Exo 5 Résoudre les EDO suivantes :

1) $3y'' + 4y' - y = 0$, $y(0) = 1$, $y'(0) = 1$ $\quad y = y(x)$

2) $2y'' - 5y' = 3y$, $y(0) = 2$, $y'(0) = 1$ $\quad y = y(\theta)$

3) $z + 1 = z'' - x^2$ $\quad z = z(x)$

4) $2u'' - u' - u = \cos(2x) + \sin(2x)$ $\quad u = u(x)$

5) $y'' - 9y = 4\cos(3\theta)$ $\quad y = y(\theta)$

6) $x'' - 4x' = e^{2t} - 4x$ $\quad x = x(t)$

$(a \neq 0)$

$$ay'' + by' + cy = 0$$

$$1y'' + \frac{b}{a}y' + \frac{c}{a}y = 0$$

$a \qquad b$

$$r^2 + ar + b = 0$$

$$\begin{cases} 3y'' + 4y' - y = 0 \\ y(0) = 1 \ ; \ y'(0) = 1 \end{cases}$$

$y = e^{rt}$

edoL. 2nd ordre sans. 2nd membre

équa. cara : $3r^2 + 4r - 1 = 0$

$\Delta = 28$ $\qquad r_1 = \frac{-4 - \sqrt{28}}{6} = \frac{-2 - \sqrt{7}}{3}$

$$r_2 = \frac{-4 + \sqrt{28}}{6} = \frac{-2 + \sqrt{7}}{3}$$

$$y_0(t) = C_1 e^{\left(\frac{-2 - \sqrt{7}}{3}\right)t} + C_2 e^{\left(\frac{-2 + \sqrt{7}}{3}\right)t} = y(t)$$

$$\begin{aligned} y(0) = 1 &\iff \begin{cases} C_1 + C_2 = 1 \\ \left(\frac{-2 - \sqrt{7}}{3}\right)C_1 + \left(\frac{-2 + \sqrt{7}}{3}\right)C_2 = 1 \end{cases} \\ y'(0) = 1 &\iff \end{aligned}$$

$$\frac{-2 - \sqrt{7}}{3}(1 - C_2) + \frac{-2 + \sqrt{7}}{3}C_2 = 1$$

$$\frac{2\sqrt{7}}{3}C_2 = 1 + \frac{2 + \sqrt{7}}{3}$$

$$C_2 = \frac{5 + \sqrt{7}}{3} \cdot \frac{3}{2\sqrt{7}}$$

$$C_2 = \frac{5 + \sqrt{7}}{2\sqrt{7}} = \frac{5\sqrt{7} + 7}{14}$$

$$C_1 = 1 - \frac{5\sqrt{7} + 7}{14} = \frac{7 - 5\sqrt{7}}{14}$$

$C_1 =$ $\qquad C_2$

$$y(t) = \left(\frac{7 - 5\sqrt{7}}{14}\right) e^{\left(\frac{-2 - \sqrt{7}}{3}\right)t} + \left(\frac{5\sqrt{7} + 7}{14}\right) e^{\left(\frac{-2 + \sqrt{7}}{3}\right)t}$$

$y(t) =$

Exo 6 Donner la solution générale des edps suivantes

$\frac{\partial^2 z}{\partial t^2} = 0$; $\frac{\partial^2 v}{\partial y \partial t} = a$ $(a \in \mathbb{R})$ $z = z(t,x)$, $v = v(y,t)$

$\frac{\partial^2 z}{\partial t^2} = 0$ edp linéaire d'ordre 2.

$$\int \frac{\partial^2 z}{\partial t^2} dt = \int 0 \, dt$$

$$\int f''(t) dt = f'(t) + c$$

$$\frac{\partial z}{\partial t} = c(x)$$

$$z(t,x) = \int \frac{\partial z}{\partial t} dt = \int c(x) \, dt$$

$$\boxed{z(t,x) = c(x) t + B(x)}$$

$c(x) = e^x$

$B(x) = \sin x$

$$\frac{\partial^2 v}{\partial y \partial t} \neq \frac{\partial^2 v}{\partial t \partial y}$$

$$\frac{\partial^2 v}{\partial y \partial t} = \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial t}\right) = a$$

h

$$\int \frac{\partial^2 v}{\partial y \partial t} dy = \int \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial t}\right) dy = \int a \, dy$$

$$\frac{\partial v}{\partial t} = ay + c(t)$$

$$\int \frac{\partial v}{\partial t} dt = \int (ay + c(t)) dt$$

$$v(t,y) = ayt + \left[\int c(t) dt\right] + B(y)$$

Si $c(t) = t$
$B(y) = y$

$$\boxed{v(t,y) = ayt + \frac{t^2}{2} + y}$$

Exo 7 Donner la classification des EDPs suivantes.

