CHAPTER 2

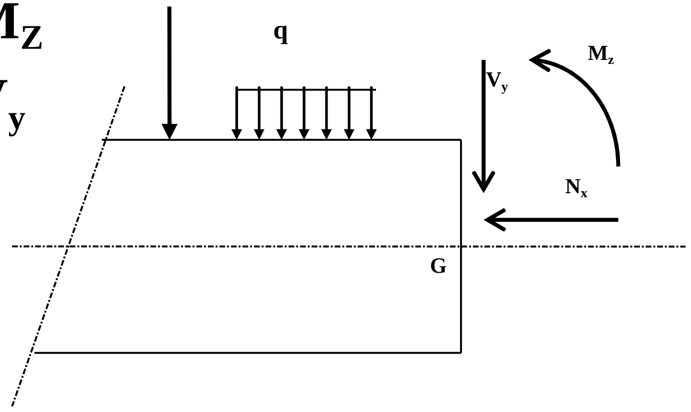
DIMENSIONING IN SIMPLE BENDING

Justifications for normal loads

1) Definition

A medium-plane beam is subject to simple plane bending if the loads are reduced to :

- **A bending moment : M_Z**
- **And a shear force : V_y**



2) Justifications

In reinforced concrete, a distinction is made between :

- ➢ **The action of the bending moment which leads to the dimensioning of the longitudinal reinforcements.**
- ➢ **The action of the shear force, which concerns the design of the transverse reinforcement.**
- ➢ **These two calculations are carried out separately and in this section we will limited to the bending moment calculations.**

3) Beam spans

In reinforced concrete, the span of the beams to be taken into account is :

- **The span between support centres when there are support devices or when the beam rests on masonry walls,**

- **The span between bare supports when the supports are in reinforced concrete (principal beam, column or wall).**

Justifications for bending moment

Three limit states are to be considered for the justification of deflected beams:

- ✓ **Ultimate resistance limit state**
- ✓ **Service limit state with regards to durability**
- ✓ **Service limit state with regards to deformation**

I. Ultimate Limit State of Resistance

We need to check that:

$$M_u \leq M_{ur}$$

Where:

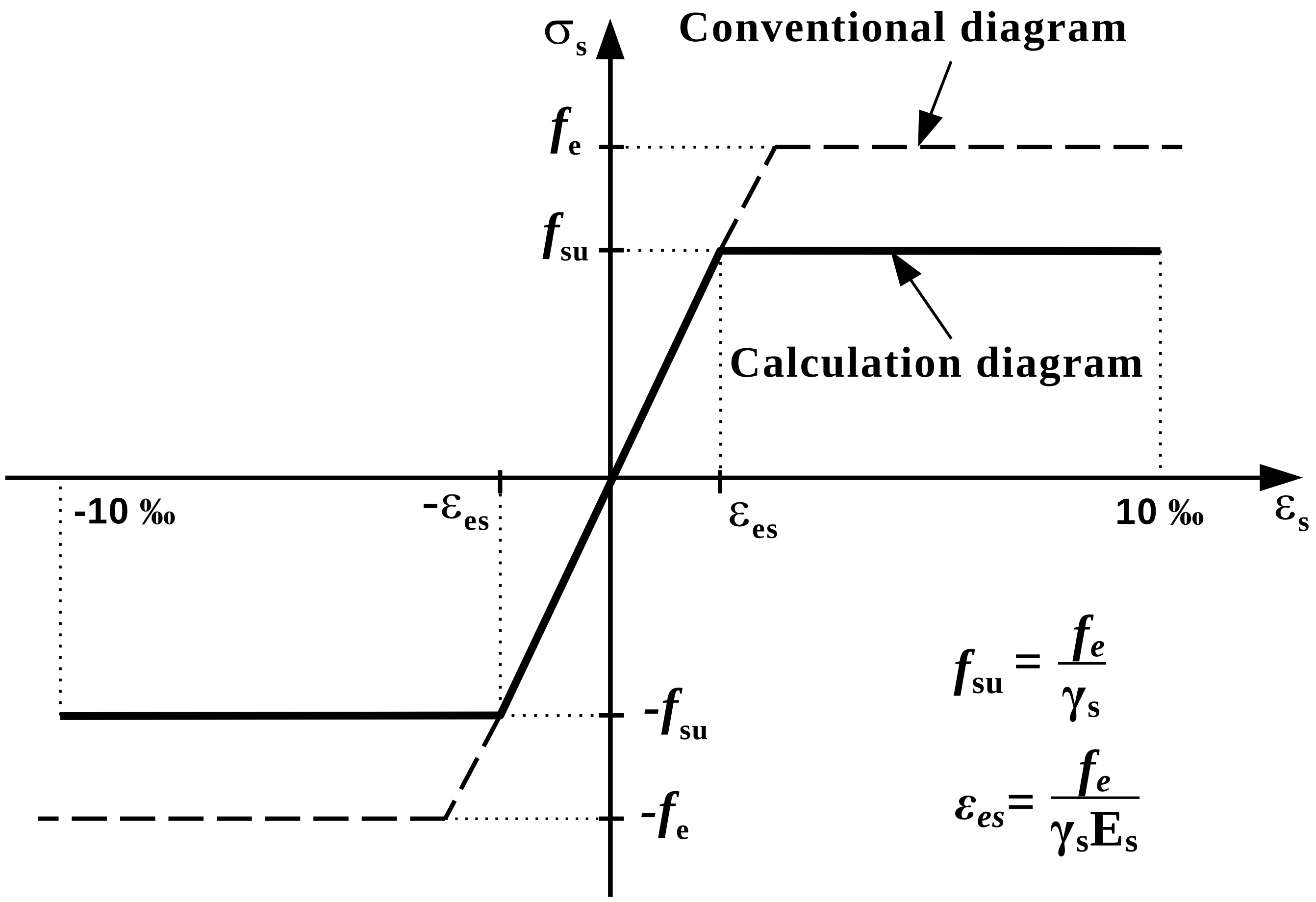
M_u : is the applied moment (design moment)

