: $Q = e = -1,6 \cdot 10^{-19}$ Coulomb (C).

1.3. The conventional direction of electric current

The conventional direction of current chosen by Ampère is opposite to that of electrons. In fact, current flows from the positive pole to the negative pole of an electric generator.

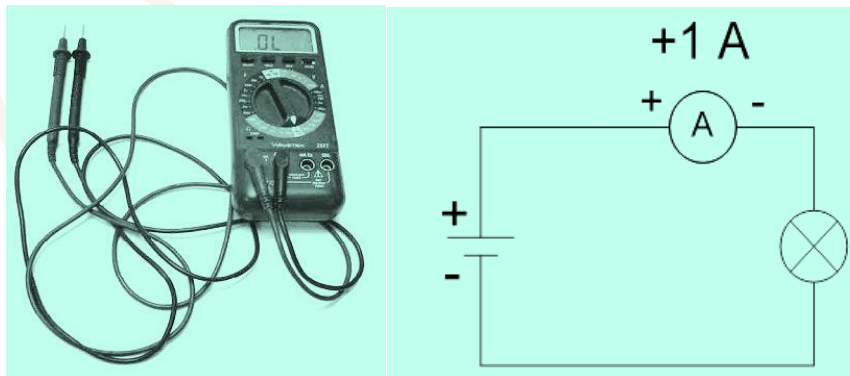


1.4. Electric current intensity شدة التيار

Current intensity measures the algebraic quantity of electricity (charge carriers) passing through a section of a conductor per unit time. $I = \frac{dq}{dt}$

The unit of current is: Ampere (A)

The current measurement tool is an ammeter placed in series in the circuit.



1.5. Electric current density كثافة التيار الكهربائيا

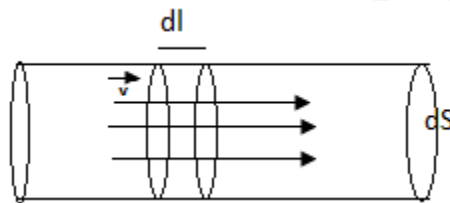
Current density, \vec{j} , represents the quantity of charge passing through a unit area per unit time. It is a vector quantity, characterizing the motion of a set of moving charges with an average velocity \vec{v} and the volume density of the charges ρ .

$$\vec{j} = \rho \vec{v} \quad \text{with} \quad \rho = nq = ne$$

with : n is the number of electrons and \vec{v} is the average velocity of the charges.

we can therefore write $\vec{j} = -ne\vec{v}$

In a conductor with a cross-section (S), $dl = dx$



$$\vec{v} = \frac{d\vec{x}}{dt} \quad \text{with} \quad d\vec{x} = \vec{v} dt$$

$$\left\{ \begin{array}{l} dq = \rho dv = \rho \vec{ds} d\vec{x} = \rho \vec{ds} \vec{v} dt \\ \vec{j} = \rho \vec{v} \end{array} \right. \quad \text{so} \quad dq = \vec{j} \vec{ds} dt \quad \text{then} \quad I = \frac{dq}{dt} = \iint \vec{j} \vec{ds}$$

In a cylindrical conductor, $I = \iint \vec{j} \vec{ds}$

Definition: Current intensity represents the flow of the current density vector \vec{j} through a section "S" of a conductor; its unit is A/m².

In general, for a wire, the relationship between the current intensity and its density will be:

$$\vec{j} = I/s$$

Reminder: Voltage is the potential difference between two points A and B (noted ddp) or $U_{AB} = V_A - V_B$ (the unit of U is Volt).

1. Ohm's law

2.1. Electrical conductivity السماحية الكهربائية

In the presence of an electric field \vec{E} , there is a current density through a conducting wire of length l and cross-section S given by the relation:

$$\vec{j} = \sigma \vec{E} \quad \text{with} \quad \sigma = \frac{n \tau e^2}{m_e}$$

With σ : electrical conductivity.

τ : average time between two successive shocks between charges.

n : number of charges per unit volume.

m_e : electron mass.

Note: Electrical conductivity is expressed in Siemens/m (Sm^{-1}).

2.2. Electrical resistivity المقاومية الكهربائية

Electrical resistivity is the inverse of electrical conductivity, noted as, $\rho = \frac{1}{\sigma} = \frac{m_e}{n \tau e^2}$

It is expressed in Ω/m .

