



Corrigé de TD N° 01 d'Electricité

1^{ère} partie : ELECTROSTATIQUE « charges ponctuelles »

Exercice 1 :

- Le potentiel au point O

$$V_O = V_A + V_B + V_C = k \frac{q_A}{OA} + k \frac{q_B}{OB} + k \frac{q_C}{OC}$$

$$OA=OB=OC=R$$

$$V_O = k \frac{(-q)}{R} + k \frac{(+q)}{R} + k \frac{(+q)}{R} \Rightarrow V_O = k \frac{q}{R}$$

- Le champ électrique au point O

$$\vec{E}_O = \vec{E}_A + \vec{E}_B + \vec{E}_C$$

Avec

$$\vec{E}_A = k \frac{q_A}{(OA)^2} \vec{u}_{AO}, \quad \vec{E}_B = k \frac{q_B}{(OB)^2} \vec{u}_{BO}$$

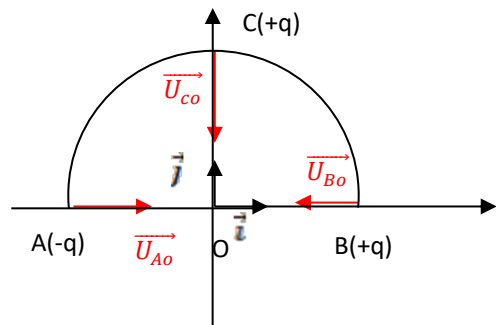
$$\vec{E}_C = k \frac{q_C}{(OC)^2} \vec{u}_{CO}$$

$$\vec{u}_{AO} = \vec{i}, \quad \vec{u}_{BO} = -\vec{i}, \quad \vec{u}_{CO} = -\vec{j}$$

$$\text{Donc } \vec{E}_O = k \frac{(-q)}{R^2} \vec{i} + k \frac{q}{R^2} (-\vec{i}) + k \frac{q}{R^2} (-\vec{j}) \Rightarrow \vec{E}_O = -k \frac{q}{R^2} (2\vec{i} + \vec{j})$$

La force électrostatique au point O

$$\vec{F}_O = q' \vec{E}_O = q \vec{E}_O = -k \frac{q^2}{R^2} (2\vec{i} + \vec{j})$$



Exercice 2

- Force électrostatique exercée sur la charge q_A

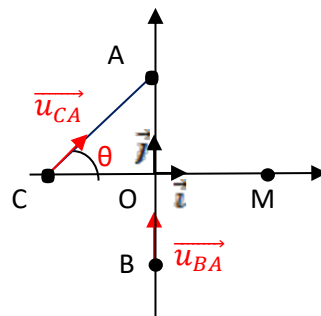
$$\vec{F}_A = \vec{F}_{BA} + \vec{F}_{CA}$$

$$\vec{F}_{BA} = k \frac{q_A q_B}{(BA)^2} \vec{u}_{BA}, \quad \vec{F}_{CA} = k \frac{q_A q_C}{(CA)^2} \vec{u}_{CA}$$

Avec $q_A=+q$, $q_B=+q$, $q_C= -2q$ et $BO=AO=a$ et $CO=b$

$BA=a+a=2a$ et $CA= ?$

D'après la règle de Pitagore dans le triangle droit (ACO) ; $CA^2=CO^2+OA^2$





$$CA^2 = a^2 + b^2$$

$$\vec{u}_{BA} = \vec{j}, \quad \vec{u}_{CA} = \cos \theta \vec{i} + \sin \theta \vec{j} \text{ avec } \cos \theta = \frac{CO}{CA} = \frac{b}{\sqrt{a^2+b^2}} \text{ et } \sin \theta = \frac{OA}{CA} = \frac{a}{\sqrt{a^2+b^2}}$$

$$\text{Donc } \vec{u}_{CA} = \frac{b}{\sqrt{a^2+b^2}} \vec{i} + \frac{a}{\sqrt{a^2+b^2}} \vec{j}$$

$$\vec{F}_A = k \frac{q^2}{4a^2} \vec{j} + k \frac{(-2q^2)}{a^2+b^2} \left(\frac{b}{\sqrt{a^2+b^2}} \vec{i} + \frac{a}{\sqrt{a^2+b^2}} \vec{j} \right) \Rightarrow \vec{F}_A = kq^2 \left[\frac{-2b}{(a^2+b^2)^{\frac{3}{2}}} \vec{i} + \left(\frac{1}{4a^2} - \frac{2a}{(a^2+b^2)^{\frac{3}{2}}} \right) \vec{j} \right]$$

