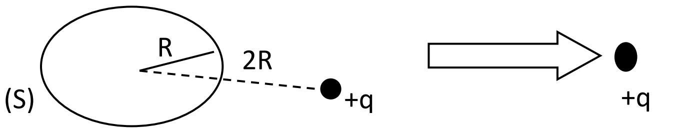
Université de Tlemcen
Faculté des Sciences
Département de Mathématiques

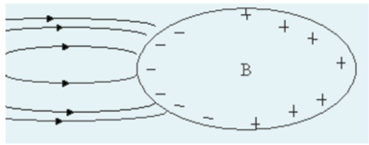
Année Universitaire 2023/2024
Durée : 01 h 30min
1ère Année LMD-MI

Corrected final exam electricity

Course questions: (6pts)

1. A metal sphere (S) of radius R, initially insulated (ΔQ=0). (1.5pts)





- When (S) is approached with a +q charge, this positive charge attracts the negative charges of the S sphere and repels the positive charges.
  The total potential will be : $V' = V_i + V_f = K\frac{Q}{R} + K\frac{+q}{2R}$ (0.5pts)
- When the potential is cancelled (grounding V=0), the positive (+) charges of S are neutralized (they flow to ground) and the sphere becomes negatively charged. (0.5pts).

Where :V'=0(0.25pts)

$\Rightarrow V' = V_i + V_f = K\frac{Q}{R} + K\frac{+q}{2R} = 0$

$$\Rightarrow Q = -\frac{q}{2} \quad (0.25pts)$$

2. Definition of a capacitor Plane: This is an assembly of two surface-charged planes under total influence. (0.5pts)

Capacitance of a Plan capacitor:

$$\mathbf{C} = \frac{\mathbf{Q}}{(\mathbf{V_1} - \mathbf{V_2})} = \frac{\mathbf{Q}}{\mathbf{U}} \ (0.5pts)$$

The field created by a plane is given by this formula E= $\frac{\sigma}{2\varepsilon_0}$.

- The field created by two planes:

$$\vec{E} = \vec{E_1} + \vec{E_2} = +\frac{\sigma}{2\varepsilon_0}(+\vec{k}) + \frac{-\sigma}{2\varepsilon_0}(-\vec{k}) \ (0.25pts)$$

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\vec{k} \ \text{Coulomb Theoreme (0.25pts)}$$

- Calculating the potential difference:

$$\begin{cases} \vec{E} = -\overrightarrow{grad}V \\ E = E(z) \end{cases} (0.5pts)$$

$$E = -\frac{dV}{dz} \Longrightarrow dV = -Edz \ (0.25pts)$$

$$V_1 - V_2 = \int Edz = \int_0^e \frac{\sigma}{\varepsilon_0} dz = \frac{\sigma}{\varepsilon_0}(z_2 - z_1) = \frac{\sigma}{\varepsilon_0} e = U \ (0.25pts)$$

- The capacitance of a spherical capacitor:

$$C = \frac{Q}{(V_1 - V_2)} = \frac{\sigma.S}{\frac{\sigma}{\varepsilon_0}e} \ \text{so} \ \mathbf{C} = \frac{\boldsymbol{\varepsilon_0 S}}{\boldsymbol{e}} \ (0.5pts)$$

3. The current density $\vec{j}$ rerepresents the amount of charge passing through the unit area per unit time (0.5pts). $\vec{j} = \sigma\vec{E}$. (0.5pts)

4. The shape of the elementary electric field $d\vec{E}$ for a linear load distribution is :

$$d\vec{E} = \frac{kdq}{r^2}\vec{u} = k\frac{\lambda dl}{r^2}\vec{u}$$ (0.5pts)

Exercise 1: (07 pts)

1- Equivalent capacity :

$C_{23} = C_2 + C_3 = 10 + 4 = 14\ \mu F$ (0.5pts)

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{2} + \frac{1}{14} + \frac{1}{7} = \frac{10}{14} \Rightarrow C_{eq} = 1{,}4\mu F$ (0.5pts)

2- Charges carried by capacitors :

In a series connection:

$Q_{eq} = Q_{C1} = Q_{C23} = Q_{C4}$ (0.5pts) **with Q_{eq} = Ceq E et $E = U_{C1} + U_{C23} + U_{C4}$** (0.5pts)

$Q_{eq} = C_{eq}E \Rightarrow Q_{eq} = 1{,}4x12 = 16{,}8\mu C$ (0.5pts)

$Q_{eq} = Q_{C_1} = Q_{C_4} = Q_{C_{23}} = 16{,}8\mu C$ (0.5pts)