2.3. Electric resistance المقاومة الكهربائية

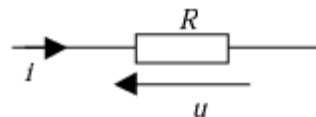
a. Definition (Ohm's law on a macroscopic scale)

The ratio between the potential difference U (ddp) between two points A and B on a metal conductor and the current I flowing through it is constant at constant temperature. It is

denoted R and defines the electrical resistance. $R = \frac{U}{I}$

The resistance of a conductor is expressed in Ω (Ohm)

Its symbol in an electric circuit is:

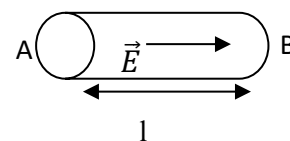


b. Calculating the resistance of a cylindrical conductor (Ohm's Law on a microscopic scale)

Consider a conductor of length l and cross-section S through which a current of intensity I flows.

The voltage across this conductor is written as:

$$U = \Delta v = v_A - v_B = RI$$



The field can be written as:

$$E = -\frac{dv}{dl} \Rightarrow \int_{v_A}^{v_B} dv = - \int E dl \quad \text{so} \quad v_B - v_A = - \int E dl \Rightarrow U = v_A - v_B = \int E dl$$

$$I = \iint \vec{j} d\vec{s} = Js \quad \text{with } J = \sigma E$$

$$R = \frac{U}{I} = \frac{\int E dl}{js} = \frac{E \int_A^B dl}{\sigma Es} = \frac{El}{\sigma Es} \quad \text{So } \begin{cases} R = \frac{l}{\sigma s} \\ \rho = \frac{1}{\sigma} \end{cases} \quad \text{or } R = \rho \frac{l}{s}$$

b. Power calculation

It is written as : $P=UI=R I^2$ its unit is “watt”

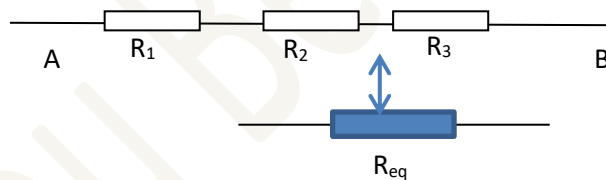
This power corresponds to the power consumed by a receiver and the power supplied by a generator.

2.5. Resistance grouping

There are two different ways of connecting resistors in an electrical circuit.

a. Grouping in series

Three resistors connected in series.



$$I_{AB} = I_{R1} = I_{R2} = I_{R3} = I \quad \text{with } U = RI$$

$$U_{AB} = U_{R1} + U_{R2} + U_{R3} \Rightarrow R_{eq}I = R_1I_1 + R_2I_2 + R_3I_3 = R_1I + R_2I + R_3I$$

$$\text{So } R_{eq} = R_1 + R_2 + R_3$$

For « n » resistors we have :

$$R_{eq} = \sum_{i=1}^n R_i$$

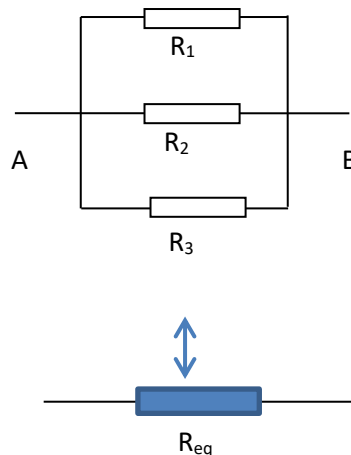
b. Grouping in parallel

Three resistors connected in parallel.