and **M_{ur}** : is the resisting moment of the section

1) Calculation hypotheses

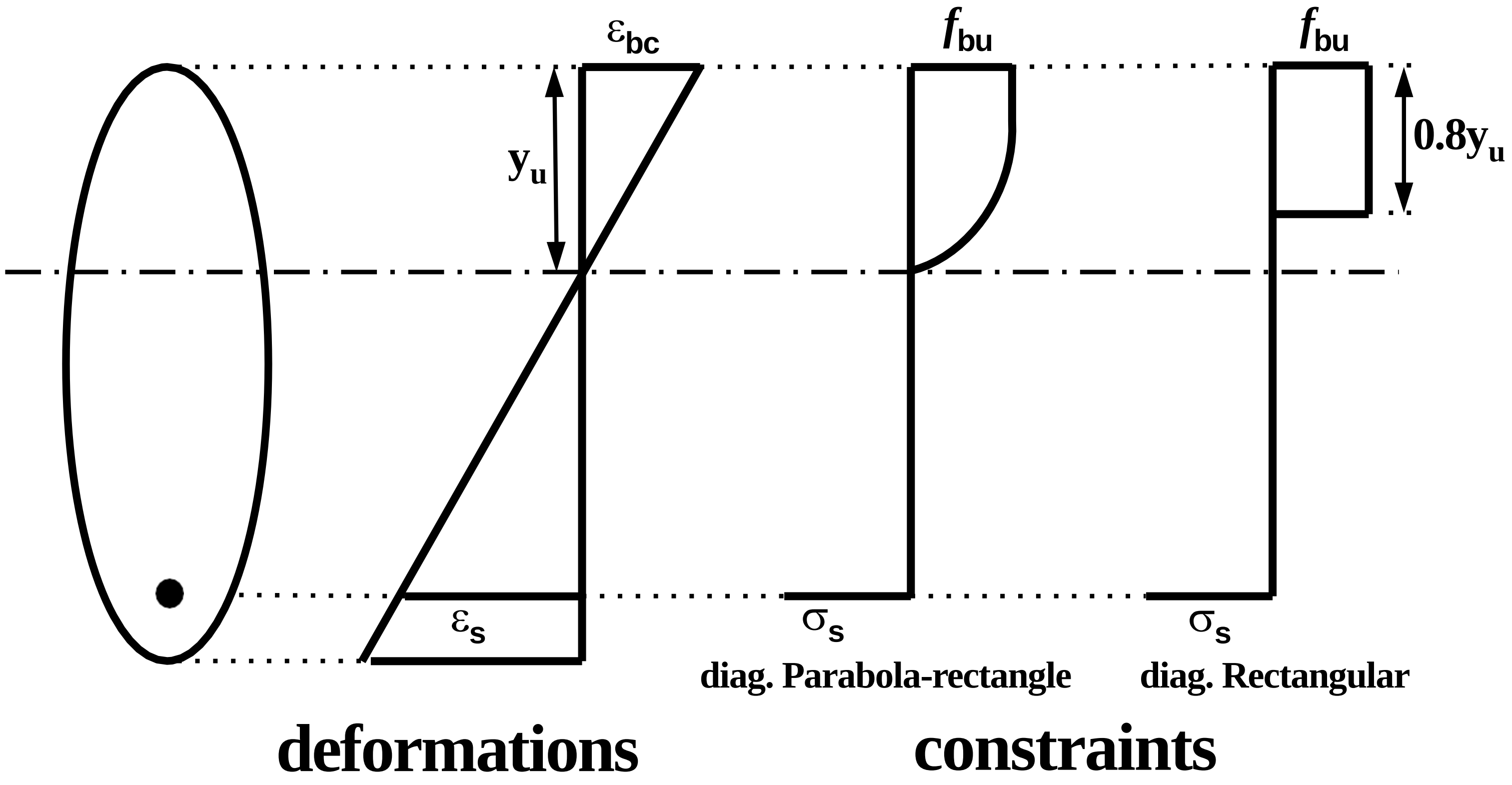
The main assumptions for the ULS (ELU) design of the RC sections subjected to simple bending are as follows:

