Chapter II : Conductors in electrostatic equuilibre and capacitors

Part I : Conductors in electrostatic equilibrium

1. General properties of conductors

1.1. Definition of a conductor

A conductor is a material that contains free charged particles that can move freely under the action of an electrostatic field.

Examples: Metals such as copper and iron.

1.2. Electrostatic Influence in a Conductor Placed in an Electric Field

When a conductor is placed in an external electric field $\vec{E}_{ext}$:

- The electric field exerts a force on the free electrons.
- The electrons move in the opposite direction to the external electric field.
- This movement causes a redistribution of charges on the surface of the conductor (this phenomenon is called electrostatic influence).
- This redistribution creates an induced electric field $\vec{E}_{ind}$, which is opposite to the external electric field $\vec{E}_{ext}$ inside the conductor.
- At equilibrium, the two fields cancel each other inside the conductor.

$$\vec{E}_{ext} + \vec{E}_{ind} = \vec{0}$$

Therefore:

$$\vec{E}_{int} = \vec{0}$$

1.3. Electrostatic Equilibrium

A conductor is in electrostatic equilibrium when its free electric charges no longer undergo any macroscopic movement inside the conductor.

This equilibrium is reached when the resulting electric field inside the conducting material becomes zero.

Properties of a conductor in electrostatic equilibrium:

1) The electric field is zero inside the conducting material:

$$\vec{E}_{int} = \vec{0}$$

2) The electric potential is the same at every point of the conductor. Therefore, the conductor is equipotential.

$$V = constante$$

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The potential difference between any two points A and B inside the conductor is therefore zero:

$$V_A - V_B = 0$$

3) In electrostatic equilibrium, any excess electric charge on a conductor is located on its outer surface.
4) The electric field just outside a conductor is perpendicular to its surface. It is given by the following relation:

$$\vec{E}_{ext} = \frac{\sigma}{\varepsilon_0}\vec{n}$$

Where:
σ : Surface charge density.
ε_0 : Permittivity of free space.
$\vec{n}$: Unit normal vector to the surface, directed outward from the conductor.

2. Electrostatic Induction

Electrostatic induction is a phenomenon that occurs when a charged conductor causes a redistribution of electric charges in another conductor, without any direct contact between them.

When a charged conductor is brought close to a neutral conductor, the free charges in the latter move under the action of the electric field created by the first conductor.

There are two types of electrostatic induction: **total induction** and **partial induction**.

2.1. Total Electrostatic Induction

Electrostatic induction is said to be total when a charged conductor (called the *inducing conductor*) is completely surrounded by another hollow conductor (called the *induced conductor*).

This phenomenon results in a complete redistribution of induced charges on the surfaces of the hollow conductor.

Consider two conductors:

C_1 : The inner conductor carries a charge $+Q$

C_2 : The hollow conductor completely surrounds C_1.

For the electric field inside the material of C_2 to be zero, an opposite charge appears on the inner surface of C_2 :

$$Q_{int} = -Q$$

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The charge on the outer surface depends on the electrical condition of C_2. Three cases can be distinguished:

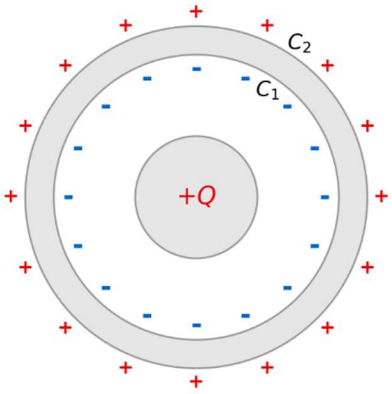
Case 1: C_2 is isolated and initially neutral

C_2 is initially neutral. Its inner surface carries a charge $-Q$.

To maintain the total charge of C_2 equal to zero, a charge $+Q$ appears on its outer surface.

$$Q_{int} = -Q$$

$$Q_{ext} = +Q$$



Case 2: C_2 initially carries a charge Q_0

The inner surface carries a charge $-Q$.

Since the isolated conductor C_2 initially carries a charge Q_0, a charge appears on its outer surface equal to:

$$Q_{int} = -Q$$

$$Q_{ext} = Q_0 + Q$$

Case 3: C_2 is connected to the ground

When C_2 is connected to the ground, its outer surface carries no charge in this configuration.

We obtain:

$$Q_{int} = -Q$$

$$Q_{ext} = 0$$

2.2. Partial Electrostatic Induction

Electrostatic induction is said to be partial when a charged conductor is placed near another conductor. In this case, the electric field of the charged conductor causes a redistribution of free charges within the nearby conductor.

Case of two conductors placed close to each other:

Consider two metallic conductors, C_1 and C_2, placed close to each other. Conductor C_1 carries a positive charge $+Q$, while conductor C_2 is initially neutral.

Under the action of the electric field created by C_1, the free electrons in conductor C_2 are attracted toward the region closest to C_1.

As a result:

- Negative charges accumulate on the side closest to C_1.
- Positive charges accumulate on the side farthest from C_1.

Conductor C_2 remains electrically neutral overall if it is initially neutral, isolated, and does not exchange charges with its surroundings.

The closer the conductors are to each other, the stronger the electrostatic induction generally becomes.

