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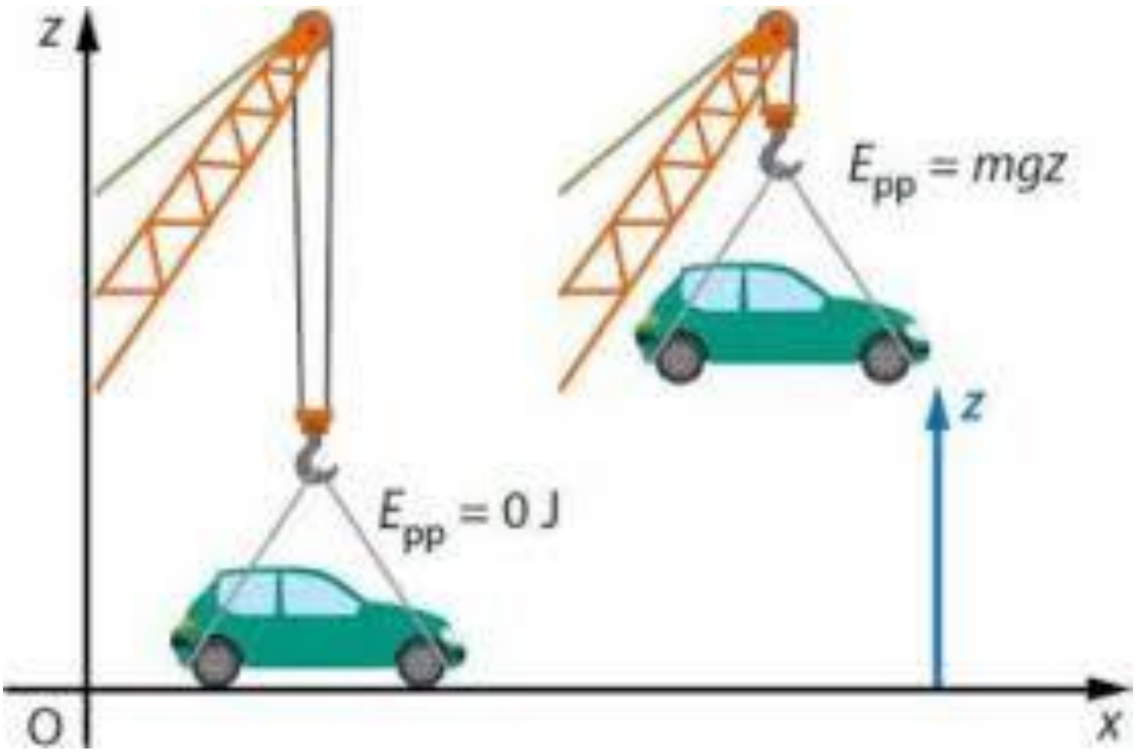
COURSE OF MECHANICS

OF THE MATERIAL POINT

Chapter VI : Work and Energy

Prepared by : Ms Hadjou Belaid Zakia and Ms Bachir Nadia



Sommary

Introduction

The aim of this chapter is to present the energy tools used in mechanics to solve problems. Indeed, sometimes the fundamental principle of dynamics is not enough to solve a problem. Newton's laws can be used to solve all the problems of classical mechanics. If we know the position and initial velocity of the particles in a system, as well as all the forces acting on them. But in practice, we don't always know all the forces at play, and even if we do, the equations to be solved are too complex. In this case, other concepts such as work and energy must be used. Before describing the different types of energy (kinetic, potential and mechanical) and using them in energy theorems, we'll introduce the notions of power and work of a force.

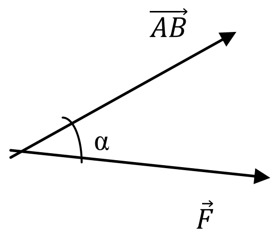
I. The work العمل

All motion under the action of external forces $\vec{F}$, implies work by these forces. In other words; work supplied by a force moves a body in its own direction and creates motion.

