Chapter I : ELECTROSTATICS

Electrostatics is the study of phenomena caused by electric charges at rest and the interactions between them.

1. Electric Particles and Charges

Atoms consist of nuclei containing positively charged particles called protons, around which negatively charged particles called electrons move. There are also particles called neutrons, which are present in the nucleus of atoms and carry no electric charge.

The values of the electric charges and masses of the atomic constituents in the International System of Units (SI) are:

Electron : $q_e = -e = -1.602\times10^{-19}$ C $m_e = 9.109\times10^{-31}$ kg
Proton : $q_p = +e = 1.602\times10^{-19}$ C $m_p = 1.672\times10^{-27}$ kg
Neutron : $q_n = 0$ C $m_n = 1.674\times10^{-27}$ kg

In the International System of Units (SI), electric charge is measured in coulombs (C).

2. Discrete Charge Distributions

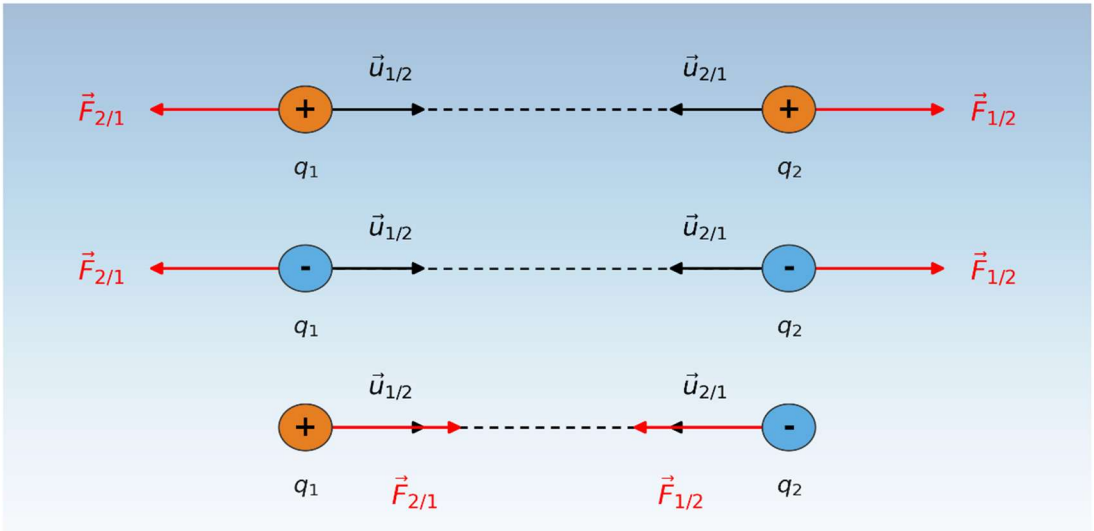
A discrete charge distribution is used when electric charges are located at specific, well-defined points in space and are separated from one another.

2.1. Electrostatic Forces: Coulomb's Law

Attraction and Repulsion Between Two Charges

Consider two electric charges q_1 and q_2 separated by a distance r. These charges may have the same sign or opposite signs.
Two types of interactions can occur between electric charges: attractive forces and repulsive forces.

- Charges with the same sign repel each other.
- Charges with opposite signs attract each other.



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Expressions of the Electrostatic Force

The electrostatic force $\vec{F}_{1/2}$exerted by charge q_1 on charge $q_2$is given by:

$$\vec{F}_{1/2} = K\frac{q_1 q_2}{r^2}\vec{u}_{1/2}$$

The electrostatic force exerted by charge q_2 on charge q_1 is given by:

$$\vec{F}_{2/1} = K\frac{q_1 q_2}{r^2}\vec{u}_{2/1}$$

Where:

K is Coulomb's constant: $\left(K = \frac{1}{4\pi\varepsilon_0} \approx 9\times 10^9\ N.m^2/C^2\right)$

$\varepsilon_0$is the permittivity of free space:$(\varepsilon_0 = 8.85\times 10^{-12}\ F/m)$.