- **Straight sections remain flat after deformation (Navier-Bernoulli hypothesis),**
- **There is no relative sliding between the reinforcement and the concrete, and the tension in the concrete is neglected,**



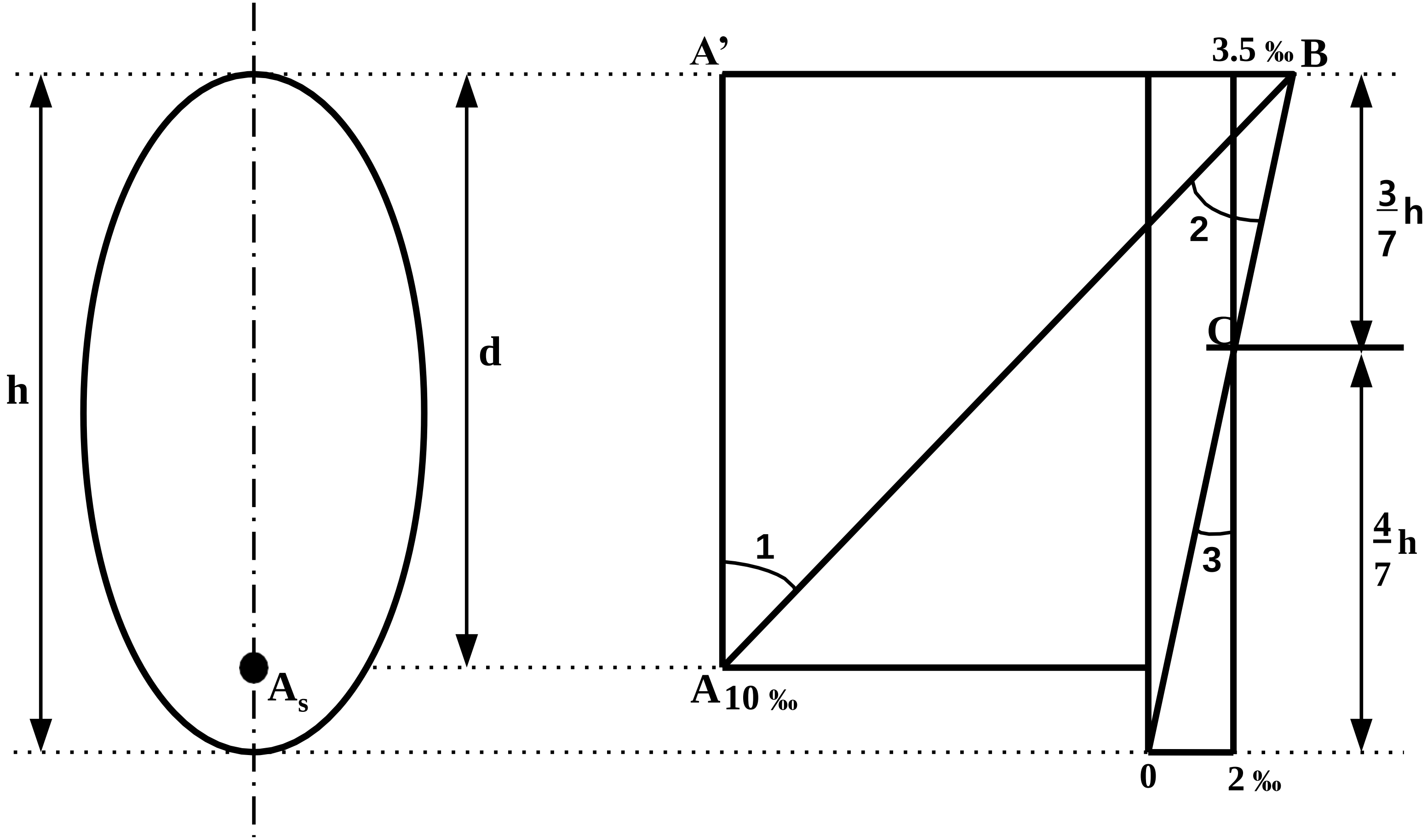
Stress-strain diagram for steel calculations