I.1. Work performed by a constant force

Let a particle subjected to a constant force $\vec{F}$ move this body a distance d=AB, the mechanical work W performed by the force $\vec{F}$ is defined as:

$$\boldsymbol{W_{AB} = \vec{F}.\overrightarrow{AB} = |\vec{F}|.|\overrightarrow{AB}|.cos\alpha}$$



α is the angle between the two vectors $\vec{F}$ and $\overrightarrow{AB}$.

- For α=0 $\quad W = |\vec{F}|.|\overrightarrow{AB}|$ because $cos0 = 1$
- For $\alpha < \frac{\pi}{2}$ with have $W > 0$ It's a driving work.
- For $\alpha = \frac{\pi}{2}$ with have $W = 0$ because $cos\frac{\pi}{2} = 0$
- For $\frac{\pi}{2} < \alpha < \pi$ $\quad W < 0$ It's a resistive work.

Unity of work in the system MKSA is « **Joule** ».

Note :

Note that work is a scalar quantity, unlike force and displacement, which are vectors.

Example 1 :

The muscular effort required to lift an object depends on both its weight (the force of gravity exerted on it), and the height h from which it is lifted.

In this case, the force of the weight is directed downwards, the displacement upwards and θ is 180°.

W = - P.h = - mgh.

The force of the weight is negative, since muscular work must be done against the force of gravity.

Example 2 :

To lift a car with a mass of one and a half tons, a force F of 15,000N vertical to the car is required.

Calculate the work done by this force to move the car by a height (AB) of 3 meters.

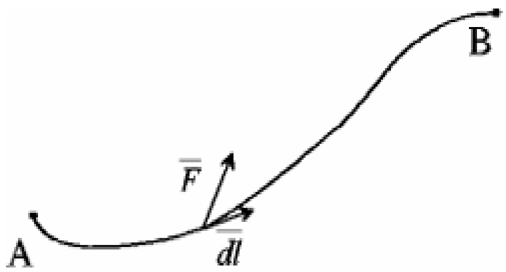
$\boldsymbol{W_{AB}}(\vec{\mathbf{F}}) = |\vec{\boldsymbol{F}}|.|\overrightarrow{\boldsymbol{AB}}|.\boldsymbol{cos\alpha}$=**F.d.cosα** =1.5 10^4 . 3=4.5 10^4 J

I.2. The work performed by a variable force

If the force varies in intensity and/or direction during displacement, and if the displacement has any form whatsoever, we need to use integral calculus to generalize the definition of work. Generally speaking, the work of a force depends on the path followed, which is why this elementary work is necessary.

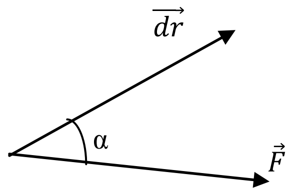
$$\mathbf{dW}=\vec{\mathbf{F}}.\overrightarrow{\mathbf{dr}} = \vec{\mathbf{F}}.\overrightarrow{\mathbf{dl}}$$

where dl is an infinitesimal displacement along the trajectory, tangential to it.



The elementary work dW performed by a force $\vec{F}$ on a point mass m during an elementary displacement dr= dl is given by:

$$\mathbf{dW}=\vec{\mathbf{F}}.\overrightarrow{\mathbf{dr}}=|\vec{\mathbf{F}}|.|\overrightarrow{\mathbf{dr}}| \cos(\vec{\mathbf{F}}, \overrightarrow{\mathbf{dr}})$$



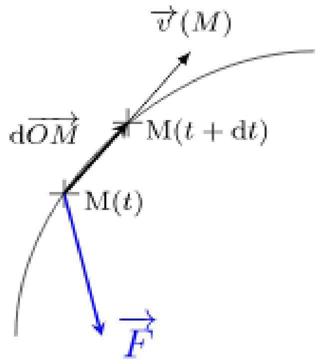
To obtain the work on an AB displacement, we integrate this elementary work :

$$\mathbf{W}=\int \mathbf{dW} = \int_A^B \vec{\mathbf{F}}.\overrightarrow{\mathbf{dr}} = \int \boldsymbol{F.dr.cos\alpha}$$

