Exo 2 Résoudre les EDO suivantes :

1) $4yx = (1+y^2)x'$ $\qquad x = x(y)$

2) $z'\cos(t) = z$ $\qquad z = z(t)$

3) $y' - ty = 0$ $\qquad y = y(t)$

4) $(1+t^2)x' = 1+x^2$ $\qquad x = x(t)$

1) $4yx = (1+y^2)x'$

On utilisant la méthode de séparation dv :

on pose : $x' = \dfrac{dx}{dy}$

$$4yx = (1+y^2)\frac{dx}{dy}$$

$$\frac{4y\,dy}{1+y^2} = \frac{dx}{x}$$

$$\int \frac{4y\,dy}{1+y^2} = \int \frac{dx}{x}$$

$$2\int \frac{2y}{1+y^2}dy = \int \frac{1}{x}dx$$

$$2\ln(1+y^2) = \ln|x| + c$$

$$\ln(1+y^2)^2 - c = \ln|x|$$

$$\boxed{x = K(1+y^2)^2}$$

K : constante

$$\int \frac{f'(x)}{f(x)} = \ln|f(x)| + c$$

$$\int \frac{f'}{1+(f(t))^2}dt = \arctan(f(t)) + c$$

$$\int \frac{-\sin(t)}{\cos(t)}\, d$$

$$\int \frac{f'}{f} \rightarrow$$

Exo1 *Intégrer les edo suivantes :*

1) $z'\cos(t) + z\sin(t) = 1 \quad t \in [0, \frac{\pi}{2}] \quad z = z(t)$

2) $x' = 1 + \tan^2(t) \quad x = x(t)$

3) $y' = e^t \cos(t) \quad y(0) = 1 \quad y = y(t)$

$$(\cos(t))z' + (\sin(t))z = 1 \quad (*)$$

$$\cos(t)z' + \sin(t)z = 0$$

$$z_0(t) = C\exp\left(-\int \frac{\sin(t)}{\cos(t)} dt\right)$$

$$z_0(t) = C\exp(\ln|\cos(t)|)$$

$$z_0(t) = C|\cos(t)|$$

$$\boxed{z_0(t) = C\cos(t)} \qquad t \in [0, \frac{\pi}{2}]$$

$a(t)y' + b(t)y = g(t)$

$a(t)y' + b(t)y = 0$ ESSM

$y_0(t) = c\exp\left(-\int \frac{b(t)}{a(t)} dt\right)$

$y(t) = C(t)\exp(A(t))$

y'

$$z(t) = C(t)\cos(t)$$

$$z'(t) = C'(t)\cos(t) - C(t)\sin(t)$$

$(\operatorname{tg}(t))' = \left(\frac{\sin(t)}{\cos(t)}\right)' = \frac{\cos^2 + \sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t}$

On remplace z, z' dans $(*)$

$$\cos(t)\big(C'(t)\cos(t) - C(t)\sin(t)\big) + \sin(t)C(t)\cos(t) = 1$$

$$C'(t)\cos^2(t) - C(t)\cancel{\cos(t)\sin(t)} + C(t)\cancel{\sin(t)\cos(t)} = 1$$

$$C'(t)\cos^2(t) = 1$$

$$C'(t) = \frac{1}{\cos^2(t)}$$

$$C(t) = \int \frac{1}{\cos^2(t)} dt$$

$$\boxed{C(t) = \operatorname{tg}(t) + k}$$

$$z(t) = (\operatorname{tg}(t) + k)\cos(t)$$

$$\boxed{z(t) = \underbrace{\sin(t)}_{y_p} + \underbrace{k\cos(t)}_{y_0}}$$

$$y = y_0 + y_p$$

3) $y' = e^t \cos(t) \quad y(0) = 1 \qquad \cdot y = y(t)$

pb de Cauchy $\begin{cases} \text{edo} \\ \boxed{y(0) = 1} \end{cases}$ c.I

$$y' = e^t \cos(t)$$

$$\frac{dy}{dt} = e^t \cos(t)$$

$$dy = e^t \cos(t) dt$$

$$\int dy = \int e^t \cos(t) dt$$

$$y = \int e^t \cos(t) dt$$

$\int \underbrace{e^{ht}}_{U} \underbrace{-h}_{V'} dt$

$\left\langle \begin{matrix} U = e^t & U' = e^t \\ V' = \cos(t) & V = \sin(t) \end{matrix} \right\rangle$

