Correction of SW N°3 of Mechanics

Kinematics of a Material Point

Exercice 1

a- we have $x(t)=2t^3+5t^2+5$ so :

The velocity : $v(t)=\frac{dx}{dt}=6t^2+10t$

The acceleration: $a(t)=\frac{dv(t)}{dt}=12t+10$

b- The body's position at time t_1=2s, as well as its instantaneous velocity and acceleration:

The position : $x(2)=2(2)^3+5(2)^2+5=41m$

Instantaneous speed: $v(2)=6(2)^2+10(2)=44m/s$

Instantaneous acceleration : $a(2)=12(2)+10=34m/s^2$

- The body's position at time t2=3s, as well as its instantaneous velocity and acceleration:

Position : $x(3)=2(3)^3+5(3)^2+5=104m$

Instantaneous speed: $v(3)=6(3)^2+10(3)=84m/s$

Instantaneous acceleration : $a(3)=12(3)+10=46m/s^2$

c- We deduce the speed and average acceleration of the body between t_1 et t_2:

Average speed: $v_{moy}=\frac{\Delta x}{\Delta t}=\frac{x(t_2)-x(t_1)}{t_2-t_1}\Rightarrow v_{moy}=\frac{104-41}{3-2}=63m/s$

Average acceleration :

$$a_{moy}=\frac{\Delta v}{\Delta t}=\frac{v(t_2)-v(t_1)}{t_2-t_1}\Rightarrow a_{moy}=\frac{84-44}{3-2}=40m/s^2$$

Exercice 2

The coordinates of a moving point M in the plane (oxy) are written as :

$x(t)=t+1$ et $y(t)=(t^2/2)+2$

a- The equation of the trajectory is then written :

(To find the equation of the trajectory, simply find the relationship between x(t) and y(t).

To do this, deduce the time from one equation, x(t) or y(t), and replace it in the other equation).

Here, we'll write t as a function of x :

$t=x-1$ so $y=\frac{(x-1)^2}{2}+2=\frac{x^2}{2}-x+\frac{5}{2}$

The equation of the trajectory is : $\mathbf{y(x)=\frac{x^2}{2}-x+\frac{5}{2}}$

b- Components of velocity and acceleration vectors:

- The velocity : $\overrightarrow{\mathrm{v(t)}} = \mathrm{v_x(t)}\vec{\mathrm{i}} + \mathrm{v_y(t)}\vec{\mathrm{j}}$

$$\begin{cases} v_x(t) = \dfrac{dx(t)}{dt} = 1 \\ v_y(t) = \dfrac{dy(t)}{dt} = t \end{cases}$$

The velocity is written by $\overrightarrow{\mathbf{v(t)}} = \vec{\mathbf{i}} + \mathbf{t}\vec{\mathbf{j}}$

The velocity module: $|\vec{v}(t)| = \sqrt{1+t^2}$

-The acceleration: $\overrightarrow{\mathrm{a(t)}} = \mathrm{a_x(t)}\vec{\mathrm{i}} + \mathrm{a_y(t)}\vec{\mathrm{j}}$

$$\begin{cases} a_x(t) = \dfrac{dv_x(t)}{dt} = 0 \\ a_y(t) = \dfrac{dv_y(t)}{dt} = 1 \end{cases}$$

So $\overrightarrow{\mathbf{a(t)}} = \vec{\mathbf{j}}$

The acceleration module $|\vec{a}(t)| = 1$

c- Normal and tangential acceleration:

-Tangential acceleration

$a_T = \frac{d|\overrightarrow{v(t)}|}{dt}$ with $|\overrightarrow{v(t)}| = \sqrt{v_x^2 + v_y^2} = \sqrt{1+t^2}$

$$a_T = \frac{d(\sqrt{1+t^2})}{dt} = \frac{2t}{2\sqrt{1+t^2}}$$

$$a_T = \frac{t}{\sqrt{1+t^2}} \quad \text{because} \quad \boldsymbol{(U^n)' = n\,U'\,U^{n-1}}$$