• **Force électrostatique exercée sur la charge q_M**

$$\vec{F}_M = \vec{F}_{BM} + \vec{F}_{CM} + \vec{F}_{AM}$$

$$\vec{F}_{BM} = k \frac{q_M q_B}{(MA)^2} \vec{u}_{BM}, \quad \vec{F}_{CM} = k \frac{q_M q_C}{(CM)^2} \vec{u}_{CM}, \quad \vec{F}_{AM} = k \frac{q_M q_A}{(AM)^2} \vec{u}_{AM}$$

Avec $q_A=+q$, $q_B=+q$, $q_C=-2q$ et $q_M=+q$
BO=AO=a, CO=b et OM=x

$$CM = b + x \Rightarrow CM^2 = (b + x)^2 \text{ et } AM = ?, BM = ?$$

D'après la règle de Pitagore dans le triangle droit (AMO) ;

$$AM^2 = MO^2 + OA^2 \Rightarrow AM^2 = a^2 + x^2$$

D'après la règle de Pitagore dans le triangle droit (BMO) ;

$$BM^2 = MO^2 + OB^2 \Rightarrow BM^2 = a^2 + x^2$$

$$\vec{u}_{CM} = \vec{i}, \quad \vec{u}_{AM} = \sin \alpha \vec{i} - \cos \alpha \vec{j} \text{ avec } \cos \alpha = \frac{OA}{AM} = \frac{a}{\sqrt{a^2+x^2}} \text{ et } \sin \alpha = \frac{OM}{AM} = \frac{x}{\sqrt{a^2+x^2}}$$

$$\text{Donc } \vec{u}_{AM} = \frac{x}{\sqrt{a^2+x^2}} \vec{i} - \frac{a}{\sqrt{a^2+x^2}} \vec{j} \text{ et } \vec{F}_{AM} = k \frac{(q^2)}{a^2+x^2} \left(\frac{x}{\sqrt{a^2+x^2}} \vec{i} - \frac{a}{\sqrt{a^2+x^2}} \vec{j} \right)$$

$$\vec{u}_{BM} = \sin \alpha' \vec{i} + \cos \alpha' \vec{j} \text{ avec } \cos \alpha' = \frac{OB}{BM} = \frac{a}{\sqrt{a^2+x^2}} \text{ et } \sin \alpha' = \frac{OM}{BM} = \frac{x}{\sqrt{a^2+x^2}}$$

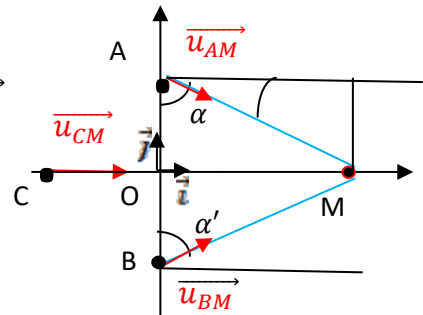
$$\text{Donc } \vec{u}_{BM} = \frac{x}{\sqrt{a^2+x^2}} \vec{i} + \frac{a}{\sqrt{a^2+x^2}} \vec{j} \text{ et } \vec{F}_{BM} = k \frac{(q^2)}{a^2+x^2} \left(\frac{x}{\sqrt{a^2+x^2}} \vec{i} + \frac{a}{\sqrt{a^2+x^2}} \vec{j} \right)$$

$$\vec{F}_M = k \frac{-2q^2}{(b+x)^2} \vec{i} + k \frac{(q^2)}{a^2+b^2} \left(\frac{x}{\sqrt{a^2+x^2}} \vec{i} - \frac{a}{\sqrt{a^2+x^2}} \vec{j} \right) + \frac{x}{\sqrt{a^2+x^2}} \vec{i} + \frac{a}{\sqrt{a^2+x^2}} \vec{j}$$

$$\Rightarrow \vec{F}_M = kq^2 \left[\left(\frac{2x}{(a^2+x^2)^{\frac{3}{2}}} - \frac{2}{(b+x)^2} \right) \vec{i} \right]$$

• **Le champ électrique au point M**

$$\vec{F}_M = q_M \vec{E}_M \Rightarrow \vec{E}_M = \frac{\vec{F}_M}{q_M}$$





$$\Rightarrow \vec{E}_M = kq \left[\left(\frac{2x}{(a^2+x^2)^{3/2}} - \frac{2}{(b+x)^2} \right) \vec{i} \right]$$