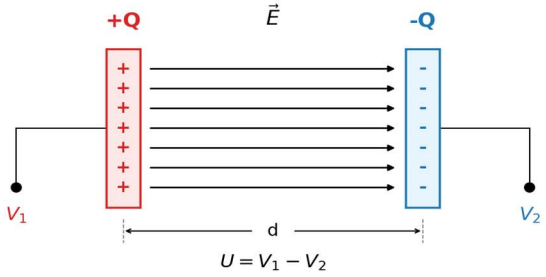
Part I : Capacitors

1. Definition of a capacitor

A capacitor is an electrical component consisting of two conductors, called *plates* or *electrodes*, in a state of total electrostatic induction and separated by an insulating material (air, vacuum, or dielectric).

When a capacitor is charged, it has:

- Two plates carrying opposite charges ($+Q$ and $-Q$).
- A potential difference (voltage) between the plates.
- An electric field in the insulating material.



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2. Capacitance of a capacitor

The electrical capacitance characterizes the ability of a capacitor to store electric charge. It is defined by:

$$C = \frac{Q}{U}$$

Where:

C : capacitance, measured in Farads (F).

Q : absolute value of the charge on one plate (C).

U : voltage between the plates (V).

$$U = V_1 - V_2$$

V_1 : potential of the positive plate.

V_2 : potential of the negative plate.

Since the electric field is directed from higher potential to lower potential:

$$V_1 > V_2$$

Therefore:

$$Q = C(V_1 - V_2)$$

The capacitance depends on the geometry of the capacitor and the nature of the insulating material placed between its plates.

3. Parallel-Plate capacitor

3.1. Definition

A parallel-plate capacitor is a device consisting of two flat, parallel metallic plates separated by an insulating material (air, vacuum, or dielectric).

When the capacitor is connected to a voltage source:

- One plate acquires a positive charge $+Q$.
- The other plate acquires a negative charge $-Q$.

The two plates carry charges that are equal in magnitude and opposite in sign.

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3.2.Electric field between the plates

Neglecting edge effects, the electric field between the two plates is uniform.

Characteristics of the electric field:

- **Direction:** Perpendicular to the plates.
- **Orientation:** From the positive plate toward the negative plate.

The magnitude of the electric field is given by:

$$E = \frac{U}{d} = \frac{V_1 - V_2}{d}$$

Where:

U : potential difference between the plates.

V_1 : potential of the positive plate.

V_2 : potential of the negative plate.

d : distance between the plates.

Relationship with surface charge density

For two parallel plates under total electrostatic induction, carrying opposite surface charge densities $+\sigma$ and $-\sigma$, the electric field between the plates is:

$$E = \frac{\sigma}{\varepsilon_0}$$

This relationship results from the superposition of the electric fields created by the two charged plates:

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

3.3. Capacitance of a Parallel-Plate Capacitor

For a parallel-plate capacitor, the capacitance is given by:

$$C = \frac{\varepsilon_0 S}{d}$$

ε_0 : permittivity of free space (vacuum).

S : surface area of one plate (m^2).

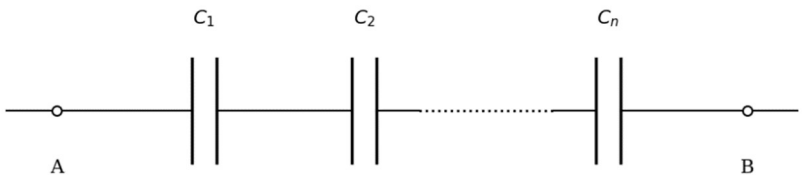
d : distance between the plates (m).

4. Capacitor Combinations

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4.1.Capacitors in Series

Capacitors are said to be connected in series when they are connected one after another along the same electrical path.



In this case, the same electric charge is stored on each capacitor:

$$Q = Q_1 = Q_2 = \cdots = Q_n$$

However, the total voltage is distributed among the different capacitors:

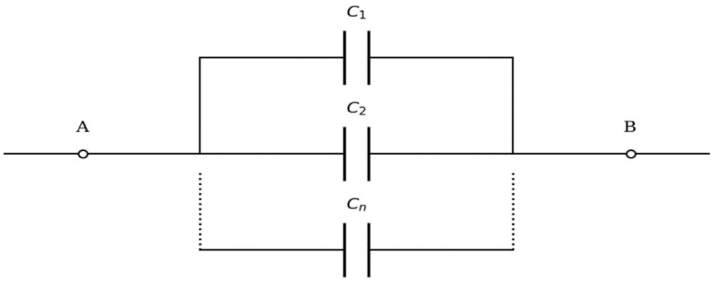
$$U = U_1 + U_2 + \cdots + U_n$$

Equivalent capacitance (C_{eq})

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

4.2.Capacitors in parallel

Capacitors are said to be connected in parallel when they are connected to the same two nodes of an electrical circuit.



Each capacitor is subjected to the same voltage:

$$U = U_1 = U_2 = \cdots = U_n$$

However, the total electric charge is distributed among the different capacitors:

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$$Q = Q_1 + Q_2 + \cdots + Q_n$$

Equivalent capacitance(C_{eq})

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_n$$

5. Energy stored in the capacitor

For a capacitor with capacitance C, carrying a charge Q and subjected to a voltage U, the stored energy is:

$$W = \frac{1}{2} C U^2$$

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