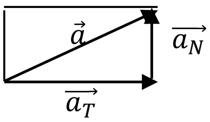
-Normal acceleration :

The accelerations a_N and a_T are the normal and tangential components of the acceleration.$\vec{a}$

$(\vec{a} = a_T\overrightarrow{U_T} + a_N\overrightarrow{U_N} \quad \Rightarrow |\vec{a}| = \sqrt{\boldsymbol{a_T^2 + a_N^2}})$

We have the shape of a right triangle, by applying Pitagort's relation.

$\boldsymbol{a^2 = a_T^2 + a_N^2}$



So $a_N^2 = a^2 - a_T^2$ ou $|\vec{a}| = \sqrt{\boldsymbol{a_T^2 + a_N^2}}$

$$a_N^2 = 1 - \left(\frac{t}{\sqrt{1+t^2}}\right)^2 = 1 - \frac{t^2}{1+t^2}$$

$$a_N^2 = \frac{1}{1+t^2}$$

So $a_N = \frac{1}{\sqrt{1+t^2}} = \frac{1}{v}$

-The radius of curvature: $a_N = \frac{v^2}{R} = \frac{1}{v} \Rightarrow R = v^3 = (1+t^2)^{\frac{3}{2}}$

c- The nature of movement

$$\overrightarrow{a(t)}.\overrightarrow{v(t)} = \begin{pmatrix}0\\1\end{pmatrix}.\begin{pmatrix}1\\t\end{pmatrix} = 1(0) + t(1) = t > 0$$

The motion is then uniformly accelerated.

<u>Exercice 3 :</u>

A particle moves along a trajectory whose equation is $y=x^2$ so that at each instant $v_x=v_0=cst$. If t=0, $x_0=0$.

a- Let's find the particle's x(t) and y(t) coordinates.

We have the following (Ox) :$v_x = v_0 = \frac{dx}{dt} \Rightarrow \int_0^x dx = \int_0^t v_0 dt$

$$\Rightarrow \boldsymbol{x(t)} = v_0 t$$

On the other hand: $y=x^2 \Rightarrow y(t) = v_0^2 t^2$

So $\begin{cases} \boldsymbol{x(t)} = v_0 t \\ y(t) = v_0^2 t^2 \end{cases}$

b- The velocity and acceleration of the particle.

The velocity

$$\begin{cases} v_x = \dfrac{dx}{dt} = v_0 \\ v_y = \dfrac{dy}{dt} = 2v_0^2 t \end{cases} \Rightarrow \overrightarrow{v(t)} = v_0\vec{\imath} + 2v_0^2 t\vec{\jmath}$$

The velocity module: $\left|\overrightarrow{v(t)}\right| = \sqrt{v_0^2 + 4v_0^4 t^2}$

The acceleration: $\begin{cases} a_x = \frac{dv_x}{dt} = 0 \\ a_y = \frac{dv_y}{dt} = 2v_0^2 \end{cases} \Rightarrow \overrightarrow{a(t)} = 2v_0^2\vec{\jmath}$

The acceleration module: $\left|\overrightarrow{a(t)}\right| = \sqrt{(2v_0^2)^2} = 2v_0^2$

- Normal and tangential accelerations :

$$a_T = \frac{d\left|\overrightarrow{v(t)}\right|}{dt} = \frac{4v_0^4 t}{\sqrt{v_0^2 + 4v_0^4 t^2}}$$

$$a_N^2 = a^2 - a_T^2 \Rightarrow a_N^2 = 4v_0^4 - \frac{16v_0^8 t^2}{v_0^2 + 4v_0^4 t^2}$$

$$\Rightarrow a_N^2 = \frac{4v_0^6}{v_0^2 + 4v_0^4 t^2}$$

So $\quad a_N = \frac{2v_0^3}{\sqrt{v_0^2+4v_0^4t^2}} = \frac{2v_0^3}{v}$

$((a^x)^y = a^{x.y} \quad et \quad a^x.a^y = a^{x+y})$

The radius of curvature: $\quad a_N = \frac{v^2}{R} = \frac{2v_0^3}{v} \Rightarrow R = \frac{v^3}{2v_0^3}$