f) for concrete ⇒ **simplified rectangular diagram (Article A.4.3,42).**

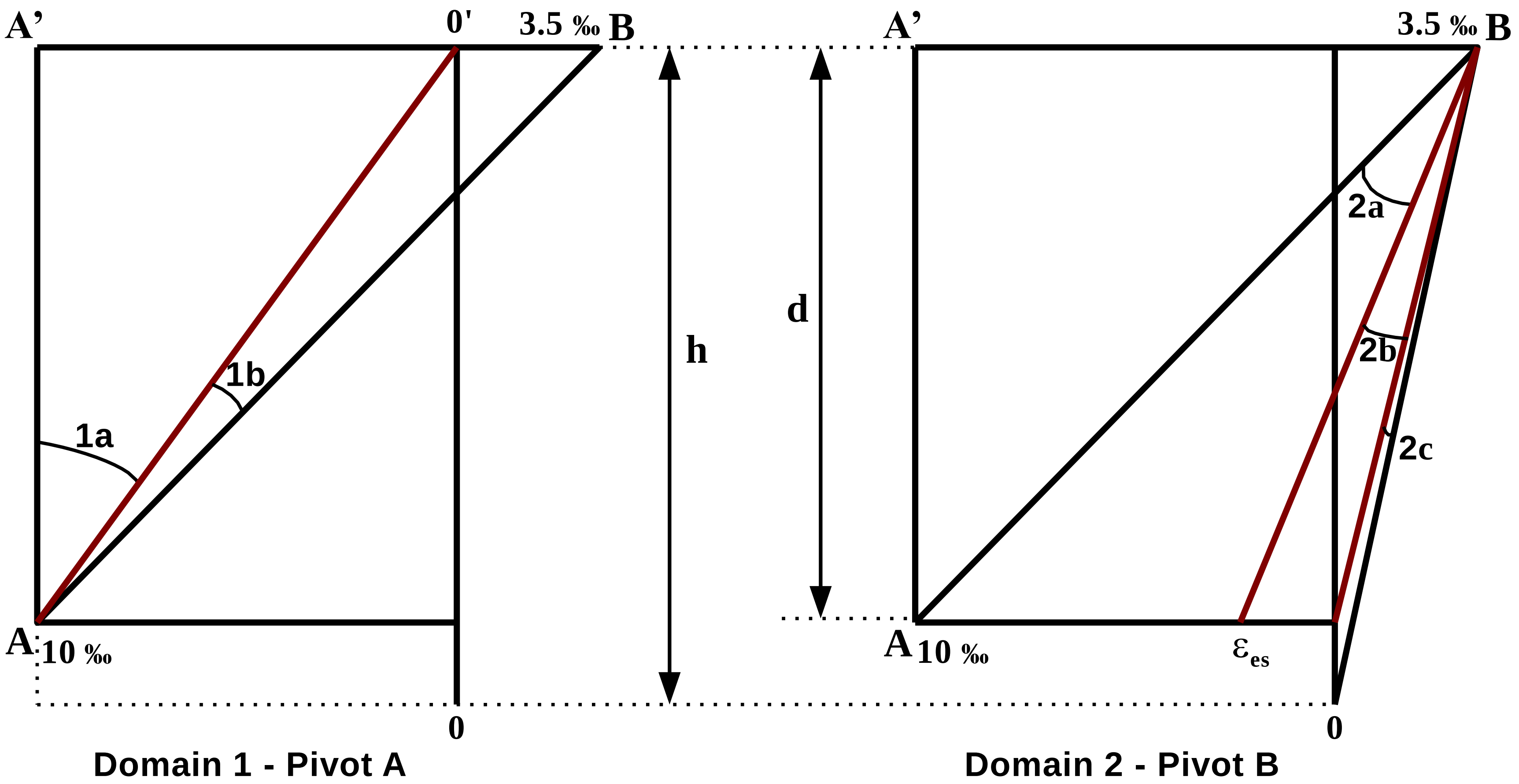


g) $\varepsilon_b \leq$ 3.5‰ in flexion et 2‰ in simple compression
$\varepsilon_s \leq$ 10‰

The deformation diagram of the section at E.L.U.R (Article A.4.3.3) therefore passes