University of Tlemcen
Faculty of Science
Thursday: 07/12/2023
Duration: 01 h 30mn

Academic Year 2023/2024
1st Year LMD-M and MI

Continuous Mechanics Test

(Calculator allowed)

Exercise 1: (05 Pts)

A. The momentum P (P=mϑ where m is a mass and ϑ is a velocity) associated with a photon depends on its frequency f according to the following expression:

$$P = \sigma^{\alpha} f^{\beta} c^{\gamma}$$

Where c is the speed of light and σ has the following dimension $[\sigma] = M.L^2.T^{-1}$.

Using dimensional analysis, find the exponents α, β and γ.

B. The average velocity of the molecules of a gas is written in the following formula:

$$\vartheta = \sqrt{\frac{PV}{m}}$$

m being the mass of the molecule, V the volume, and p the pressure of the gas.
Calculate the relative uncertainty in ϑ as a function of **Δp, Δm and ΔV**.

Exercise 2: (05 Pts)

A. $\vec{i}, \vec{j}$ *and* $\vec{k}$ being the unit vectors of an orthonormal reference frame (Oxyz), consider the vectors. $\vec{r_1} = 2\,\vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{r_2} = \vec{i} + \vec{j} + \vec{k}$

1- Calculate the vector product $\vec{r_1} \wedge \vec{r_2}$.

2- Deduce the angle θ formed by the two vectors $\vec{r_1}$ and $\vec{r_2}$.

B. Let be a polar coordinate system with origin O and unit vectors $\vec{\mathbf{u}_\rho}, \vec{\mathbf{u}_\theta}$.

M is a point with coordinates $\begin{cases} \rho = 2t^3 + 1 \\ \theta = \omega t \end{cases}$ (ω constant).

1- Using a detailed diagram, give the expression of the position vector $\overrightarrow{\mathbf{OM}}$ and calculate the velocity vector of point M in polar coordinates.

2- Write this velocity vector $\vec{\mathbf{v}}$ (M) in cartesian coordinates $(\vec{\mathbf{i}}, \vec{\mathbf{j}}, \vec{\mathbf{k}})$.

Exercise 3: (05 Pts)

A particle moves along a trajectory whose equation is $\mathbf{x^2 + y^2 = 4}$ such that **x(t)=2 sin(ωt).**

Knowing that ω is constant and at t=0, the mobile is at point M (0, R), Determine:

1) The component y(t).
2) Velocity and acceleration vector components and their moduli.
3) Tangential and normal accelerations.
4) The nature of the motion.

Bon courage

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Correction of Continuous Mechanics Test

Exercise 1: (5 pts)

A- The momentum P is given by the following expression: 2.5 pts

$$P = \sigma^\alpha f^\beta c^\gamma = Mv \quad \text{so } [P] = M.L^1.T^{-1} \textbf{(0.5 pts)}$$

$$\text{We have } \begin{cases} [v] = [c] = L.T^{-1} \\ [f] = T^{-1} \\ [M] = M \end{cases} \textbf{(0.75 pts)} \text{ and } [\sigma] = M.L^2.T^{-1}$$

$$\Rightarrow [P] = [\sigma]^\alpha [f]^\beta [c]^\gamma = (M.L^2.T^{-1})^\alpha (T^{-1})^\beta (L\ .T^{-1})^\gamma \textbf{(0.25 pts)}$$

$$\Rightarrow [P] = M^1 L^1 T^{-1} = M^\alpha L^{2\alpha+\gamma} T^{-\alpha-\beta-\gamma} \textbf{ (0.25 pts)}$$

$$\Rightarrow \begin{cases} \alpha = 1 \\ 2\alpha + \gamma = 1 \\ -(\alpha+\beta+\gamma) = -1 \end{cases} \Rightarrow \begin{cases} \alpha = 1 \\ \gamma = -1 \\ \beta = 1 \end{cases} \textbf{(0.75 pts)}$$

$$\Rightarrow \boldsymbol{P = \sigma . f . c^{-1}}$$

B- Relative uncertainty about v. (2.5pts)

$$\boldsymbol{\vartheta = \sqrt{\frac{PV}{m}}}$$

$$\Rightarrow \boldsymbol{\vartheta^2} = \frac{\boldsymbol{PV}}{\boldsymbol{m}} \Rightarrow \log(\boldsymbol{\vartheta^2}) = \log\frac{\boldsymbol{PV}}{\boldsymbol{m}} \textbf{ (0.5 pts)}$$

$$\Rightarrow 2\log\vartheta = \log P + \log V - \log m \textbf{ (0.5 pts)}$$

$$\Rightarrow 2\frac{d\vartheta}{\vartheta} = \frac{dP}{P} + \frac{dV}{V} + \frac{dm}{m} \textbf{ (0.5 pts)}$$

$$\Rightarrow 2\frac{\Delta\vartheta}{\vartheta} = \frac{\Delta P}{P} + \frac{\Delta V}{V} + \frac{\Delta m}{m} \textbf{(0.5 pts)}$$

$$\Rightarrow \frac{\Delta\vartheta}{\vartheta} = \frac{1}{2}\left(\frac{\Delta P}{P} + \frac{\Delta V}{V} + \frac{\Delta m}{m}\right) \textbf{(0.5 pts)}$$

Exercise 2 : (05 pts)

A- $\vec{r_1} = 2\,\vec{\imath} - 2\vec{\jmath} + 3\vec{k}$, $\quad \vec{r_2} = \vec{\imath} + \vec{\jmath} + \vec{k}$

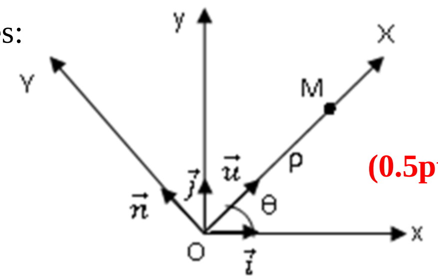
1- Calculatation of vector product $\vec{r_1} \wedge \vec{r_2}$.