1) $\frac{\partial^2 U}{\partial t^2} - y\frac{\partial^2 U}{\partial y^2} + \frac{\partial U}{\partial t} + 4t = 0$ $\qquad U = U(t,y)$

2) $x\frac{\partial^2 \vartheta}{\partial x^2} - y\frac{\partial^2 \vartheta}{\partial y^2} + \frac{\partial \vartheta}{\partial y} = 0$ $\qquad \vartheta = \vartheta(x,y)$

edp d'ordre 2.

$$\underbrace{A\frac{\partial^2 U}{\partial x^2} + B\frac{\partial^2 U}{\partial x \partial y} + C\frac{\partial^2 U}{\partial y^2}}_{p.p} + \cdots = 0$$

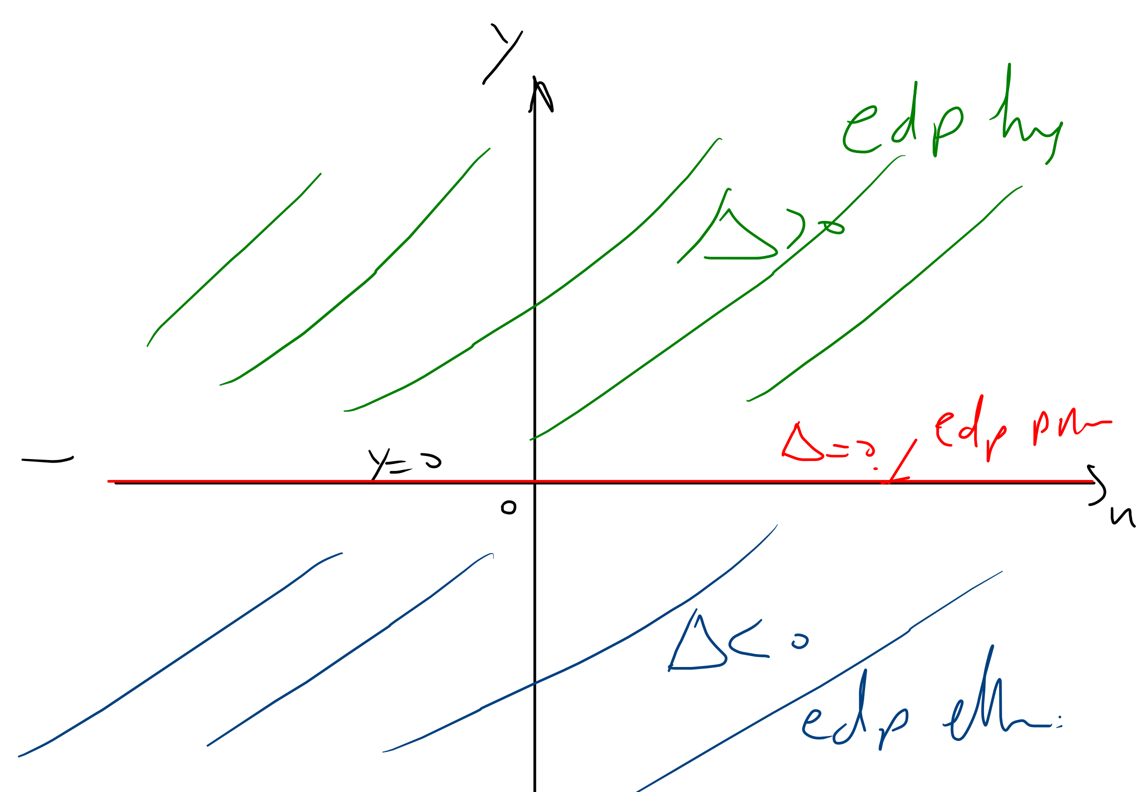
$\Delta = \boxed{B^2 - 4AC}$ $\quad > 0$, < 0, $= 0$ $\quad (x,y) \in \mathbb{R}^2$

$4 > 0$

$\frac{\partial^2 U}{\partial t^2} - y\frac{\partial^2 U}{\partial y^2} + \frac{\partial U}{\partial t} + 4t = 0$ $\qquad U = U(t,y)$

$A = 1$, $B = 0$, $C = -y$

$\Delta = 4y$
- $\to y > 0 \to \Delta > 0$ edp hyp
- $\to y = 0 \to \Delta = 0$ " para
- $\to y < 0 \to \Delta < 0$ " ellip —



$2u'' - u' - u = x+1$

$2u'' - u' - u = 0$ ESSM

e.c: $2r^2 - r - 1 = 0$

$\Delta = 9 > 0$ $r_1 = \frac{1-3}{4} = -\frac{1}{2}$

$r_2 = \frac{1+3}{4} = 1$

$U_0(x) = C_1 e^{-\frac{1}{2}x} + C_2 e^{+x}$

poly $d^\circ 0 = a$

" $d^\circ 1 = ax+b$

" $d^\circ 2 = ax^2+bx+1$

$2u'' - u' - u = x+1$

$U_p = ax+b$

$U_p' = a$

$U_p'' = 0$

$-a - ax - b = x+1$

$-ax - (a+b) = x+1$

$\begin{cases} -a = 1 \\ -(a+b) = 1 \end{cases}$ $\boxed{a = -1}$ $b = 0$

$\boxed{U_p = -x}$ $U(x) = C_1 e^{-\frac{1}{2}x} + C_2 e^{+x} - x$

1) edol a.c.c. 2 m. $\begin{cases} \to \text{Lag} \\ \to \text{iden.} \end{cases}$

poh

$$\boxed{y = y_0 + y_p}$$

$= p^{o}y \longrightarrow y_p^1$

$= \sim (x \longrightarrow y_p^2$

$y_p = y_p^1 + y_p^2$

$y = y_0 + y_p^1 + y_p^2$