$\vec{u}_{1/2}$is the unit vector directed from q_1 toward q_2

$\vec{u}_{2/1}$is the unit vector directed from q_2 toward q_1

Since $\vec{u}_{1/2} = -\vec{u}_{2/1}$the two forces have equal magnitudes and opposite directions. This illustrates Newton's third law of action and reaction.

2.2. Principle of Superposition

If a charge q, located at point M, is subjected to the influence of n charges q_1, q_2,……, q_n, located at points M_1, M_2,…. M_n , respectively, the total electrostatic force acting on charge q is the vector sum of the individual forces:

$$\vec{F}_M = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_n$$

Exemple

Consider three electric charges q_1, q_2, and q_3 located at the vertices of a triangle ABC, which is right-angled at A.

Calculate the resultant electrostatic force acting on charge q_1 due to the presence of charges $q_2$and q_3.

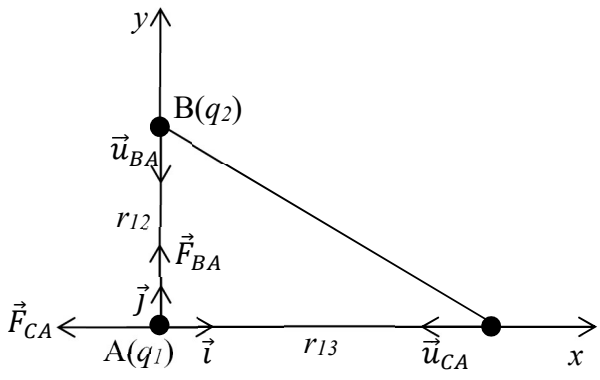
Given: q_1=10µc , q_2= -20µc , q_3=30µc , AB=1m , AC=2m.

Let $\vec{F}_{BA}$be the force exerted by charge q_2 on q_1,

and $\vec{F}_{CA}$be the force exerted by charge q_3 on q_1

Using Coulomb's law, we obtain:

$$\vec{F}_{BA} = K\frac{q_B q_A}{(BA)^2}\vec{u}_{BA}$$



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$$\vec{F}_{CA} = K\frac{q_C q_A}{(CA)^2}\vec{u}_{CA}$$

Let us choose a coordinate system with the x and y axes, in which each force is expressed in terms of its components and the unit vectors $\vec{\imath}$ and $\vec{\jmath}$ (see the figure).

Therefore: $\overrightarrow{u_2} = -\vec{\jmath}$ et $\overrightarrow{u_3} = -\vec{\imath}$

$$\vec{F}_{BA} = K\frac{q_B q_A}{(BA)^2}(-\vec{\jmath}) = -9\times 10^9\frac{10\times 10^{-6}\times(-20)\times 10^{-6}}{(1)^2}\vec{\jmath} = 1.8\vec{\jmath}(N)$$
$$\vec{F}_{CA} = K\frac{q_C q_A}{(CA)^2}(-\vec{\imath}) = -9\times 10^9\frac{10\times 10^{-6}\times 30\times 10^{-6}}{(2)^2}\vec{\imath} = -0.675\vec{\imath}(N)$$

By performing the vector sum of the two forces:

$$\vec{F}_A = \vec{F}_{BA} + \vec{F}_{CA} = -0.675\vec{\imath} + 1.8\vec{\jmath}(N)$$

The magnitude of this force is:

$$F = \sqrt{(-0.675)^2 + (1.8)^2} = 1.922\ N$$

Remark

- ✓ Coulomb's law is valid only for stationary electric charges in a vacuum.
- ✓ This law has the same vector properties as Newton's law of universal gravitation:

$$\vec{F} = G\frac{m_A m_B}{(AB)^2}\vec{u}$$

The fundamental difference is that gravitational forces are always attractive, whereas electrostatic forces can be either attractive or repulsive.