α is the angle between the two vectors $\vec{F}$ *and* $\overrightarrow{dr}$; $\alpha = (\vec{F}, \overrightarrow{dr})$

I.3. The power الاستطاعة

Let a point M move along its trajectory at a velocity $\vec{v}$ (M) relative to the reference frame of study, It experiences a force $\vec{F}$(M) as shown in the figure opposite :



The power of a force $\vec{F}$ is the work per unit time.

We have two types:

- The average power $P_{avr} = \frac{\Delta W}{\Delta t}$
- The instantaneous power $P = \frac{dW}{dt}$

Then the instantaneous power of the $\vec{F}$ is:

$$P(\vec{F}) = \frac{dW}{dt} = \frac{|\vec{F}|.|\vec{dr}|}{dt} = \vec{F} \cdot \vec{v}\,(M) = \|\vec{F}\| \times \|\vec{v}\,(M)\| \times \cos\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Note :

- ✓ The unit of power is the « Watt ».
- ✓ This force can be classified into three types:
  - It is driving, if its power is positive which corresponds to an angle $\alpha<\pi/2$.
  - It is resistive, if its power is negative which corresponds to an angle $\alpha>\pi/2$.
  - Finally, it can be of zero power, in which case $\alpha=\pi/2$.

II. Energy الطاقة

In physics, energy is defined as the capacity of a system to produce work. Energy is not a material substance: it is a physical quantity that characterizes the state of a system; it can be stored and exists in many forms.

II.1. Kinetic energy الطاقة الحركية

In order to accelerate a point mass to a defined speed, work must be done. This work is then stored in the point mass in the form of kinetic energy.

Suppose the object's initial velocity is v_0 and the force F is applied in the direction of v_0, producing a displacement d=dr.

We have : dW = $F.\, dr$ and $F = ma = m\frac{dv}{dt}$

From this expression we can deduce the following :

$$dW = F\, dr = m\frac{dv}{dt}\, dr$$

$$\Rightarrow dW = m\frac{dr}{dt}dv \quad Then \quad dW = mvdv$$

Let's integrate the expression of elementary work, and derive the definition of kinetic energy:

$$W = m\int_A^B vdv \quad \Rightarrow W = \frac{1}{2}\,m(v_B^2 - v_A^2) = \frac{1}{2}\,mv_B^2 - \frac{1}{2}\,mv_A^2 \quad \text{..................(2)}$$

Where $\mathbf{v_A}$ is the velocity of the moving body at point A and $\mathbf{v_B}$ its velocity at point B.

The kinetic energy of a material point of mass **m** and instantaneous velocity $\vec{v}$ is given by the expression:

$$Ec = \frac{1}{2}mv^2 \quad \text{...(3)}$$

(2) and (3) gives us : $\boldsymbol{W_{\vec{F}(A\to B)} = E_{C_B} - E_{C_A} = \Delta E_c}$

And since *p=mv*, we can also write :

$$Ec = \frac{P^2}{2m}$$

Statement of the Kinetic Energy Theorem : نضرية الطاقة الحركية

The variation in kinetic energy of a material point subjected to a set of external forces between two positions A and B is equal to the sum of the work of these forces between these two points.

$$\boldsymbol{W_{\vec{F}(A\to B)} = E_{C_B} - E_{C_A} = \Delta E_c \quad \Rightarrow \sum_i W_i = \Delta E_C}$$

II.2. Conservatives forces القوة المنحفضة

A force is said to be conservative, or to derive from a potential, if its work is independent of the path taken, whatever the probable displacement between the starting point and the end point.

Conservative forces include the force of gravity, spring return force and the tension force of a wire.

