Exo 5 Résoudre les EDO suivantes :

1) $3y'' + 4y' - y = 0$, $y(1) = 1$, $y'(2) = 1$ $\quad y = y(x)$

2) $2y'' - 5y' = 3y$, $y(0) = 2$, $y'(0) = 1$ $\quad y = y(\theta)$

3) $z + 1 = z'' - x^2$ $\quad z = z(x)$

4) $2u'' - u' - u = \cos(2x) + \sin(2x)$ $\quad u = u(x)$

5) $y'' - 9y = 4\cos(3\theta)$ $\quad y = y(\theta)$

6) $x'' - 4x' = e^{2t} - 4x$ $\quad x = x(t)$

edo 2^{nd} ordre sans 2^{nd} membre. $\quad ay'' + by' + cy = 0 \quad (a \neq 0)$

$y'' \to r^2, \quad y' \to r, \quad y \to 1$

e.c $\quad ar^2 + br + c = 0 \to \Delta$

$$\begin{cases} 2y'' - 5y' = 3y \\ y(0) = 2 \ ; \ y'(0) = 1 \end{cases}$$

$$2y'' - 5y' - 3y = 0$$

e.c : $2r^2 - 5r - 3 = 0 \quad \to \Delta = 49 > 0 \quad r_1 = \frac{5 - 7}{4} = -\frac{1}{2}$

$$r_2 = \frac{5 + 7}{4} = 3$$

$$y_0(t) = y(t) = C_1 e^{-\frac{t}{2}} + C_2 e^{3t}$$

$$y'(t) = -\frac{1}{2} C_1 e^{-\frac{t}{2}} + 3 C_2 e^{3t}$$

$\Delta = 0 \quad C_1 t + C_2$

$y = (C_1 t + C_2) e^{rt}$

$$y(0) = 2 \iff \boxed{C_1 + C_2 = 2}$$

$$y'(0) = 1 \iff \boxed{-\frac{1}{2} C_1 + 3 C_2 = 1}$$

$\Rightarrow C_1 = \frac{10}{7}$

$C_2 = \frac{4}{7}$

$$y(t) = \frac{10}{7} e^{-\frac{t}{2}} + \frac{4}{7} e^{3t}$$

$ay'' + by' + cy = g(t)$ — la la / la id — pv / tr / e — y

$ay'' + by' + cy = 0 \quad \to y_0(t)$

$\to y_p(t)$

$$y = y_H + y_p$$

$2u'' - u' - u = 2t - 3$ ⊛

$d^\circ = 0 \to a$

$d^\circ = 1 \to ax + b$

$d^\circ = 2 \to ax^2 + bx + c$

$2U'' - U' - U = 0$

e.c: $2r^2 - r - 1 = 0$

$\Delta = 9 > 0$ $\quad r_1 = -\frac{1}{2}$ $\quad r_2 = 1$

$$U_H(t) = U_0(t) = C_1 e^{-\frac{t}{2}} + C_2 e^{t}$$

$U_p = ?$

$$\left.\begin{array}{l} U_p = at + b \\ U_p' = a \\ U_p'' = 0 \end{array}\right) \to ⊛ \quad 2(0) - a - at - b = 2t - 3$$

$$-at - (a+b) = 2t - 3$$

$$\begin{cases} -a = 2 \\ -a - b = -3 \end{cases} \qquad \boxed{a = -2} \quad \boxed{b = 5}$$

$$\boxed{U_p = -2t + 5}$$

$$U(t) = U_H(t) + U_p(t) = C_1 e^{-\frac{t}{2}} + C_2 e^{t} - 2t + 5$$

$$U'(t) = -\frac{1}{2} C_1 e^{-t/2} + C_2 e^{t} - 2$$

$$\begin{cases} U(0) = 1 \\ U'(0) = 1 \end{cases}$$

$$1 = U'(0) = -\frac{1}{2} C_1 + C_2 - 2$$

$$U(0) = C_1 + C_2 + 5 = 1$$

$$\begin{cases} -C_1 + 2C_2 = 6 \\ C_1 + C_2 = -4 \end{cases} \qquad \boxed{C_2 = \frac{2}{3}} \quad \boxed{C_1 = \frac{-14}{3}}$$

$= \text{poly} + f.\text{tri}$

Ae^{kx}

$Y_p = \text{poly} + f.\text{tr}$

$2y'' + y' + y = x^2 + 1 + \sin(2x)$

$y_p^1 = ax^2 + bx + c$

$y_p^2 = A\sin(2x) + B\cos(2x)$

$Y_p = y_p^1 + y_p^2$

$Y = Y_p + Y_0$

$= \cos( )$

$\sin( )$

$\cos + \sin$

$Y_p = A\cos + B\sin( )$

$A\cos( ) + B\sin$

$= \frac{\cos}{\sin}$

$z'' + z' + z = 2\sin(3x) + \cos(x)$ | A

$z_p = A\sin(3x) + B\cos(3x)$

$z_p' = 3A\cos(3x) - 3B\sin(3x)$

$z_p'' = -9A\sin(3x) - 9B\cos(3x)$

$( )\sin$

$+ ( )\cos$

Exo 6 Donner la solution générale des edps suivantes

$$\frac{\partial^2 z}{\partial t^2}=0 \quad ; \quad \frac{\partial^2 v}{\partial y \partial t}=a \quad (a\in\mathbb{R}) \qquad z=z(t,x) \quad v=v(y,t)$$

$$\frac{\partial^2 z}{\partial t^2}=0 \iff \frac{\partial}{\partial t}\left(\frac{\partial z}{\partial t}\right)=0 \qquad \int f'(t)dt = f(t)+c$$