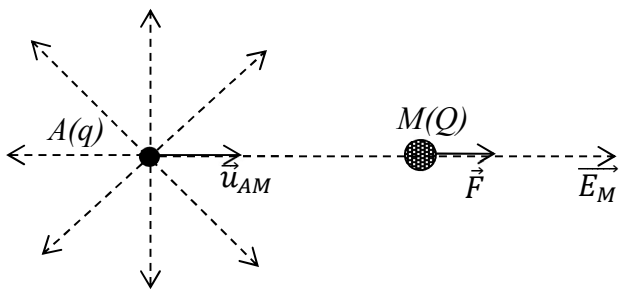
2.3.Electrostatic field

The electrostatic field created at point M by a charge q located at point A is expressed by the following relation:

$$\vec{E}_M = K\frac{q}{(AM)^2}\vec{u}_{AM}$$

$\vec{u}_{AM}$ is the radial unit vector directed from A toward M

The charge q creates an electric field $\vec{E}_M$ throughout the surrounding space.



Electrostatic Field Due to Charge q

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The electric field is a vector quantity.

In the International System of Units (SI), the electric field is expressed in newtons per coulomb (N/C) or volts per meter (V/m).

Electric Field Created by a Discrete Charge Distribution

The total electric field created at point M by a system of n charges q_1, q_2,……, q_n, located at points M_1, M_2,…. M_n, respectively, is given by:

$$\vec{E}_M = \vec{E}_1(M) + \vec{E}_2(M) + \cdots + \vec{E}_n(M)$$

Relationship Between Electrostatic Force and Electric Field

When a charge Q is placed at point M, where an electrostatic field $\vec{E}_M$ exists, the charge experiences a force given by:

$$\vec{F}_M = Q\vec{E}_M$$

Remarks

The electrostatic field produced by a point charge q is:

- ✓ Radial: the field lines are straight rays originating from the charge.
- ✓ Directed outward if $q>0$.
- ✓ Directed inward if $q<0$.

2.4.Electrostatic Potential

The electrostatic potential created at point M by a charge q located at point A is expressed by the relation:

$$V_M = K\frac{q}{AM}$$

The electrostatic potential is a scalar quantity.

In the International System of Units (SI), the electrostatic potential is expressed in volts (V).

Electric potential created by a discrete charge distribution

The total electric potential created at point M by a system of n charges q_1, q_2,……, q_n, located at points M_1, M_2,…. M_n, respectively, is given by:

$$V_M = V_1(M) + V_2(M) + \cdots + V_n(M)$$

Remark

- ✓ The electrostatic potential is positive if $q>0$.

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- ✓ The electrostatic potential is negative if $q<0$.
- ✓ The electrostatic potential produced by a point charge q has the same value at all points located at the same distance r from the charge. Therefore, the potential is constant on spherical surfaces of radius r, which are called equipotential surfaces.
- ✓ An equipotential surface is a surface on which the electrostatic potential is identical at every point.
- ✓ The electrostatic field $\vec{E}$ is derived from the potential V. Therefore, the electric field is always perpendicular to equipotential surfaces.

Relationship Between the Electric Field and the Electrostatic Potential

The relationship between the electric field and the electrostatic potential is given by:

$$\vec{E}_M = -\overrightarrow{grad}\, V(M)$$

This result applies to all coordinate systems. In Cartesian coordinates, it means:

$$E_x = -\frac{\partial V}{\partial x} \quad ; \quad E_y = -\frac{\partial V}{\partial y} \quad ; \quad E_z = -\frac{\partial V}{\partial z}$$

The electrostatic field $\vec{E}_M$ is said to be derived from the potential $V(M)$.

3. Continuous Charge Distributions

Continuous charge distributions are characterized by an average electric charge density, defined at every point in space.

3.1.Electrostatic Field and Potential

In continuous charge distributions, charged particles can be distributed uniformly along a line, over a surface, or throughout a volume.

For such a charge distribution, the infinitesimal electric field $\overrightarrow{dE}(M)$ at a point M, created by an infinitesimal charge dq, is calculated. The vector sum is then replaced by an integral.

Thus, the total electrostatic field created at point M by this charge distribution is:

$$\vec{E}(M) = \int \overrightarrow{dE}(M)$$

Where

$$\overrightarrow{dE}(M) = K\frac{dq}{r^2}\vec{u}$$

Similarly, the total electrostatic potential is:

$$V(M) = \int dV(M) = \int K\frac{dq}{r}$$

hree types of continuous charge distributions are defined.

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3.1.1. Linear Charge Distribution

For a linear charge distribution:

$$dq = \lambda . dl$$

dl is an infinitesimal element of the wire.