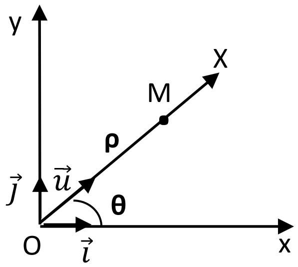
Exercice 4

A) A material point M is identified by its Cartesian coordinates (x,y): Find x and y in terms of polar coordinates ρ and θ??

$$\overrightarrow{OM} = x\vec{\imath} + y\vec{\jmath} \quad (1)$$

In the othor hand $\overrightarrow{OM}$ is written by projection as:

$$\overrightarrow{OM} = \rho cos\theta\vec{\imath} + \rho sin\theta\vec{\jmath} \quad (2)$$

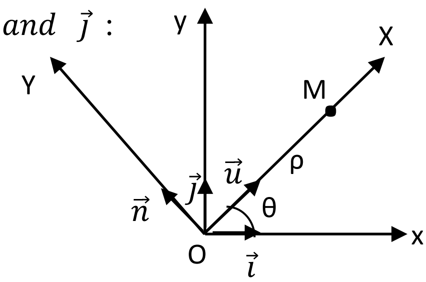


$$(1) \text{ and } (2) \Rightarrow \begin{cases} x = \rho\, cos\theta \\ y = \rho\, sin\theta \end{cases}$$

A. The unit vector $\vec{u}$ as a function of the unit vectors $\vec{\imath}$ *and* $\vec{\jmath}$:

we have $\overrightarrow{OM} = \left|\overrightarrow{OM}\right|\vec{u} = \rho\vec{u} = \rho cos\theta\vec{\imath} + \rho sin\theta\vec{\jmath}$

so $\vec{u} = cos\theta\vec{\imath} + sin\theta\vec{\jmath}$



and $\vec{n} = -sin\theta\vec{\imath} + cos\vec{\jmath}$

$\vec{n}$ and $\vec{u}$ represent the unit vectors of the polar coordinate basis.

2- Calculate the expression of $d\vec{u}/_{d\theta}$, which this vector represents?

$$\frac{d\vec{u}}{d\theta} = \frac{d(cos\theta\vec{\imath} + sin\theta\vec{\jmath})}{d\theta} = -sin\theta\vec{\imath} + cos\vec{\jmath} = \vec{n}$$

$\boldsymbol{\frac{d\vec{u}}{d\theta}}$ represents a unit vector perpendicular to $\vec{u}$ in the direct direction .

B. The position of point M is given by $\begin{cases} \overrightarrow{OM} = t^2\vec{u} \\ \theta = \omega t \end{cases}$ $(\omega \text{ constant})$

The expression of the velocity vector $\vec{v}$ in polar coordinates is :

$$\vec{v} = \frac{d\overrightarrow{OM}}{dt} = \frac{d(t^2\vec{u})}{dt} = 2t\vec{u} + t^2\frac{d\vec{u}}{dt}$$

$$\frac{d\vec{u}}{dt} = \frac{d\vec{u}}{d\theta}.\frac{d\theta}{dt} = \vec{n}.\omega$$

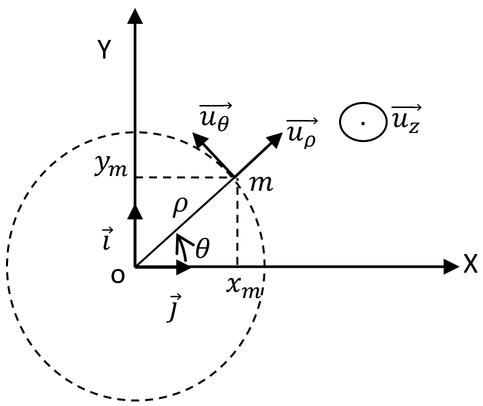
$$\boldsymbol{\vec{v} = 2t.\vec{u} + t^2.\omega.\vec{n}}$$