• **Le potentiel au point M**

$$V_M = V_A + V_B + V_C = k \frac{q_A}{AM} + k \frac{q_B}{BM} + k \frac{q_C}{CM}$$

$$\Rightarrow V_M = k \frac{q}{\sqrt{a^2+x^2}} + k \frac{q}{\sqrt{a^2+x^2}} - 2k \frac{q}{b+x}$$

$$\Rightarrow V_M = 2kq \left(\frac{1}{\sqrt{a^2+x^2}} - \frac{1}{b+x} \right)$$

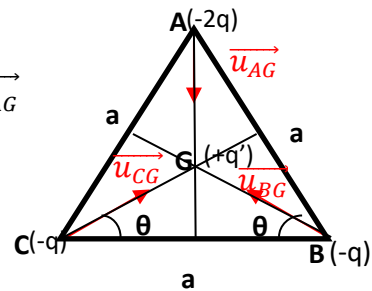
Exercice 3 :

• **La force électrostatique au point G :**

$$\vec{F}_G = \vec{F}_{BG} + \vec{F}_{CG} + \vec{F}_{AG}$$

$$\vec{F}_{BG} = k \frac{q_G q_B}{(GB)^2} \vec{u}_{BG}, \quad \vec{F}_{CG} = k \frac{q_G q_C}{(CG)^2} \vec{u}_{CG}, \quad \vec{F}_{AG} = k \frac{q_G q_A}{(AG)^2} \vec{u}_{AG}$$

Avec $q_A = -2q, q_B = -q, q_C = -q$ et $q_M = +q'$
 $AG^2 = BG^2 = CG^2 = a^2/3$



$$\vec{u}_{AG} = -\vec{j}, \quad \vec{u}_{BG} = -\cos \theta \vec{i} + \sin \theta \vec{j} \text{ et } \vec{u}_{CG} = \cos \theta \vec{i} + \sin \theta \vec{j}$$

$$\theta = \frac{\pi}{6} (30^\circ) \text{ donc } \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \text{ et } \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\vec{u}_{BG} = -\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2} \vec{j} \text{ et avec } \vec{u}_{CG} = \frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2} \vec{j}$$

$$\vec{F}_{BG} = k \frac{(-qq')}{a^2/3} \left(-\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2} \vec{j} \right), \quad \vec{F}_{CG} = k \frac{(-qq')}{a^2/3} \left(\frac{\sqrt{3}}{2} \vec{i} + \frac{1}{2} \vec{j} \right), \quad \vec{F}_{AG} = k \frac{(-2qq')}{a^2/3} (-\vec{j})$$

$$\vec{F}_G = -3k \frac{qq'}{a^2} \left(\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) \vec{i} + \left(\frac{1}{2} + \frac{1}{2} - 2 \right) \vec{j} \right)$$

$$\vec{F}_G = -3k \frac{qq'}{a^2} (-\vec{j})$$

$$\vec{F}_G = 3k \frac{qq'}{a^2} \vec{j}$$

• **Le champ électrique au point G**



$$\vec{F}_G = q_G \vec{E}_G \Rightarrow \vec{E}_G = \frac{\vec{F}_G}{q_G}$$

$$q_G = q'$$

$$\Rightarrow \vec{E}_G = \frac{\vec{F}_G}{q'} = 3k \frac{q}{a^2} \vec{j}$$

• **Le potentiel au point M**

$$V_G = V_A + V_B + V_C = k \frac{q_A}{AG} + k \frac{q_B}{BG} + k \frac{q_C}{CG}$$

$$\Rightarrow V_G = k \frac{(-2q)}{a/\sqrt{3}} + k \frac{(-q)}{a/\sqrt{3}} + k \frac{(-q)}{a/\sqrt{3}}$$

$$\Rightarrow V_G = -4 \sqrt{3} \frac{kq}{a}$$

Exercice 4 :

• **La force électrostatique au point D :**

$$\vec{F}_D = \vec{F}_{BD} + \vec{F}_{CD} + \vec{F}_{AD}$$

$$\vec{F}_{BD} = k \frac{q_D q_B}{(DB)^2} \vec{u}_{BD}, \quad \vec{F}_{CD} = k \frac{q_D q_C}{(CD)^2} \vec{u}_{CD}, \quad \vec{F}_{AD} = k \frac{q_D q_A}{(AD)^2} \vec{u}_{AD}$$