λ is the linear charge density.

The total electrostatic field and potential created at point M by this charge distribution are therefore:

$$\vec{E}(M) = \int K \frac{\lambda . dl}{r^2} \vec{u}$$
$$V(M) = \int K \frac{\lambda . dl}{r}$$

3.1.2. Surface Charge Distribution

For a surface charge distribution:

$$dq = \sigma . ds$$

ds is an infinitesimal surface element.

σ is the surface charge density.

The total electrostatic field and potential created by a surface charge distribution are:

$$\vec{E}(M) = \iint K \frac{\sigma . ds}{r^2} \vec{u}$$
$$V(M) = \iint K \frac{\sigma . ds}{r}$$

3.1.3. Volume Charge Distribution

For a volume charge distribution:

$$dq = \rho . dV$$

dV is an infinitesimal volume element.

ρ is the volume charge density.

In this case, the total electrostatic field and potential are:

$$\vec{E}(M) = \iiint K \frac{\rho . dV}{r^2} \vec{u}$$
$$V(M) = \iiint K \frac{\rho . dV}{r}$$

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4. Electrostatic dipole

An electrostatic dipole consists of two point charges of equal magnitude and opposite signs ($-q$ located at A and $+q$ located at B), separated by a very small distance a.

This dipole is characterized by a vector quantity called the electric dipole moment:

$$\vec{p} = q.a\vec{\imath} = q.\overrightarrow{AB}$$

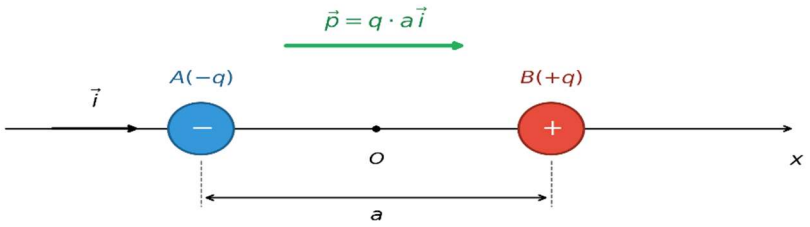
Where:

a is the distance between the two charges (in m).

q is the positive magnitude of the charge (in C).

$\vec{\imath}$ is a unit vector along the dipole axis (Ox), always directed from the negative charge ($-q$) toward the positive charge ($+q$).

In the International System of Units (SI), the electric dipole moment is expressed in coulomb-metres (C·m).



Electrostatic Dipole

For an arbitrary point in space located at a distance r) from the centre O) and at an angle θ with respect to the dipole axis, the following expressions are used:

- Electrostatic potential:

$$V(M) = \frac{Kp\cos\theta}{r^2}$$

- Magnitude of the electrostatic field:

$$E(M) = \frac{Kp}{r^3}\sqrt{1 + 3\cos^2\theta}$$

5. Gauss's theorem

5.1. Introduction

To calculate the electric field, we can always use Coulomb's law. However, when charges are distributed symmetrically, the calculations can become lengthy.

Gauss's theorem provides a much simpler method when the charge distribution has planar, cylindrical, or spherical symmetry.

5.2.Electric flux

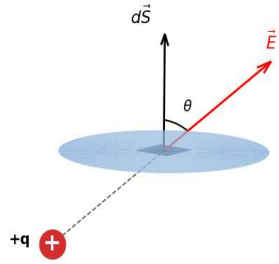
Electric flux measures the amount of electric field passing through a surface.

Consider a point charge $+q$ and an infinitesimal surface element ds,

oriented by its surface vector $\overrightarrow{dS}$, such that:

$$\overrightarrow{dS} = \vec{n}.dS$$

$\vec{n}$ is the unit vector normal to the surface.



The infinitesimal electric flux through this surface is therefore:

$$d\Phi = \vec{E}.\overrightarrow{dS} = E.dS.\cos\theta$$

Where θ is the angle between $\vec{E}$ and $\overrightarrow{dS}$.