Exercice 5

1. A material point M is identified by its Cartesian coordinates (x, y, z).

Write the relationship between Cartesian coordinates and polar coordinates.

$$\begin{cases} x = \rho cos\theta \\ y = \rho sin\theta \\ z = z_M \end{cases}$$



2. Find the expression of the position vector and deduce the velocity $\vec{v}$ of point M in cylindrical coordinates.

$$\overrightarrow{OM} = \rho\overrightarrow{U_\rho} + z\overrightarrow{U_z}$$

$$\Rightarrow \vec{v} = \frac{\mathrm{d}\overrightarrow{OM}}{\mathrm{dt}} = \frac{\mathrm{d}\rho}{\mathrm{dt}}\overrightarrow{U_\rho} + \rho\frac{d\overrightarrow{U_\rho}}{dt} + \frac{\mathrm{dz}}{\mathrm{dt}}\overrightarrow{U_z} + z\frac{d\overrightarrow{U_z}}{dt}$$

$$on\ a\ \frac{d\overrightarrow{U_z}}{dt} = 0 \Rightarrow \vec{v} = \dot{\rho}\overrightarrow{U_\rho} + \rho\frac{d\theta}{dt}\frac{d\overrightarrow{U_\rho}}{d\theta} + \dot{z}\overrightarrow{U_z}$$

$$\Rightarrow \vec{v} = \dot{\rho}\overrightarrow{U_\rho} + \rho\dot{\theta}\overrightarrow{U_\theta} + \dot{z}\overrightarrow{U_z}$$

3. A velocity vector $\vec{v}$ of point M in cylindrical coordinates:

We have $\begin{cases} \rho = 4t^2 \\ \theta = \omega t \\ z = \sqrt{t} \end{cases}$ Hence $\begin{cases} \frac{d\rho}{dt} = 8t \\ \frac{d\theta}{dt} = \omega \\ \frac{dz}{dt} = \frac{1}{2\sqrt{t}} \end{cases}$

$$\Rightarrow \vec{v} = \dot{\rho}\overrightarrow{U_\rho} + \rho\dot{\theta}\frac{d\overrightarrow{U_\rho}}{d\theta} + \dot{z}\overrightarrow{U_z} = 8t\overrightarrow{U_\rho} + 4t^2.\omega.\overrightarrow{U_\theta} + \frac{1}{2\sqrt{t}}\overrightarrow{U_z}$$

<u>Exercice 6</u>

$$\begin{cases} v_x = R\omega\cos(\omega t) \\ v_y = R\omega\sin(\omega t) \end{cases}$$

Knowing that at t=0, the moving body is at the origin O (0,0),

1. The components of the acceleration vector and its modulus. $\begin{cases} a_x = \frac{dv_x}{dt} = -R\omega^2 sin\,(\omega t) \\ a_y = \frac{dv_y}{dt} = R\omega^2\cos(\omega t) \end{cases}$

$$[\vec{a}] = \sqrt{(-R\omega^2 sin\,(\omega t))^2 + (R\omega^2 cos\,(\omega t))^2} = R\omega^2$$

2. The tangential and normal components of acceleration and deduce the radius of curvature.

Tangential acceleration:

$$[\vec{v}] = \sqrt{(R\omega\ \ cos(\omega t))^2 + (R\omega\ sin\,(\omega t))^2} = R\omega$$

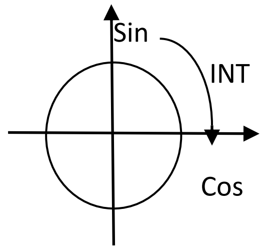
$$a_T = \frac{dv}{dt} = \frac{dR\omega}{dt} \Rightarrow a_T = 0$$

Normale acceleration:

$a_N = \frac{v^2}{R} = a = R\omega^2$ car $a_T = 0$ et $R = \frac{v^2}{a_N} = \frac{R^2\omega^2}{R\omega^2}$=R

Radius of curvature is R.