Avec $q_A = +q$, $q_B = -2q$, $q_C = +2q$ et $q_D = -q$

$$AD = BD = a$$

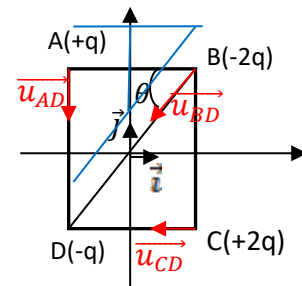
$$BD^2 = CD^2 + BC^2 = a^2 + a^2 = 2a^2$$

$$BD = a\sqrt{2}$$

$$\cos \theta = \frac{AB}{BD} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \sin \theta = \frac{AD}{BD} = \frac{a}{a\sqrt{2}} = \frac{\sqrt{2}}{2} \text{ donc } \theta = \frac{\pi}{4}$$

$$\cos\left(\frac{\pi}{2} - \varphi\right) = \sin \varphi$$

$$\sin\left(\frac{\pi}{2} - \varphi\right) = \cos \varphi$$



$$\vec{u}_{AD} = -\vec{j}, \quad \vec{u}_{CD} = -\vec{i} \text{ et } \vec{u}_{BD} = -\cos \theta \vec{i} - \sin \theta \vec{j}$$

$$\theta = \frac{\pi}{4} \text{ donc } \cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\vec{u}_{BD} = -\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j}$$



$$\vec{F}_{BD} = k \frac{(-2q)(-q)}{2a^2} \left(-\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j} \right) = k \frac{q^2}{a^2} \left(-\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j} \right),$$

$$\vec{F}_{CD} = k \frac{(+2q)(-q)}{a^2} (-\vec{i}) = 2k \frac{q^2}{a^2} \vec{i},$$

$$\vec{F}_{AD} = k \frac{(+q)(-q)}{a^2} (-\vec{j}) = k \frac{q^2}{a^2} \vec{j}$$

$$\vec{F}_D = k \frac{q^2}{a^2} \left(-\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j} \right) + 2k \frac{q^2}{a^2} \vec{i} + k \frac{q^2}{a^2} \vec{j}$$

$$\vec{F}_D = k \frac{q^2}{a^2} \left(\left(-\frac{\sqrt{2}}{2} + 2 \right) \vec{i} + \left(-\frac{\sqrt{2}}{2} + 1 \right) \vec{j} \right)$$

• Le champ électrique au point O :

$$\vec{E}_O = \vec{E}_{BO} + \vec{E}_{CO} + \vec{E}_{AO} + \vec{E}_{DO}$$

$$\vec{E}_{BO} = k \frac{q_B}{(OB)^2} \vec{u}_{BO}, \quad \vec{E}_{CO} = k \frac{q_C}{(CO)^2} \vec{u}_{CO}, \quad \vec{E}_{AO} = k \frac{q_A}{(AO)^2} \vec{u}_{AO}, \quad \vec{E}_{DO} = k \frac{q_D}{(DO)^2} \vec{u}_{DO}$$

Avec $q_A = +q$, $q_B = -2q$, $q_C = +2q$ et $q_D = -q$

$$AO^2 = \left(\frac{a}{2} \right)^2 + \left(\frac{a}{2} \right)^2 = \frac{a^2}{2} \text{ donc } AO^2 = BO^2 = CO^2 = DO^2 = \frac{a^2}{2}$$

($BO = a\sqrt{2}/2$ donc $BO^2 = a^2/2$)

$$\vec{u}_{BO} = -\cos \theta \vec{i} - \sin \theta \vec{j} = -\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j}$$

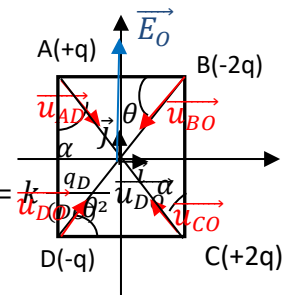
$$\vec{u}_{AO} = \sin \alpha \vec{i} - \cos \alpha \vec{j} = \frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j},$$

$$\vec{u}_{CO} = -\sin \alpha \vec{i} + \cos \alpha \vec{j} = -\frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j}$$

$$\vec{u}_{DO} = \cos \theta \vec{i} + \sin \theta \vec{j} = \frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \quad (\text{avec } \theta = \alpha = \frac{\pi}{4} \text{ donc } \cos \frac{\pi}{4} = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2})$$