And $\boldsymbol{U_{23}} = \boldsymbol{U_2} = \boldsymbol{U_3}$ (0.5pts) $\Rightarrow \frac{Q_{C_{23}}}{C_{23}} = \frac{Q_{C_2}}{C_2} = \frac{Q_{C_3}}{C_3}$

$\Rightarrow Q_{C_2} = \frac{Q_{C_{23}} x\ C_2}{C_{23}} = \frac{16{,}8x4}{14} = 4.8\mu C$ (0.5pts) and $Q_{C_3} = \frac{Q_{C_{23}} x\ C_3}{C_{23}} = \frac{16{,}8x10}{14} = 12\mu C$ (0.5pts)

3- the ddp of capacitors

$U_1 = \frac{Q_{C_1}}{C_1} = \frac{16{,}8}{2} = 8{,}4Volt$ (0.5pts) and $U_4 = \frac{Q_{C_4}}{C_4} = \frac{16{,}8}{7} = 2{,}4Volt$ (0.5pts)

And $U_3 = U_2 = 12 - 8{,}4 - 2{,}4 = 1{,}2Volt$ (0.5pts)

4. The energy carried by C_1 is:

$E_p = \frac{1}{2}CU^2 = \frac{1}{2}.QU$ (0.5pts) so $E_p = 18\ 10^{-9}\ J$ (0.5pts)

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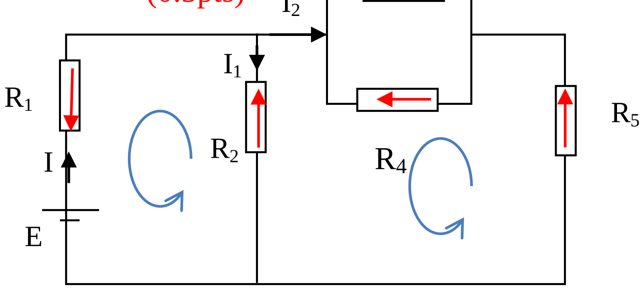
Exercise 2: (07 pts)

1- Current intensity I using Kirchoff's laws :

with: $R_1=2\Omega$, $R_2=20\Omega$, $R_3=12\Omega$, $R_4=6\Omega$, $R_5=16\Omega$ and $E=24$ V

Nods law: $I=I_1+I_2$ (0.5pts)

(0.5pts)



Lope law:

$E-R_1I-R_2I_1=0$ (0.5pts)

$R_2I_1-R_{34}I_2-R_5I_2=0$ (0.5pts)

$$\frac{1}{R_{34}}=\frac{1}{R_3}+\frac{1}{R_4}=\frac{1}{6}+\frac{1}{12}=\frac{3}{12}\Rightarrow R_{34}=4\Omega \text{ (0.5pts)}$$

$$\begin{cases}24-2(I_1+I_2)-20\,I_1=0\\ 20I_1-16I_2-4I_2=0\end{cases}\Rightarrow\begin{cases}12-2I_2-22I_1=0\\ 20I_1-20I_2=0\end{cases}$$

$I_2=I_1$ so $24-24I_2=0$ then $I_2=I_1=1$ A (0.5pts) and $I=2$A (0.5pts)

2- The current I using the equivalent resistance:

$R_{345}=16+4=20\ \Omega$(0.25pts) ,

$$\frac{1}{R_{2345}}=\frac{1}{R_2}+\frac{1}{R_{345}}\text{(0.25pts)}=\frac{1}{20}+\frac{1}{20}=\frac{2}{20}\Rightarrow R_{2345}=10\ \Omega \text{ (0.25pts)}$$

$R_{eq}=R_1+R_{2345}$(0.25pts)$=2+10=12\ \Omega$ with $E-R_{eq}I_1=0$ (0.5pts)

so $I_1=\frac{E}{R_{eq}}=\frac{24}{12}=2A$ (0.5pts)

3- Circulating currents in resistors R_3 and R_4 :

$U_{34}=R_{34}I_2=4x1=4V$ (0.25pts) with $U_{34}=U_3=U_4\Rightarrow U_{34}=R_3I_2'=R_4I_2''$ (0.25pts)

So $I_2'=\frac{U_{34}}{R_3}=\frac{4}{12}=\frac{1}{3}A$ (0.25pts) and $I_2''=\frac{U_{34}}{R_4}=\frac{4}{6}=\frac{2}{3}A$ (0.25pts)

4- we have $P_T = U*I=Req.\ I^2= 12*4= 48$ W (0.5pts)

In the other hand: P = EI = 24V x 2A = 48W (0.5pts)

Conclusion: $P_T = P$ (0.5pts)