

Tutorial Sheet 02

Exercise 1 Using the fundamental counting principle, response to the following questions:

- 1) Lily is trying to decide what to wear. She has shirts in the following colors: red, purple, and blue, and she has pants in the following colors: black and white. How many different outfits can Lily choose from (assuming she selects one shirt and one pair of pants)?
- 2) You go to check out three books at the library, and you want one history book, one science book, and one fantasy book. The library has 50 history books, 95 fantasy novels, and 30 books about science. How many combinations of books do you have to choose from?
- 3) There are 88 daily newspapers and 55 weekly magazines published in Chicago. If Colin wants to subscribe to exactly one daily newspaper and one weekly magazine, how many different choices does he have?
- 4) Calvin wants to go to Milwaukee. He can choose from 3 bus services or 2 train services to head from home to downtown Chicago. From there, he can choose from 2 bus services or 3 train services to head to Milwaukee. How many ways are there for him to get to Milwaukee?
- 5) Six friends Andy, Bandy, Candy, Dandy, Endy, and Fandy want to sit in a row at the cinema. If there are only six seats available, how many ways can we seat these friends?
- 6) How many positive divisors does 2000 have?

Exercise 2: 1) Using the principle of mathematical induction, prove the Binomial Expansion Formula:

$$(x + y)^n = \sum_{k=0}^n C_n^k x^k y^{n-k}$$

- 2) Show that an n-element set has exactly 2^n distinct subsets.
- 3) How many words of length n can be formed from the letters A,B,C, such that the number of As is even?

Exercise 3 With the alphabet of 26 letters,

- 1) How many different words of 7 letters can we write, the same letter being able to appear more than once.
- 2) How many different words of 7 letters can we write, the same letter being able to appear only once.

Exercise 4 A commission of 5 members including 3 economists and 2 lawyers must be made up of 13 candidates divided into 7 economists and 6 lawyers.

- a) In how many different ways can this commission be constituted?
- b) Same question if an economist designated by the economists must absolutely be part of the commission.
- c) Same question if one of the economists and one of the lawyers, both individualized, cannot be part of the commission together.

Exercise 5 the 12 delegates of an association must elect a representative from among themselves to the establishment board, there are only two candidates A and B. a candidate is elected if he obtains an absolute majority; all delegates must vote for one or the other.

- a) Determine the number of possible votes.
- b) Determine the number of possible votes for A to be elected, for B to be elected.
- c) Determine the number of votes possible for there to be a runoff.

Exercise 6: Let $1 \leq r \leq n$ and consider all subsets of size r from the set $\{1, 2, \dots, n\}$. Each of these subsets has a smallest member. Define $F(n, r)$ as the arithmetic mean of these smallest elements. Prove that:

$$F(n, r) = \frac{n+1}{r+1}$$