$$I_{AB} = I_{R1} + I_{R2} + I_{R3} \quad \text{with } I = U/R$$

$$U_{AB} = U_{R1} = U_{R2} = U_{R3} \Rightarrow \frac{U_{AB}}{R_{eq}} = \frac{U_1}{R_1} + \frac{U_2}{R_2} + \frac{U_3}{R_3}$$

$$\text{So } \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



For « n » resistors we have :

$$\frac{1}{R_{eq}} = \sum_{i=1}^n \frac{1}{R_i}$$

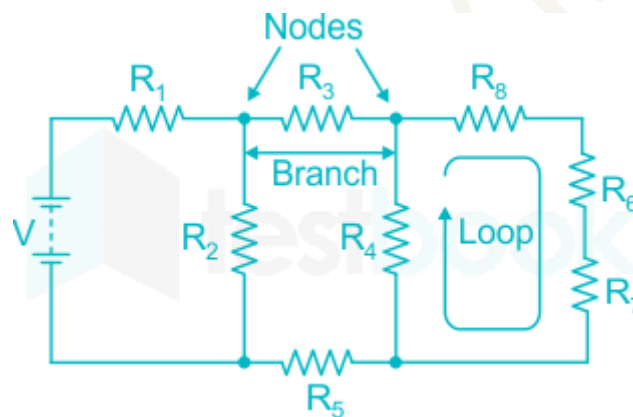
3. Electrical network

An electrical circuit or network is a set of electrical devices interconnected by perfect conductor wires. In a circuit, we distinguish several types of electrical devices, such as resistors, generators, capacitors, etc.

A node (عقدة) is a point in the circuit connected to three or more devices.

A network branch (فرع) is the part of the circuit between two nodes.

A mesh (Loop حلقة) is a closed path of branches passing at most once through a given node.



The various types of electrical devices can be classified into two categories:

Generator (مولد): This is the element that generates the current; the current in this element flows out of the positive pole and in through the negative pole. The conventional direction of the potential difference U (d.d.p) is from negative pole to positive pole for a generator.

Receptor (مستقبل): In a receiver, current flows in through the positive pole and out through the negative pole.

Note:

* In a receptor: Current and voltage flow in opposite directions.



* In a generator: Current and voltage have the same direction.



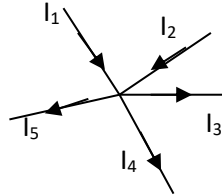
4. Kirchoff's laws

Kirchoff's two laws allow the analysis of electrical networks.

4.1. Law of nodes

A node is a point where several electrical conductors meet. The sum of the currents flowing into a node is equal to the sum of the currents flowing out of the node:

$$\sum I_{incoming} = \sum I_{outgoing}$$

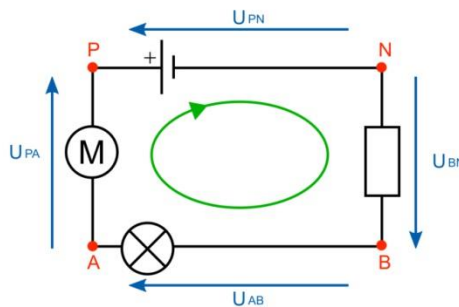


$$I_1 + I_2 = I_3 + I_4 + I_5$$

4.1. Law of meshes

The algebraic sum of potential differences in a mesh is equal to zero:

$$\sum U_{maille} = 0$$



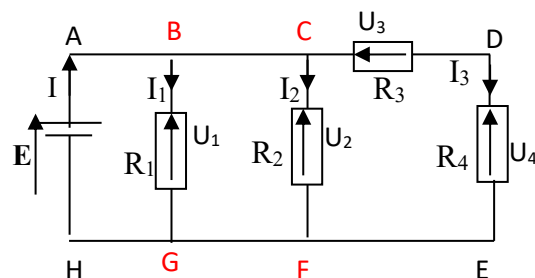
$$U_{PA} + U_{AB} + U_{BN} + U_{PN} = 0$$

Application example:

Consider the circuit shown below.

1. Calculate the value of the current I using Kirchoff's two laws.
2. Find the value of I , using the equivalent circuit resistance.

We give : $E=24V$; $R_1=R_2=20\Omega$; $R_3=R_4=5\Omega$.



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The answer: In this montage, we have:

4 nodes which are : B, C, F, G

6 meshes : ABGHA ; BCFGB ; CDEFC ; ACFHA ; BDEGB ; ADEHA.