$$y = e^t \sin(t) - \int e^t \sin(t) dt$$

$\left\langle \begin{matrix} U = e^t \to U' = e^t \\ V' = \sin(t) \to V = -\cos(t) \end{matrix} \right\rangle$

$$y = e^t \sin(t) - \left(-e^t \cos(t) + \int e^t \cos(t) dt\right) + c$$

$$y = e^t(\sin(t) + \cos(t)) - \underbrace{\int e^t \cos(t) dt}_{y} + c$$

$$2y = e^t(\sin(t) + \cos(t)) + c$$

$$y(t) = \frac{1}{2} e^t(\sin(t) + \cos(t)) + c$$

$$1 = y(0) = \frac{1}{2} + c \iff \boxed{c = \frac{1}{2}}$$

$$y(t) = \frac{1}{2} e^t(\sin(t) + \cos(t)) + \frac{1}{2}$$

La méthode d'identification pour résoudre une EDO linéaire 1 ordre :

$a(t)y' + b(t)y = g(t)$ EDO 1ʳᵉ linéaire

1) $a(t)$, $b(t)$. cont.

2) $g(t)$: poly, exp. trig

$y_0 + \boxed{y_p} = y$

$g(t)$	y_p
$P_n(t)$	$Q_n(t)$
$P_n(t)e^{kt}$	$x Q_n(t) e^{kt}$ $y_0 = e^{kt}$
$A\sin(kt) + B\cos(kt)$	$A'\sin(kt) + B'\cos(kt)$

sin(M
cos

$y' + 2y = t^2$ $\quad$ $y' + 2y = 1e^{x}$ $\quad$ $y' + 2y = \sin(t)$

$y_0 = c\exp(-\int 2dt)$

$y_0 = ce^{-2t}$

$g(t)$. poly d°2

$y_p = at^2 + bt + c$

$y_p' = 2at + b$

$y_p' + 2y_p = t^2$

$(2at + b) + 2(at^2 + bt + c) = t^2$

$2at^2 + (2a + 2b)t + b + 2c = t^2 + 0t + 0$

par ident

$$\begin{cases} 2a = 1 \\ 2a + 2b = 0 \\ b + 2c = 0 \end{cases}$$

$\boxed{a = \frac{1}{2}}$ $\boxed{b = -\frac{1}{2}}$ $\boxed{c = \frac{1}{4}}$

$\boxed{y_p = \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{4}}$

$y = y_p + y_0 = \frac{1}{2}t^2 - \frac{1}{2}t + \frac{1}{4} + ce^{-2t}$

$y_p = ae^{t}$

$y_p' = ae^{t}$

$ae^{t} + 2ae^{t} = e^{t}$

$3a = 1$

$\boxed{a = \frac{1}{3}}$

$y_p = \frac{1}{3}e^{t}$

$y_G = ce^{-2t} + \frac{1}{3}e^{t}$

$y' + 2y = e^{-2t}$

$y_p = ate^{-2t}$

$y_p' = ae^{-2t} - 2ate^{-2t}$

$ae^{-2t} - 2ate^{-2t} + 2ate^{-2t} = e^{-2t}$

$\boxed{a = 1}$

$y_p = te^{-2t}$

$y_p = A'\sin(t) + B'\cos(t)$

y_p'

$P_0(t) = a$

$P_1(t) = at + b$

$P_2(t) = at^2 + bt + c$