Equations différentielles

$$F(x, f(x), f'(x), f'' \dots f^{(n)}) = 0$$

$F(x, f(x), f'(x)) = 0$ eq. diff d'ordn 1

$f'(x) + f(x) + x = 0$ eq. d. 1

$y = y(x)$

$z = z(t)$

$f(x) = y$, z. W.

$y' + y + x = 0 \longrightarrow y = y(x)$

$y'' + y' = 3$ eq. diff. d'ordn 2.

$2y' + y + 2t = 0$

$\underbrace{y = y(t)}$

$f(x)$

edo

ed 1^{e} ordre

ed 2^{nd} ordre

ed linéaire

ed. aux séparées

edp

$f(x,y)$

$\frac{\partial f}{\partial x}$ $\frac{\partial^2 f}{\partial x^2}$

$\frac{\partial f}{\partial y}$ $\frac{\partial^2 f}{\partial y^2}$

EDO 1er ordre

$$F(x, y, y') = 0$$

EDO à variables séparées :

$$\frac{dy}{dx} = y' = f(x) \cdot g(y)$$

$g(y) \neq 0$

$$\int \frac{dy}{g(y)} = \int f(x)\, dx$$

$$\int \frac{1}{g(y)}\, dy = \int f(x)\, dx$$

$$\boxed{G(y) = F(x) + C}$$

$y = (x .)$

$$e^{y} = x^2 + C$$

$$y = \ln|x^2 + C|$$

$$F(\theta, z, z') = 0$$

$$\frac{a}{b} = cd$$

$$\frac{a}{d} = bc$$

$$z = z(\theta)$$

$$F(t, x, x') = 0$$

$$x = x(t)$$

EDO 1ère ordre

$F(x, y, y') = 0$

EDO à variables séparées:

$\frac{dy}{dx} = y' = f(x) \cdot g(y)$

$\frac{a}{b} = \frac{c}{d}$; $\frac{a}{d} = bc$

$g(y) \neq 0$

$\int \frac{dy}{g(y)} = \int f(x)dx$

$\int \frac{1}{g(y)} dy = \int f(x)dx$

$\boxed{G(y) = F(x) + c}$

$y = (x.)$

$e^{y} = x^2 + c$

$y = \ln|x^2 + c|$

EDO à v.s

$\frac{dy}{dx} = y' = \frac{f(x)}{g(y)}$ $g(y) \neq 0$

$\int g(y)dy = \int f(x)dx$

$\boxed{G(y) = F(x) + C}$

$\int \frac{1}{y} dy = \int \frac{1}{x} dx$

$e^{a+b} = e^a e^b$

$\ln|y| = \ln|x| + c$

$|y| = e^{\ln|x| + c}$ $\Rightarrow y = \pm e^{\ln|x| + c} = \underbrace{\pm e^{c}}_{k} e^{\ln|x|}$

$\boxed{y(x) = k|x|}$

Exp 1

$y' - ye^{x} = 0$ $y = y(x)$

$2y' + \frac{\ln(t)}{y^2} = 0$ $y = y(t)$

$y' - ye^{x} = 0$

$y' = ye^{x}$

$\frac{dy}{dx} = ye^{x}$

$\int \frac{dy}{y} = \int e^{x} dx$

$\ln|y| = e^{x} + c$

$|y| = e^{(e^{x} + c)}$

$y = \pm e^{e^{x}} \cdot e^{c}$

$\boxed{y = k e^{e^{x}}}$

$2y' + \frac{\ln(t)}{y^2} = 0$

$2\frac{dy}{dt} = -\frac{\ln(t)}{y^2}$

$2y^2 dy = -\ln(t) dt$

$\int 2y^2 dy = \int -\ln(t) dt$

$\frac{2y^3}{3} = -\int 1 \cdot \ln t \, dt$

$u = \ln(t) \longrightarrow u' = \frac{1}{t}$

$v' = 1 \longrightarrow v = t$

$\frac{2y^3}{3} = -(t\ln(t) - t) + c$

$y = \sqrt[3]{-\frac{3}{2}(t\ln(t) - t) + c}$

EDO → linéaire 1^{er} ordre

2nd membre (2nd M)

$$\underline{a(t)}y' + \underline{b(t)}y = \overbrace{g(t)}^{} \quad -- \; (2) \qquad a(t) \neq 0$$

a. b. g : sont des fonction continues sur $I \subset \mathbb{R}$ dépend de t.

1/ Solution générale de l'edo. sans 2nd me (3)

$$a(t)y' + b(t)y = 0 \quad — \; (3)$$

$$y_0(t)$$

2/ Solution particulière $y_p(t)$

Sol général de (2) : $y(t) = y_0(t) + y_p(t)$

$$a(t)y' + b(t)y = 0$$ E.D.O.S.H
E.D.O. Homogênea

$y_0(t)$

$$a(t)y' = -b(t)y$$

$$\frac{dy}{dt} = y' = \frac{-b(t)}{a(t)} \cdot y$$

$$\int \frac{dy}{y} = \int -\frac{b(t)}{a(t)}\,dt$$

$$\ln|y| = \int -\frac{b(t)}{a(t)}\,dt$$

$$y_0(t) = \pm \exp\left(\int -\frac{b(t)}{a(t)}\,dt + C\right)$$

$$y_0(t) = K \exp\left(\int \frac{-b(t)}{a(t)}\,dt\right)$$