In the International System of Units (SI), the unit of electric flux is:

$$Nm^2/C$$

Interpretation

- If $\vec{E}$ points outward through the surface ($\theta < 90°$), then $d\Phi > 0$.
- If $\vec{E}$ points inward through the surface ($\theta > 90°$), then $d\Phi < 0$.
- If $\vec{E}$ is tangent to the surface ($\theta = 90°$), then $d\Phi = 0$.
- If $\vec{E}$ is perpendicular to the surface and points in the same direction as $\overrightarrow{dS}$ ($\theta = 0°$), then $d\Phi = E.dS$.

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5.3.Statement of Gauss's Theorem

5.3.1. Statement

Gauss's theorem establishes a simple relationship between the electric flux through a closed surface and the electric charge enclosed within that surface.

The total electric flux through a closed surface is equal to the total charge enclosed by that surface divided by the permittivity of free space, ε_0.

$\Phi = \oiint_{S_G} \vec{E}.\overrightarrow{dS} = \frac{Q_{int}}{\varepsilon_0}$.

Where :

$\vec{E}$ is the electric field

$\overrightarrow{dS}$ is the infinitesimal surface vector, directed outward from the closed surface.

Q_{int} is the total electric charge enclosed within the surface.

$\varepsilon_0$is the permittivity of free space.

The symbol $\oiint$ indicates that the integration is performed over an entire closed surface.

The permittivity of free space is approximately:

$$\varepsilon_0 \approx 8.85 \times 10^{-1} \ F/m$$

Gauss's theorem applies to various charge distributions, including point, linear, surface, and volume charge distributions.

5.3.2. The Gaussian Surface

To apply Gauss's theorem, we imagine a closed surface surrounding the region under study. This surface is called the Gaussian surface, denoted by S_G.

A Gaussian surface is:

- Closed.
- Imaginary (an abstract mathematical surface).

5.3.3. Choosing the Gaussian Surface

We choose a surface that has the same symmetry as the charge distribution. This makes it easier to determine the direction of the electric field and simplifies the calculation of electric flux.

The main cases are:

1. Spherical charge distribution

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For a point charge or a distribution with spherical symmetry, we choose a sphere centered on the charge distribution.

2. Infinitely long charged wire

For an infinitely long charged wire, we choose a cylinder whose axis coincides with the axis of the wire.

Calculation of the Electric Field Created by a Point Charge Using Gauss's Theorem

Statement: Consider a positive point charge $+q$ placed at a point O. Determine the electrostatic field created by this charge at a distance r from O, using Gauss's theorem.

Step 1: Determine the direction of the electric field

By symmetry, the electric field created by the charge is radial.

Let us choose a spherical Gaussian surface of radius r, centered on the charge.

All points on this sphere are at the same distance r from the charge.

Therefore, the electric field has the same magnitude at every point on the surface.

Moreover, the electric field is perpendicular to the surface. Thus, the angle between the two vectors is zero, $\theta = 0$, and hence:

$\cos\theta = \cos 0^{\circ} = 1$

and

$$\vec{E}.\overrightarrow{dS} = E.dS$$

Step 2: Calculate the Electric Flux

The electric flux through the Gaussian surface is:

$$\Phi = \oiint_{S_G} \vec{E}.\overrightarrow{dS} = \oiint_{S_G} E.dS$$

Since the electric field E is constant over the spherical surface:

$$\Phi = E \oiint_{S_G} dS$$

The surface area of a sphere is:

$$S = 4\pi r^2$$

Therefore:

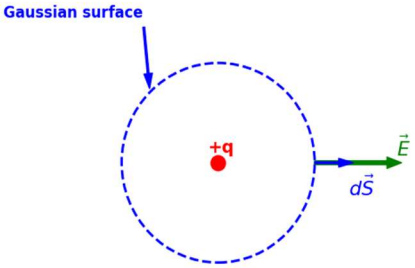
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$$\Phi = E.4\pi r^2$$

Step 3: Apply Gauss's Theorem

The charge enclosed by the Gaussian surface is:

$$Q_{enclosed} = q$$



According to Gauss's theorem:

$$E.4\pi r^2 = \frac{q}{\varepsilon_0}$$

Solving for E:

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Thus, we recover the expression for the electric fieldcreated by a point charge.

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