through one of the 3 pivots A, B or C defined below:



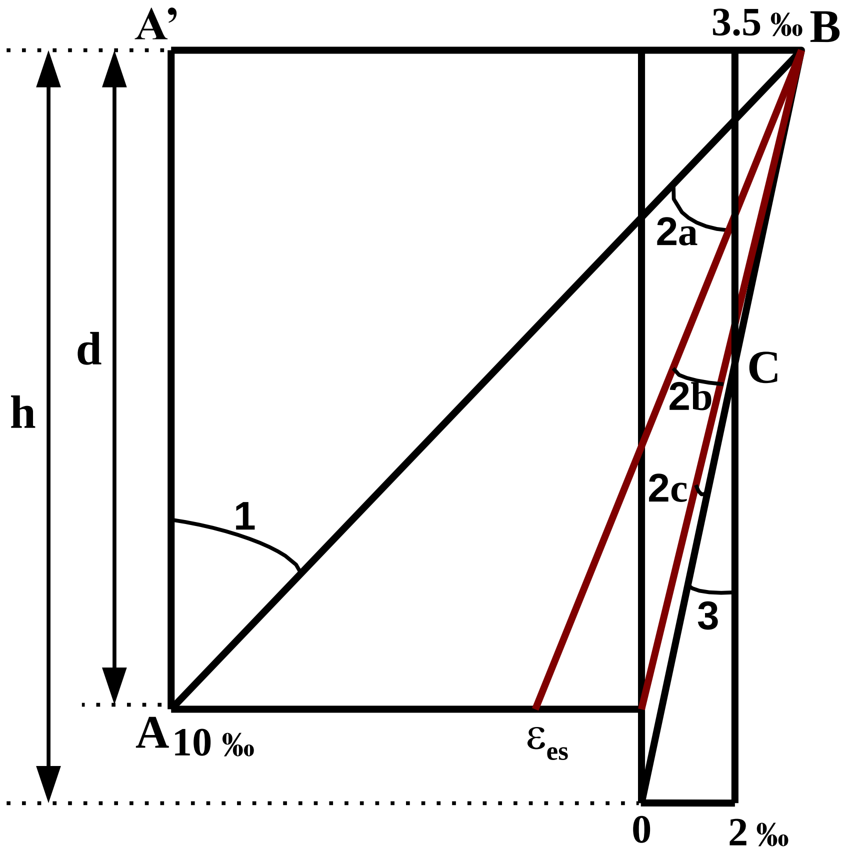
2) Possible deformation lines for simple bending



Let y_u be the depth of the neutral axis,

$\alpha_u = y_u/d$ is the relative depth of the neutral axis

Determine the bounds of α_u for each domain.