3. The components of the position vector



$$\begin{cases} v_x = R\omega\cos(\omega t) \\ v_y = R\omega\sin(\omega t) \end{cases} \Rightarrow \begin{cases} \dfrac{dx}{dt} = R\omega\cos(\omega t) \\ \dfrac{dy}{dt} = R\omega\sin(\omega t) \end{cases}$$

$$\Rightarrow \begin{cases} dx = R\omega\cos(\omega t)dt \\ dy = R\omega\sin(\omega t)\,dt \end{cases}$$

$$\Rightarrow \begin{cases} \displaystyle\int dx = R\int \omega\cos(\omega t)dt \\ \displaystyle\int dy = R\int \omega\sin(\omega t)\,dt \end{cases}$$

$$\Rightarrow \begin{cases} x = R\sin(\omega t) \\ y = -R\cos(\omega t) \end{cases}$$

The trajectory equation.

$$x^2 + y^2 = R^2 sin^2\omega t + R^2 cos^2\omega t \Rightarrow x^2 + y^2 = R^2$$

4. The nature of movement

The acceleration $a=a_N$ and the equation of the trajectory is $x^2 + y^2 = R^2$, so the motion is uniformly circular.

Exercice 7

A material point M moves along the OX axis with acceleration $\vec{a} = a\,\vec{\imath}$ with a > 0.
1- Determine the velocity vector knowing that v (t=0)= v_0 .

$$a = \frac{dv}{dt} \Rightarrow \int_{v_0}^{v} dv = a\int_0^t d\,t$$

$$\Rightarrow \boldsymbol{v - v_0 = a\,t} \quad (1)$$

so $\boldsymbol{\vec{v} = (a\,t + v_0)\vec{\imath}}$

2- The position vector $\overrightarrow{OM}$ knowing that x(t=0)=x_0.

$$v = \frac{dx}{dt} = a\,t + v_0 \quad \Rightarrow \int_{x_0}^{x} dx = \int_0^t (a\,t + v_0)d\,t = a\int_0^t t d\,t + v_0\int_0^t d\,t$$

$$\Rightarrow x - x_0 = \left[a\frac{t^2}{2} + v_0 t\right]_0^t$$

$$\boldsymbol{\int x^n\,dx = \frac{x^{n+1}}{n+1}} \quad \textbf{and} \quad \boldsymbol{\int \frac{dx}{x} = \ln x}$$

$\boldsymbol{x = \frac{1}{2}\, a t^2 + v_0 t + x_0}$ (2)

$$\Rightarrow \overrightarrow{\boldsymbol{OM}} = \left(\frac{\mathbf{1}}{\mathbf{2}}\, \boldsymbol{a\, t^2 + v_0 t + x_0}\right)\vec{\boldsymbol{\imath}}$$

3. Show that $v^2 - v_0^2 = 2a(x - x_0)$

(1)$\Rightarrow t = \frac{v - v_0}{a}$ in (2) $x - x_0 = \frac{1}{2}\, a \left(\frac{v-v_0}{a}\right)^2 + v_0\left(\frac{v-v_0}{a}\right) = \frac{v^2+v_0^2-2vv_0}{2a} + \frac{vv_0 - v_0^2}{a}$

$$\Rightarrow x - x_0 = \frac{v^2 + v_0^2 - 2vv_0}{2a} + \frac{2vv_0 - 2v_0^2}{2a}$$

$$\Rightarrow x - x_0 = \frac{v^2 - v_0^2}{2a}$$

so $2a(x - x_0) = v^2 - v_0^2$

4- For motion to be uniformly accelerated, $\vec{a}\,.\vec{v}$ must be positive.
   For motion to be uniformly retarded, $\vec{a}\,.\vec{v}$ must be negative.