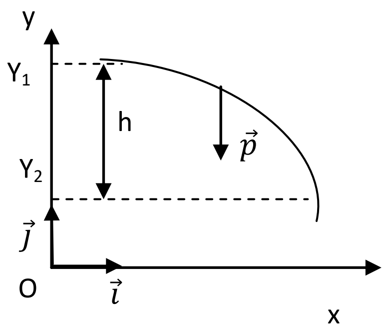
Example : Let's calculate the work of the force of gravity.

$dW = \vec{p}.\vec{dl}$ with $p = -mg\,\vec{j}$

$\vec{dl} = dx\vec{i} + dy\vec{j}$ so $dW = -mgdy$

$$W = -mg\int_{y_1}^{y_2} dy = -mg(y_2 - y_1)$$

$$\Rightarrow W = mg(y_1 - y_2) = mgh$$



So the force of gravity $\vec{p}$ is a conservative force because its work does not depend on the path followed, and it is said to derive from a potential.

Spring return force is also a conservative force.

Note:

A force is said to be non-conservative if its work depends on the path followed, as in the case of ***friction force***.

II.3. Potential energy الطاقة الكامنة

Potential energy is a function of coordinates, such as the integration between its two values at start and finish. It represents the work done by the particle to move it from its initial position to its final position.

If the force $\vec{F}$ is a force deriving from a potential (conservative), then :

$$W = \int_A^B \overrightarrow{F_C}.\overrightarrow{dr} = E_{P_A} - E_{P_B} \quad \Rightarrow dW = -dE_p$$

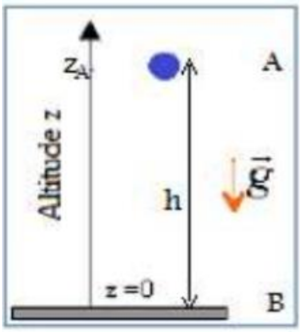
$$\text{then} \quad \boldsymbol{W_{A\to B}(\overrightarrow{F_C})) = -\Delta E_p}$$

Potential energy is always calculated relative to a reference frame (Ep=0).

The potential energy function Ep is determined to within one constant.

By identifying the two expressions dE_p and dW, we arrive at the following result: The differential of potential energy is equal to and opposite in direction to the differential of work.

Example 1: Wight force قوة الثقل



The force of weight is a conservative force, hence :

$W_{A\to B}(\overrightarrow{F_C})) = -\Delta E_p$

And $W = mg(y_1 - y_2) = mgh$

So $W_{\vec{p}} = -\Delta E_p = -(E_{pf} - E_{pi}) = Epi = mg(z_A - z_B)$

Because E_{pf} is the reference potential energy.

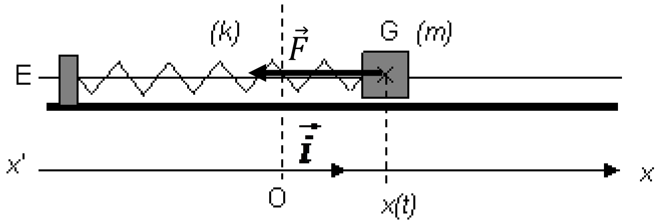
$$\text{So } Ep = mgH$$

Note :

If $Z_A > Z_B$ we have $E_p > 0$

If $Z_A < Z_B$ we have $E_p < 0$

Example 2 : Spring return force قوة الارجاع لنابض



$\vec{F} = -kx\vec{\imath}$, $\overrightarrow{dl} = dx.\vec{\imath}$ et $dW = \vec{F}.\overrightarrow{dl}$

$dW = -dE_p = -kx.dx \Rightarrow dE_p = kxdx$

$$\Rightarrow \int dEp = k\int_{x_i}^{x_f} xdx$$

$$\Rightarrow Ep = \frac{1}{2}k(x_f^2 - x_i^2) = \frac{1}{2}kx^2$$

II.4. Mechanic energy (Totale Energie) الطاقة الكلية

The mechanical energy of a material point at a given instant is equal to the sum of kinetic energy and potential energy:

$$E_M = E_C + Ep \Rightarrow E_M = E_C + E_p$$

- **Principle of conservation of mechanical energy مبدا انحفاظ الطاقة الميكانيكية**

In a conservative (or potential-derived) force field, mechanical energy is conserved over time.

$$E_M = E_C + E_p = Cte$$

This means that the variation in mechanical energy is zero $\Delta E_M = 0$, it also means that the variation in kinetic energy is equal to the opposite of the variation in potential energy:

$$\Delta Ec = -\Delta Ep$$

In other words, if the system is isolated or free, mechanical energy is conserved.