domain 1 : $\alpha_u \le 0.259$

domain 2a : $0.259 < \alpha_u \le \alpha_\ell$

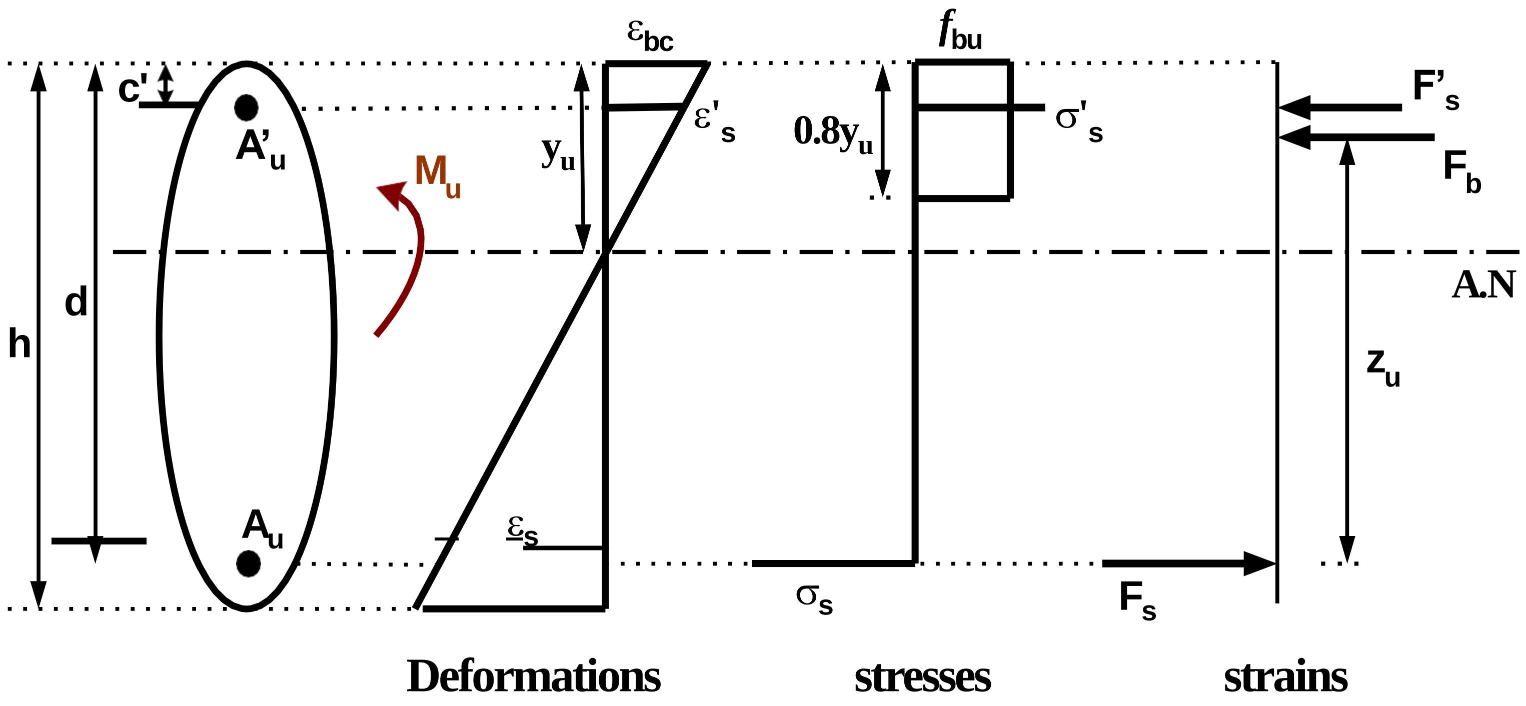
with $\alpha_\ell = 3.5 / (3.5 + 1000\,\varepsilon_{es})$