Exercice 8

The differential of vector $\vec{r}$, $d\vec{r} = d\vec{l} = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$ can be expressed in cylindrical coordinates as $d\vec{r} = \frac{\partial \vec{r}}{\partial \rho}\, d\rho + \frac{\partial \vec{r}}{\partial \theta}\, d\theta + \frac{\partial \vec{r}}{\partial z}\, dz$.

1. We are looking for the vectors $\frac{\partial \vec{r}}{\partial \rho}$, $\frac{\partial \vec{r}}{\partial \theta}$ et $\frac{\partial \vec{r}}{\partial z}$.

We are $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$

- The displacement vector in cartesian coordinates (x, y, z) :

$$d\vec{r} = d\vec{l} = dx\vec{\imath} + dy\vec{\jmath} + dz\vec{k}$$

- The displacement vector in cylindrical coordinates (ρ, θ, z) :

$$d\vec{r} = \frac{\partial \vec{r}}{\partial \rho}\, d\rho + \frac{\partial \vec{r}}{\partial \theta}\, d\theta + \frac{\partial \vec{r}}{\partial z}\, dz$$

Relationships between cartesian coordinates (x, y, z) and cylindrical coordinates (ρ, θ, z) are :

$$\begin{cases} x = \rho cos\theta \\ y = \rho sin\theta \\ z = z_M \end{cases} \Longrightarrow \begin{cases} dx = d\rho.cos\theta - \rho\,.\sin\theta.d\theta \\ dy = d\rho.sin\theta + \rho.cos\theta.d\theta \\ dz = dz_M \end{cases}$$

$$\Longrightarrow d\vec{r} = d\vec{l} = (d\rho.cos\theta - \rho\,.\sin\theta.d\theta)\vec{\imath} + (d\rho.sin\theta + \rho.cos\theta.d\theta)\vec{\jmath} + dz\vec{k}$$

$$\Longrightarrow d\vec{r} = (cos\theta.\vec{\imath} + sin\theta.\vec{\jmath})d\rho + (-\rho sin\theta\vec{\imath} + \rho.cos\theta.\vec{\jmath})d\theta + dz\vec{k}............(1)$$

$$\Longrightarrow d\vec{r} = \left(\frac{\partial\vec{r}}{\partial\rho}\right) d\rho + \left(\frac{\partial\vec{r}}{\partial\theta}\right) d\theta + \left(\frac{\partial\vec{r}}{\partial z}\right) dz..(2)$$

With identification between (1) et (2) we'll have :

$$\Longrightarrow \begin{cases} \dfrac{\partial\vec{r}}{\partial\rho} = cos\theta.\vec{\imath} + sin\theta.\vec{\jmath} \\ \dfrac{\partial\vec{r}}{\partial\theta} = -\rho sin\theta\vec{\imath} + \rho.cos\theta.\vec{\jmath} \\ \dfrac{\partial\vec{r}}{\partial z} = \vec{k} \end{cases}$$

2. Deduce Unit Vectors $\overrightarrow{U_\rho}$, $\overrightarrow{U_\theta}$ et $\overrightarrow{U_z}$ (cylindrical coordinates) as function of $\vec{\imath}$, $\vec{\jmath}$ and $\vec{k}$ (Cartesian coordinates) :

The displacement vector in cylindrical coordinates is written:

$$d\vec{r} = d\rho\overrightarrow{U_\rho} + \rho d\theta\overrightarrow{U_\theta} + dz\vec{k}..............................(3)$$

$$(2) \text{ and } (3) \Longrightarrow \begin{cases} \overrightarrow{U_\rho} = \frac{\partial\vec{r}}{\partial\rho} = cos\theta.\vec{\imath} + sin\theta.\vec{\jmath} \\ \overrightarrow{U_\theta} = \frac{1}{\rho}\frac{\partial\vec{r}}{\partial\theta} = -sin\theta\vec{\imath} + cos\theta.\vec{\jmath} \\ \overrightarrow{U_z} = \frac{\partial\vec{r}}{\partial z} = \vec{k} \end{cases}$$