1. Kirchhoff's two laws:

Law of nodes:

At the point B : $I=I_1+I'$ and at point C : $I'=I_2+I_3$

Law of meshes (in the direction of the meshes):

$$\text{ABGHA : } E - R_1 I_1 = 0 \Rightarrow 24 - 20I_1 = 0 \text{ so } I_1 = 1,2 \text{ A}$$

$$\text{BCFGB : } R_1 I_1 - R_2 I_2 = 0 \Rightarrow 20I_1 - 20I_2 = 0 \text{ so } I_1 = I_2 = 1,2 \text{ A}$$

$$\text{CDEFC : } R_2 I_2 - R_3 I_3 - R_4 I_3 = 0 \Rightarrow 20(1,2) - 5I_3 - 5I_3 = 0 \text{ so } I_3 = 2,4 \text{ A}$$

$$I=1,2+1,2+2,4=4,8 \text{ A}$$

2. The current can be calculated using another method:

R_3 is in series with R_4 so $R_{34}=R_3+R_4=5+5=10\Omega$

R_1 , R_2 and R_{34} are connected in parallel, so :

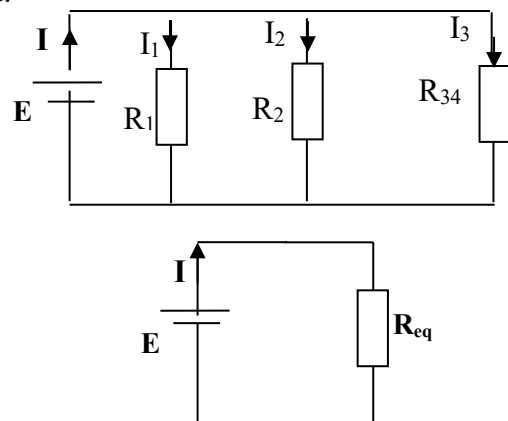
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{34}} = \frac{1}{20} + \frac{1}{20} + \frac{1}{10} = \frac{4}{20}$$

$$\Rightarrow R_{eq} = 5\Omega$$

We have the law of meshes:

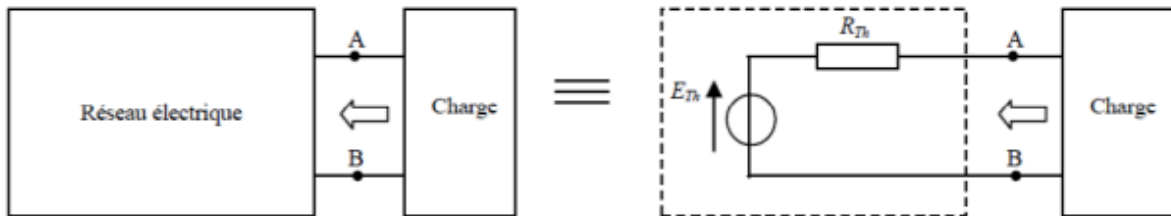
$$E = R_{eq} I$$

$$\Rightarrow I = \frac{E}{R_{eq}} = \frac{24}{5} = 4,8 \text{ A}$$



5. Thévenin's and Norton's Theorems

Thevenin's and Norton's theorems are the most important theorems in electronics. They enable complex problems to be solved in a minimum of time and with very few equations.



Thevenin circuit equivalent to an electrical circuit

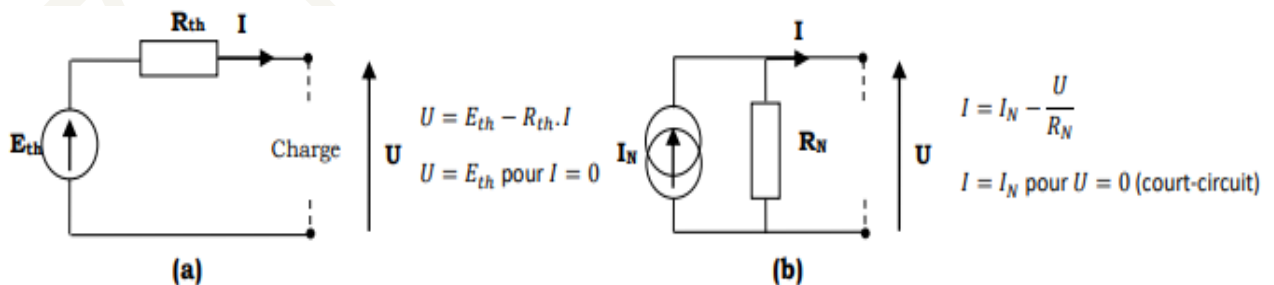
5.1. Thévenin's Theorem

Thevenin's theorem in steady-state conditions is based on the principle that any two-port linear network is equivalent to a source of electromotive force E_{th} and internal resistance R_{th} .