$f(t)$

$$\int \frac{\partial^2 z}{\partial t^2}dt = \int 0\,dt$$

$$\frac{\partial z}{\partial t} = C(x)$$

$$\int \frac{\partial z}{\partial t}dt = \int C(x)dt$$

$$z = C(x)t + B(x) \qquad C(x)=e^x \quad B(x)=x^2$$

$$\boxed{z(x,t)=te^x+x^2}$$

$$\int f''(t)dt = f'(t)+c$$

$$\int (f'(t)+c)dt = f(t)+ct+k$$

$$\frac{\partial^2 v}{\partial y \partial t}=a \iff \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial t}\right)=a$$

$$\int \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial t}\right)dy = \int a\,dy$$

$$\frac{\partial v}{\partial t} = ay+b(t)$$

$$\int \frac{\partial v}{\partial t}dt = \int (ay+b(t))dt$$

$$v(t,y) = ayt+\int b(t)dt + k(y)$$

Exo 7 Donner la classification des EDPs suivantes.

1) $\frac{\partial^2 U}{\partial t^2} - y\frac{\partial^2 U}{\partial y^2} + \frac{\partial U}{\partial t} + 4t = 0$ $\qquad U = U(t,y)$

2) $x\frac{\partial^2 \vartheta}{\partial x^2} - y\frac{\partial^2 \vartheta}{\partial y^2} + \frac{\partial \vartheta}{\partial y} = 0$ $\qquad \vartheta = \vartheta(x,y)$

edp : $d^0 = 2$

$$\underbrace{A\frac{\partial^2 U}{\partial x^2} + B\frac{\partial^2 U}{\partial x \partial y} + C\frac{\partial^2 U}{\partial y^2}}_{P.P} + \_\_\_ = -$$

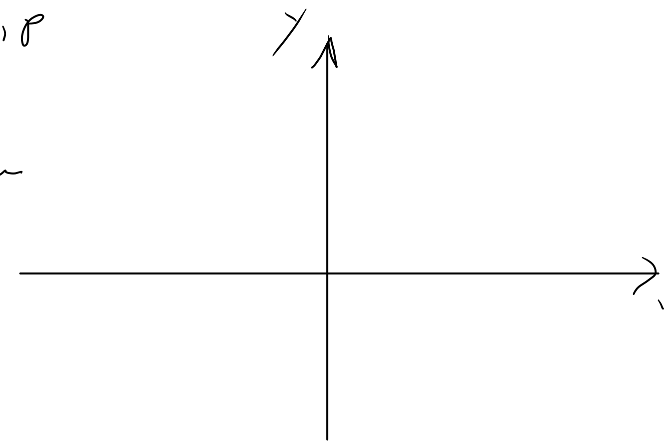
$\Delta = B^2 - 4AC > 0 \to$

$< 0 \to$

$= 0 \to$

$\Delta = -3$

edp elliptique

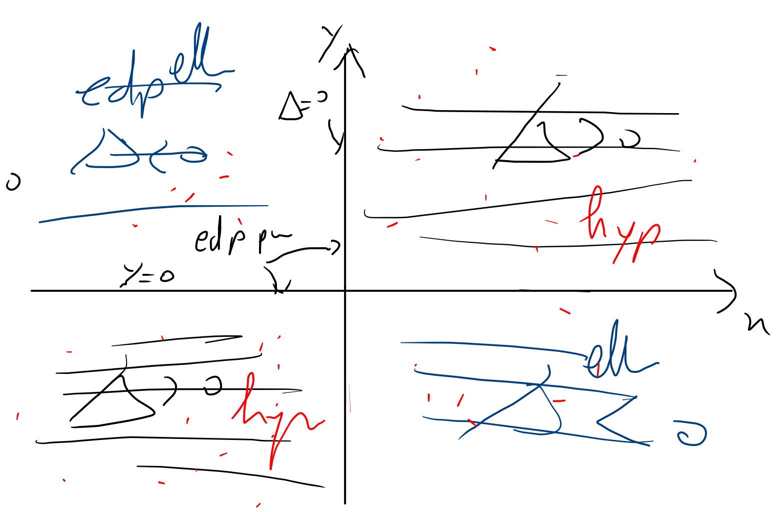


$$\underbrace{x\frac{\partial^2 \vartheta}{\partial x^2} - y\frac{\partial^2 \vartheta}{\partial y^2}}_{PP} + \frac{\partial \vartheta}{\partial y} = 0 \qquad \vartheta = \vartheta(x,y)$$

$A = x \qquad B = 0 \qquad C = -y$

$\Delta = 4xy$

$xy = 0 \to \Delta = 0$



$\Delta > 0$
$xy > 0 \begin{cases} x > 0, y > 0 \\ x < 0, y < 0 \end{cases}$

$\Delta = 0$
$xy = 0 \begin{cases} x = 0 \\ y = 0 \end{cases}$

$\Delta < 0$
$xy < 0 \begin{cases} x > 0, y < 0 \\ x < 0, y > 0 \end{cases}$

$$az'' + bz' + cz = 0$$

$$(i)^2$$

$$\underline{\Delta < 0}$$

$$\Delta = -4 < 0$$

$$= (i2)^2$$

$$\alpha + i\beta$$

$$r_1 = \frac{-b - 2i}{2a} = -\frac{b}{2a} + i$$

$$\alpha - i\beta$$

$$r_2 = \frac{-b + 2i}{2a}$$

$$y(\cdot) = e^{-\frac{b}{2a}}\left(C_1 \left(\quad \right) + C_2 \left(\quad \right)\right)$$

$$3y' + \sqrt{y} = 0$$

edoL 2a c.v =

$$y'' + 2(y')^2 + y = 0$$

$$y'' + 2\ln y + y' = 0$$