$$\vec{E}_{BO} = k \frac{(-2q)}{a^2/2} \left(-\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j} \right), \quad \vec{E}_{CO} = k \frac{2q}{a^2/2} \left(-\frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \right)$$

$$\vec{E}_{AO} = k \frac{q}{a^2/2} \left(\frac{\sqrt{2}}{2} \vec{i} - \frac{\sqrt{2}}{2} \vec{j} \right), \quad \vec{E}_{DO} = k \frac{(-q)}{a^2/2} \left(\frac{\sqrt{2}}{2} \vec{i} + \frac{\sqrt{2}}{2} \vec{j} \right)$$





$$\text{donc } \vec{E}_O = 2k \frac{q}{a^2} \left[\left(-2 \left(-\frac{\sqrt{2}}{2} \right) + 2 \left(-\frac{\sqrt{2}}{2} \right) + \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \vec{i} + \left(-2 \left(-\frac{\sqrt{2}}{2} \right) + 2 \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \right) \vec{j} \right]$$

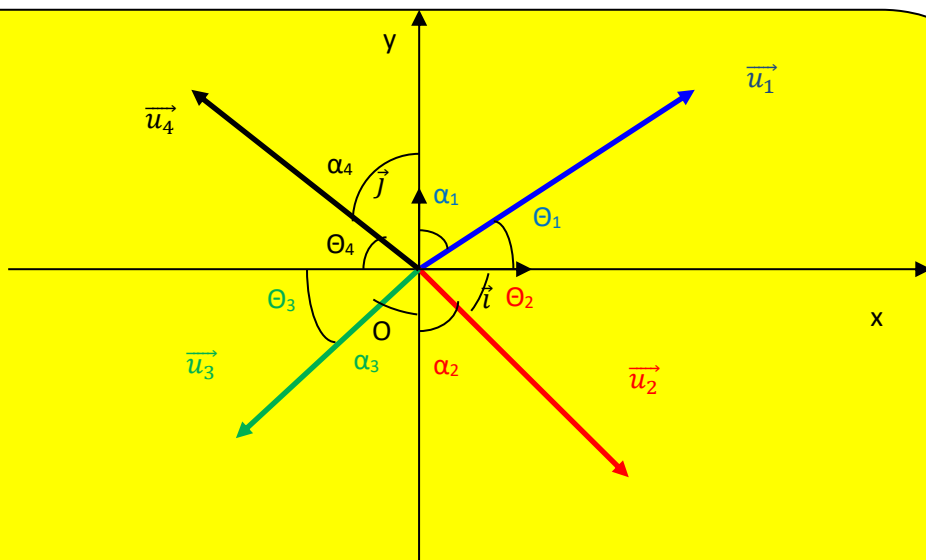
$$\Rightarrow \vec{E}_O = 2\sqrt{2}k \frac{q}{a^2} \vec{j}$$

• **Le potentiel au point O**

$$V_O = V_A + V_B + V_C + V_D = k \frac{q_A}{OA} + k \frac{q_B}{OB} + k \frac{q_C}{OC} + k \frac{q_D}{OD}$$

$$\Rightarrow V_O = k \frac{(+q)}{a/\sqrt{2}} + k \frac{(-2q)}{a/\sqrt{2}} + k \frac{(2q)}{a/\sqrt{2}} + k \frac{(-q)}{a/\sqrt{2}}$$

$$\Rightarrow V_O = 0$$



$$\vec{u}_1 = \cos \theta_1 \vec{i} + \sin \theta_1 \vec{j} \quad \text{ou} \quad \vec{u}_1 = \sin \alpha_1 \vec{i} + \cos \alpha_1 \vec{j}$$

$$\vec{u}_2 = \cos \theta_2 \vec{i} - \sin \theta_2 \vec{j} \quad \text{ou} \quad \vec{u}_2 = \sin \alpha_2 \vec{i} - \cos \alpha_2 \vec{j}$$

$$\vec{u}_3 = -\cos \theta_3 \vec{i} - \sin \theta_3 \vec{j} \quad \text{ou} \quad \vec{u}_3 = -\sin \alpha_3 \vec{i} - \cos \alpha_3 \vec{j}$$

$$\vec{u}_4 = -\cos \theta_4 \vec{i} + \sin \theta_4 \vec{j} \quad \text{ou} \quad \vec{u}_4 = -\sin \alpha_4 \vec{i} + \cos \alpha_4 \vec{j}$$