- R_{th} : is equal to the equivalent resistance of the network when all its generators are switched off;
- E_{th} : is equal to the open-circuit voltage of the network, when $I = 0$.

5.2. Norton's Theorem

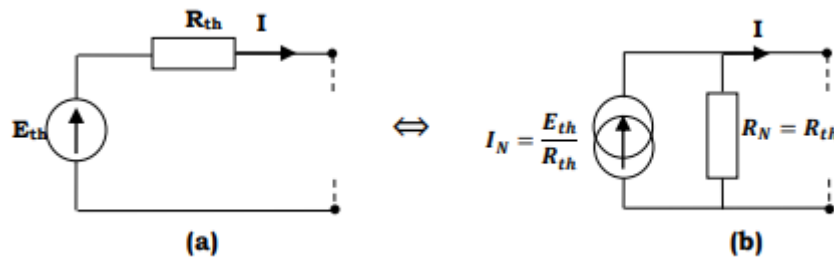
In steady-state conditions, any two-port linear network is equivalent to a Norton current source with current I_N and internal resistance R_N equal to the Thevenin resistance. The current I is equal to the source current I_N when the source is short-circuited.



Thevenin and Norton circuits

5.3. Thevenin–Norton Equivalence

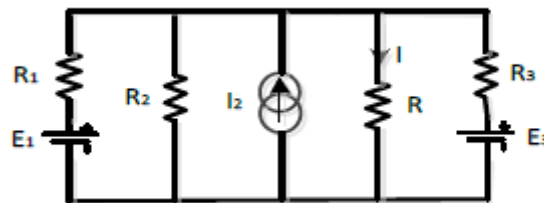
A Thevenin voltage source, with electromotive force E_{th} and internal resistance R_{th} , is equivalent to a Norton current source, with current $I_N = E_{th} / R_{th}$ and the same internal resistance R_N .



Thevenin's and Norton's theorems are useful when seeking a specific electrical quantity, such as the current through a resistor placed in a complex circuit. In such cases, we assume that this resistor is powered by the rest of the circuit, which we thereby isolate, and for which we seek the Thevenin or Norton equivalent.

Application

Determine the current I flowing through the resistor R in the following circuit using the transformation *Thevenin- Norton*.



Correction

1. Converting branches into Norton models: We convert each active branch into a Norton current generator (I_N , R_N):

- Branch 1 (E_1 , R_1 in serie) : $I_{N1} = E_1 / R_1 = 10 / 60 = 0,1667$ A and $R_{N1} = R_1 = 60 \Omega$
- Branch 2 (I_2 , R_2 in parallel) : It is already in the format Norton : $I_{N2} = I_2 = 0,1$ A and $R_{N2} = R_2 = 100 \Omega$.
- Branch 3 (E_3 , R_3 in serie) : $I_{N3} = E_3 / R_3 = 7 / 40 = 0,175$ A et $R_{N3} = R_3 = 40 \Omega$

2. Simplification of the circuit (Norton equivalent)

The circuit now consists of three current sources in parallel with their respective resistors, all feeding the load R.

- Total Norton current ($I_{N_{\text{eq}}}$): As the sources are directed in the same direction towards the upper node: $I_{N_{\text{eq}}} = I_{N1} + I_{N2} + I_{N3} = 1/6 + 0,1 + 0,175 = 53/120 = 0,4417 \text{ A}$.
- Equivalent Norton resistance ($R_{N_{\text{eq}}}$): This corresponds to the parallel connection of R_1 , R_2 et R_3 : $1/ R_{N_{\text{eq}}} = 1/60 + 1/100 + 1/40 = 31/600$ so $R_{N_{\text{eq}}} = 600/31\Omega = 19,35\Omega$.

3. Calculation of the current I through resistor R

The simplified circuit consists of a source $I_{N_{\text{eq}}}$ in parallel with $R_{N_{\text{eq}}}$ and R.

We use the current-divider rule to find I:

$$I = I_{N_{\text{eq}}} (R_{N_{\text{eq}}} / (R_{N_{\text{eq}}} + R)) = 0,1732 \text{ A}$$

Result: The current I flowing through the resistor R is approximately

173,2 mA.