$\vec{r_1} \wedge \vec{r_2} = -5\vec{\imath} + \vec{\jmath} + 4\vec{k}$ **(0.5 pts)**

2- $|\vec{r_1} \wedge \vec{r_2}| = |\vec{r_1}|.|\vec{r_1}|.sin\theta \rightarrow sin\theta = \frac{|\vec{r_1} \wedge \vec{r_2}|}{|\vec{r_1}|.|\vec{r_1}|} = 0.9 \Rightarrow \theta \sim 64°C$ **(0.5 pts)**

B- A material point M is identified by its polar coordinates:

$$\begin{cases} \rho = 2t^3 + 1 \\ \theta = \omega t \end{cases} (\omega \text{ constant})$$



(0.5pts)

1- **A position vector of the point M** is : $\overrightarrow{OM} = \rho.\vec{u_\rho} = (2t^3 + 1)\vec{u_\rho}$ **(01 pts)**

The velocity vector $\vec{v}$ of point M in polar coordinates will be:

$$\Longrightarrow \vec{v} = \frac{\mathrm{d}\overrightarrow{OM}}{\mathrm{dt}} = \frac{\mathrm{d}\rho}{\mathrm{dt}}\overrightarrow{u_\rho} + \rho' \frac{d\overrightarrow{u_\rho}}{dt} =$$

$$\Longrightarrow \vec{v} = 6t^2\overrightarrow{U_\rho} + \rho\frac{d\theta}{dt}\frac{d\overrightarrow{U_\rho}}{d\theta}$$

$$\Longrightarrow \vec{v} = 6t^2\overrightarrow{U_\rho} + (2t^3+1)\omega\overrightarrow{U_\theta}$$ **(01 pts)**

2- **$\vec{v}$ in cartesiennes coordonates.**

We have;

$$\begin{cases} \overrightarrow{u_\rho} = cos\theta\vec{\imath} + sin\theta\vec{\jmath} \\ \overrightarrow{u_\theta} = -sin\theta\vec{\imath} + cos\theta\vec{\jmath} \end{cases}$$ **(0.5 pts)**

$$\Longrightarrow \vec{v} = 6t^2(cos\theta\vec{\imath} + sin\theta\vec{\jmath}) + (2t^3+1)\omega(-sin\theta\vec{\imath} + cos\theta\vec{\jmath})$$

$$\Longrightarrow \vec{v} = (6t^2cos\theta - (2t^3+1)\omega sin\theta)\vec{\imath} + (6t^2sins\theta + (2t^3+1)\omega cos\theta)\vec{\jmath}$$ **(01 pts)**

<u>Exercise 3</u> : (05 pts)

We have **$x^2 + y^2=4$** such that **$x(t) = 2\sin(\omega t)$.**

1- Y(t)= ? :

$x^2 + y^2=4$ is an equation of the circular trajectory with radius R=2. So we have circular motion. So; **$y(t) = 2\cos(\omega t)$**

2nd method: we have **$x(t) = 2\sin(\omega t)$** and $x^2 + y^2=4$

so $y^2=4 - x^2=4- 4\sin^2(\omega t) = 4(1-\sin^2(\omega t)$ Then: **$y(t) = 2\cos(\omega t)$** **(0.5 pts)**

2- The velocity and the acceleration:

$$\begin{cases} v_x = \frac{dx}{dt} \\ v_y = \frac{dy}{dt} \end{cases} \Longrightarrow \begin{cases} v_x = 2\omega\cos(\omega t) \\ v_y = -2\omega\sin(\omega t) \end{cases}$$ **(01 pts)** ***so*** $v = \sqrt{{v_x}^2 + {v_y}^2} = 2\omega$ **(0.5 pts)**

And $\begin{cases} a_x = \frac{dv_x}{dt} \\ a_y = \frac{dv_y}{dt} \end{cases} \Longrightarrow \begin{cases} a_x = -2\omega^2 sin(\omega t) \\ a_y = -2\omega^2\cos(\omega t) \end{cases}$ **(01 pts)**

So $a = \sqrt{{a_x}^2 + {a_y}^2} = 2\omega^2$ **(0.5 pts)**

3- Calculation of a_T , a_N :

$a_T = \frac{dv}{dt} = 0$ **(0.5 pts)** *and* $a^2 = {a_T}^2 + {a_N}^2 \Rightarrow a_N = a = 2\omega^2$ **(0.5 pts)**

4- The nature of the motion**:**

We have : $\vec{a}.\vec{v} = v_x.a_x + v_y.a_y = 0$

So we have a uniform circular motion (ω=cst) **(0.5 pts)**