In the presence of **frictional forces**, the variation in mechanical energy is equal to the sum of the work of the frictional forces. W_{Ffrott}:

$$\Delta E_M = \sum W_{A\rightarrow B}(\overrightarrow{F_{NC}}) = W_{A\rightarrow B}(\overrightarrow{F_{frot}})$$

- **Friction force work** : عمل قوة الاحتكاك

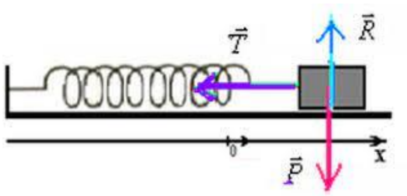
$$W_{A\rightarrow B}(\overrightarrow{F_{frot}}) = -F_f.AB$$

Example :

A mass m is attached to a spring of stiffness k, and the other end of the spring is attached to point C. The mass m can slide on the horizontal surface. Initially, the mass is at rest at point O of equilibrium.

1) Assuming no friction, move mass m from point O to point A, such that OA=a. Determine the work of the Spring return force as m moves from O to A. Then determine the speed of m at point O.

2) Same questions as question 1, but now we assume that friction exists, and give the dynamic friction coefficient µc.



Answers:

1- we have, $\vec{F} = -kx\vec{i}$ et $\overrightarrow{dl} = dx\vec{i}$

$$\Rightarrow W_{\vec{F}} = \int dW_{\vec{F}} = \int \vec{F}.\overrightarrow{dl} = -k\int_a^0 xdx = \frac{1}{2}ka^2$$

Wa also have : $\sum_i \boldsymbol{W_i} = \boldsymbol{\Delta E_C} = W_{\vec{p}} + W_{\vec{R}} + W_{\vec{F}}$

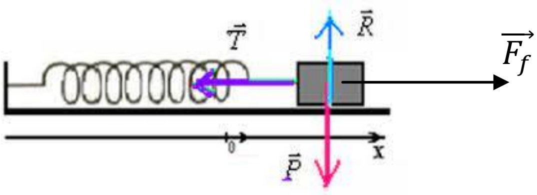
with $W_{\vec{p}} = W_{\vec{R}} = \vec{0}$ because $\vec{R}\ and\ \vec{p} \perp \overrightarrow{Ox}$

so $\boldsymbol{\Delta E_C} = W_{\vec{F}} = \frac{1}{2}ka^2 = \frac{1}{2}mv_O^2 - \frac{1}{2}mv_A^2$ with v_A=0

sence, $\boldsymbol{v_o} = \boldsymbol{a}\sqrt{\frac{k}{m}}$

2- Case of friction

We also have : $\sum_i \boldsymbol{W_i} = \boldsymbol{\Delta E_C} = W_{\vec{p}} + W_{\vec{R}} + W_{\overrightarrow{F_f}} + W_{\vec{F}}$



with : $W_{\vec{p}} = W_{\vec{R}} = \vec{0}$

so $\boldsymbol{\Delta E_C} = W_{\vec{F}} + W_{\overrightarrow{F_f}} = \frac{1}{2}ka^2 - a.F_f = \frac{1}{2}ka^2 - a.\mu_c.mg = \frac{1}{2}mv_o^2$ because v_A=0

sence , $\boldsymbol{v_o} = \sqrt{\frac{ka^2}{m} - 2\mu_c.a.g} = \boldsymbol{a}\sqrt{\frac{k}{m} - \frac{2\mu_c.g}{a}}$

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