Note :

The unit vectors of the Cartesian coordinates base can be written as a function of the unit vectors of the cylindrical coordinates base from the table below:

	$\vec{\imath}$	$\vec{\jmath}$	$\vec{k}$
$\overrightarrow{u_\rho}$	Cosθ	Sinθ	0
$\overrightarrow{u_\theta}$	-sinθ	Cosθ	0
$\overrightarrow{u_z}$	0	0	1

$$\Longrightarrow \begin{cases} \vec{\imath} = \cos\theta\overrightarrow{u_\rho} - \sin\theta\,\overrightarrow{u_\theta} \\ \vec{\jmath} = \sin\theta\overrightarrow{u_\rho} + \cos\theta\overrightarrow{u_\theta} \\ \vec{k} = \overrightarrow{u_z} \end{cases}$$

3. Checking that they are orthogonal?

$$\Longrightarrow \begin{cases} \left|\overrightarrow{U_\rho}\right| = \sqrt{cos\theta^2 + sin\theta^2} = 1 \\ \left|\overrightarrow{U_\theta}\right| = \sqrt{(-sin\theta)^2 + cos\theta^2 = 1} \\ \left|\overrightarrow{U_z}\right| = \left|\vec{k}\right| = 1 \end{cases}$$

Hence $\overrightarrow{U_\rho}, \overrightarrow{U_\theta}$ et $\overrightarrow{U_z}$, are the unit vectos.

We have $\overrightarrow{U_\rho}.\overrightarrow{U_\theta} = 0, \overrightarrow{U_\rho}.\overrightarrow{U_z} = 0$ et $\overrightarrow{U_z}.\overrightarrow{U_\theta} = 0$

So $\overrightarrow{U_\rho}, \overrightarrow{U_\theta}$, et $\overrightarrow{U_z}$ are orthogonal vectors.

Therefore the vectors $\overrightarrow{U_\rho}, \overrightarrow{U_\theta}, \overrightarrow{U_z}$ form an orthonormal reference frame.

4. Write $\vec{A} = 2x\vec{\imath} + y\vec{\jmath} - 2z\vec{k}$ in cylindrical coordonates.

We have $\begin{cases} x = \rho cos\theta \\ y = \rho sin\theta \\ z = z_M \end{cases}$ *et* $\begin{cases} \vec{\imath} = \cos\theta\overrightarrow{u_\rho} - \sin\theta\,\overrightarrow{u_\theta} \\ \vec{\jmath} = \sin\theta\overrightarrow{u_\rho} + \cos\theta\overrightarrow{u_\theta} \\ \vec{k} = \overrightarrow{u_z} \end{cases}$

So $\vec{A} = 2x\vec{\imath} + y\vec{\jmath} - 2z\vec{k}$ is wretten by :

$$\Rightarrow \vec{A} = 2\rho cos\theta\left(\cos\theta\overrightarrow{u_\rho} - \sin\theta\,\overrightarrow{u_\theta}\right) + \rho sin\theta\left(\sin\theta\overrightarrow{u_\rho} + \cos\theta\overrightarrow{u_\theta}\right) - 2z\vec{k}$$

$$\Rightarrow \vec{A} = (2\rho cos\theta^2 + \rho sin\theta^2)\overrightarrow{u_\rho} + (-2\rho\cos\theta\sin\theta + \rho os\theta\sin\theta)\overrightarrow{u_\theta} - 2z\vec{k}$$

$$\Rightarrow \boldsymbol{\vec{A} = (cos\theta^2 + 1)\rho\overrightarrow{u_\rho} - \rho\cos\theta\sin\theta\overrightarrow{u_\theta} - 2z\overrightarrow{u_z}}$$