University of Tlemcen
Faculty of Science
Department of Mathematic

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1st Year LMD-M and MI

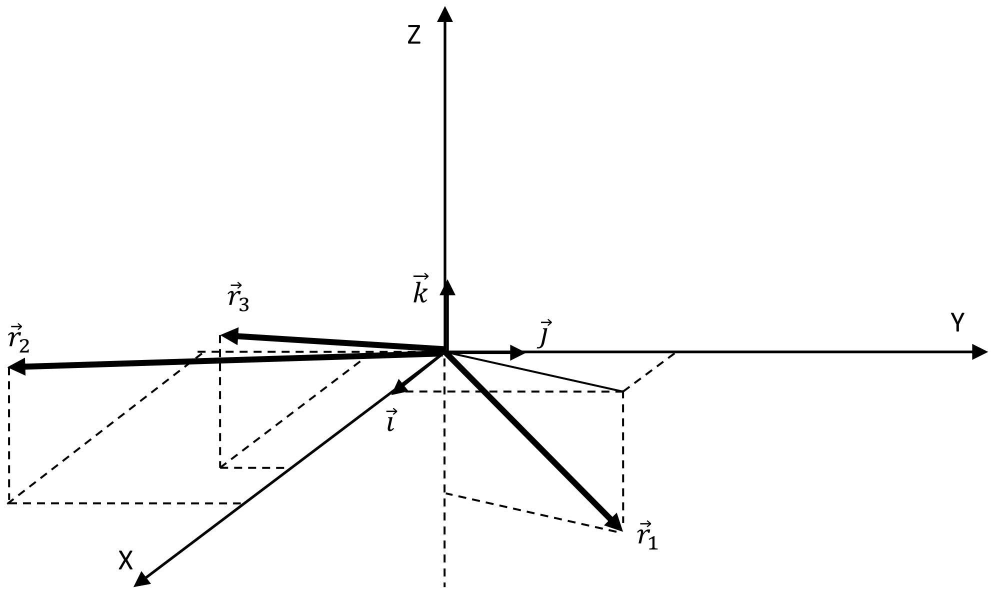
Supervised work correction N°2 of Mechanics

Vector analysis

Exercice 1:

We are $\vec{r_1}=\vec{\imath}+3\vec{\jmath}-2\vec{k}\begin{pmatrix}1\\3\\-2\end{pmatrix}$ $\vec{r_2}=4\vec{\imath}-2\vec{\jmath}+2\vec{k}\begin{pmatrix}4\\-2\\2\end{pmatrix}$ and $\vec{r_3}=3\vec{\imath}-\vec{\jmath}+2\vec{k}\begin{pmatrix}3\\-1\\2\end{pmatrix}$

1- Vector representation $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$:



2- The magnitudes of :

$$\vec{A}\begin{pmatrix}x\\y\\z\end{pmatrix}\Rightarrow|\vec{A}|=\|\vec{A}\|=\sqrt{x^2+y^2+z^2}$$

$$|\vec{r_1}|=\sqrt{x_1^2+y_1^2+z_1^2}=\sqrt{1+9+4}=\sqrt{14}$$

$$|\vec{r_2}|=\sqrt{x_2^2+y_2^2+z_2^2}=\sqrt{16+4+4}=\sqrt{24}$$

$$|\vec{r_1}|=\sqrt{x_3^2+y_3^2+z_3^2}=\sqrt{9+1+4}=\sqrt{14}$$

3- $\vec{r_1}.\vec{r_2}=x_1x_2+y_1y_2+z_1z_2=4-6-4=-6$

and $\vec{r_1}\wedge\vec{r_2}=\begin{vmatrix}\vec{\imath}&\vec{\jmath}&\vec{k}\\1&3&-2\\4&-2&2\end{vmatrix}=\begin{vmatrix}3&-2\\-2&2\end{vmatrix}\vec{\imath}-\begin{vmatrix}1&-2\\4&2\end{vmatrix}\vec{\jmath}+\begin{vmatrix}1&3\\4&-2\end{vmatrix}\vec{k}$

$$\Rightarrow \vec{r_1} \wedge \vec{r_2} = (3.2 - (-2.(-2)))\vec{\imath} - (1.2 - ((-2).4))\vec{\jmath} + (1.(-2) - (3.4))$$

$$\Rightarrow \vec{r_1} \wedge \vec{r_2} = 2\vec{\imath} - 10\vec{\jmath} - 14\vec{k}$$

Exercice 2 :

We give the three vectors $\vec{V_1}(1, 1, 0)$, $\vec{V_2}(0, 1, 0)$ and $\vec{V_3}$ (0, 0, 2).

1. Calculates the normes $\|\vec{V_1}\|$, $\|\vec{V_2}\|$ and $\|\vec{V_3}\|$:

Let's calculate the norms of the various vectors and the unit vectors of their respective directions.

$$\|\vec{V_1}\| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2} \Rightarrow \vec{v_1} = \frac{\vec{V_1}}{\|\vec{V_1}\|};\ \vec{v_1}(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$$

$$\|\vec{V_2}\| = \sqrt{0^2 + 1^2 + 0^2} = 1 \Rightarrow \vec{v_2} = \frac{\vec{V_2}}{\|\vec{V_2}\|} = \vec{\jmath};\ \vec{v_2}(0, 1, 0)$$

$$\|\vec{V_3}\| = \sqrt{0^2 + 0^2 + 2^2} = \sqrt{4} = 2 \Rightarrow \vec{v_3} = \frac{\vec{V_3}}{\|\vec{V_3}\|};\ \vec{v_3}(0, 0, 1)$$

2. Let's calculate cos $(\widehat{\vec{v_1}, \vec{v_2}})$ as follows :

$$\vec{v_1}.\vec{v_2} = \frac{\sqrt{2}}{2} \quad and\ \vec{v_1}.\vec{v_2} = \|\vec{v_1}\|.\|\vec{v_2}\|.\cos(\widehat{\vec{v_1}, \vec{v_2}})$$

$$\Rightarrow \cos(\widehat{\vec{v_1}, \vec{v_2}}) = \frac{\sqrt{2}}{2}$$

3. We have a :

$$\vec{v_1}.\vec{v_2} = \frac{\sqrt{2}}{2}$$

$$\vec{v_2} \wedge \vec{v_3} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \vec{\imath} + 0\vec{\jmath} + 0\vec{k} = \vec{\imath}$$

$$\Rightarrow \vec{v_2} \wedge \vec{v_3} = \vec{\imath}(1,0,0)$$

$$\vec{v_1}.(\vec{v_2} \wedge \vec{v_3}) = 1 \times 1 + 1 \times 0 + 0 \times 0 = 1$$

- The first term represents the scalar product between the vectors $\vec{v_1}$ et $\vec{v_2}$ is equal to the product of the projection modulus of $\vec{v_1}$ on $\vec{v_2}$ multiplied by the magnitude of $\vec{v_2}$.
- The second term is the vector product between$\vec{v_2}$ et $\vec{v_3}$.
- The last term is the mixed product between $(\vec{v_1}, \vec{v_2}, \vec{v_3})$ and is none other than the volume of the parallelepiped built on the basis of the three vectors.

Exercice 3 :

A(2, 0,0), B(2, −2, 0) and C(2, 3, −1).

1. The vector product $\overrightarrow{OA}\Lambda\overrightarrow{OB}$:

$$\overrightarrow{OA}\begin{pmatrix}2\\0\\0\end{pmatrix};\overrightarrow{OB}\begin{pmatrix}2\\-2\\0\end{pmatrix}\ so\ \overrightarrow{OA}\Lambda\overrightarrow{OB}=\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 2 & 0 & 0\\ 2 & -2 & 0\end{vmatrix}=-4\vec{k}$$

The area of the triangle (OAB) is half the area of the parallelogram formed by the two vectors $\overrightarrow{OA}$ et $\overrightarrow{OB}$

S(OAB)=$\frac{|\overrightarrow{OA}\Lambda\overrightarrow{OB}|}{2}=\frac{4}{2}=2$

2. The mixed product $(\overrightarrow{OA},\overrightarrow{OB},\overrightarrow{OC})$, and the volume of the parallelepiped built on the vectors.

$$(\overrightarrow{OA}\Lambda\overrightarrow{OB}).\overrightarrow{OC}=\begin{pmatrix}0\\0\\-4\end{pmatrix}.\begin{pmatrix}2\\3\\-1\end{pmatrix}=4$$

So the volume of the parallelepiped built on the vectors equal 4.

Exercice 4 :

Let be a vector $\vec{U}=(t\vec{\imath}+3\vec{\jmath})/(\sqrt{t^2+9})$

1- $\vec{U}$ is a unit vector ?

Check that $|\vec{U}|=1$ or $|\vec{U}|=\sqrt{\frac{1}{(t^2+9)}(t^2+9)}$=1

So $\vec{U}$ is an unit vector.

2- The derivative of $\overrightarrow{U}$:

$$\frac{d\vec{u}}{dt}=\frac{d}{dt}\left(\frac{t}{(\sqrt{t^2+9})}\right)\vec{\imath}+\frac{d}{dt}\left(\frac{3}{(\sqrt{t^2+9})}\right)\vec{\jmath}$$

$$\Rightarrow\frac{d\vec{u}}{dt}=\left(\frac{t^2-t^2+9}{(t^2+9)^{3/2}}\right)\vec{\imath}+\left(\frac{-3t}{(t^2+9)^{3/2}}\right)\vec{\jmath}$$

$$\Rightarrow\frac{d\vec{u}}{dt}=\left(\frac{9}{(t^2+9)^{3/2}}\right)\vec{\imath}+\left(\frac{-3t}{(t^2+9)^{3/2}}\right)\vec{\jmath}$$