domain 2b : $\alpha_\ell < \alpha_u \le 1$

domain 2c : $1 < \alpha_u \le \alpha_{BC} = h/d$

domain 3 : $h/d < \alpha_u$

3) Notations - Equilibrium equations

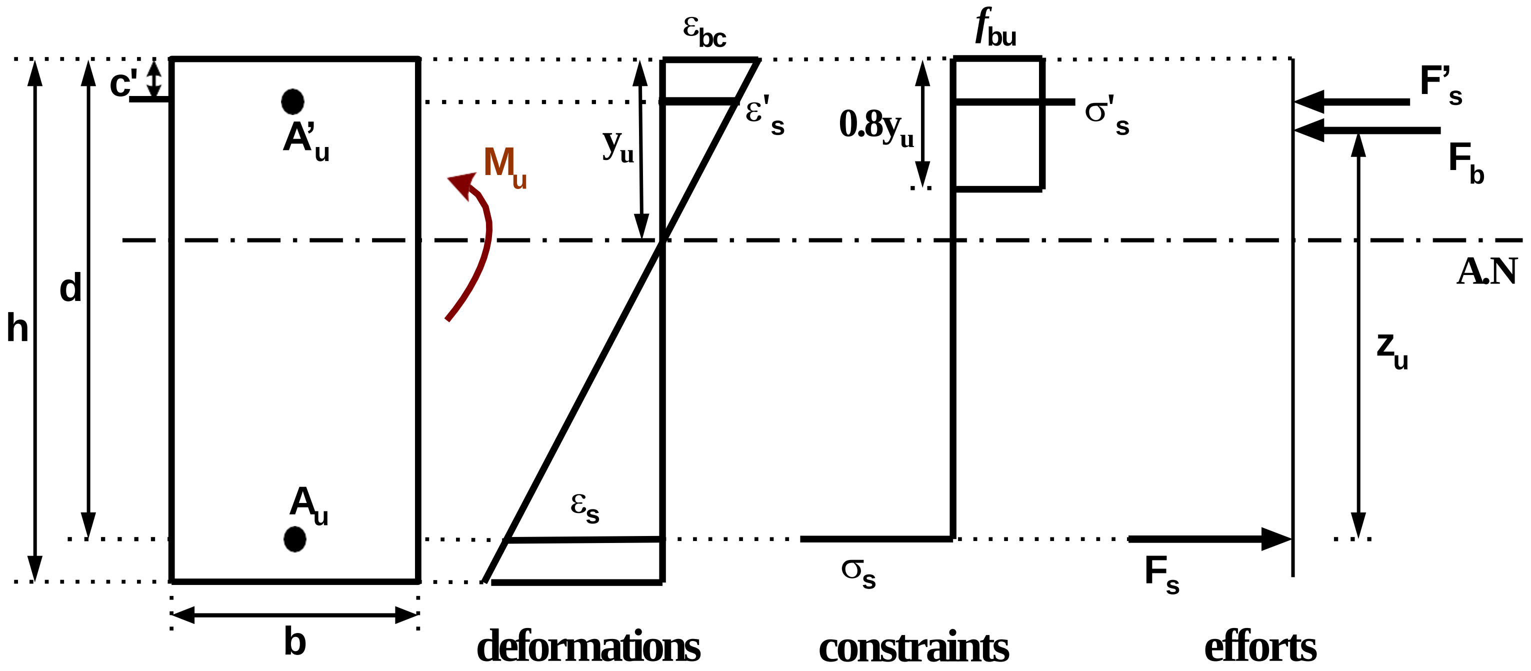


The equilibrium of the section results in the following two equations:

- **Equilibrium of normal forces :** $F_b + F'_s - F_s = 0$
- **Equilibrium of moments in relation to the centre of gravity of the tensioned steel :**

$$F_b\, z_u + F'_s\,(d - c') = M_u$$

4) Case of rectangular sections



In this case we have :

$$F_b = 0.8\, b\, y_u\, f_{bu}$$

$$F_s = A_u . \sigma_s$$

$$F'_s = A'_u . \sigma'_s$$

$$z_u = d - 0.4\, y_u$$

➢ Moment résistant béton :

$$M_b = F_b\, z_u \quad = 0.8\, b\, y_u\, f_{bu}\, (d - 0.4\, y_u)$$
$$= 0.8\, b.\, y_u/d\, .\, f_{bu}\, .d^2(1 - 0.4\, y_u/d)$$
$$= 0.8\alpha_u\, (1 - 0.4\alpha_u)\, bd^2\, .\, f_{bu} \qquad \textit{avec} \qquad \alpha_u = y_u\, /d$$

We can write: $M_b = \mu_u\, bd^2\, f_{bu}$

with $\mu_u = 0.8\alpha_u\, (1 - 0.4\alpha_u)$ **et** $\alpha_u = y_u\, /d$

- μ_u **is called the relative moment of the concrete or 'reduced ultimate moment'.**
- α_u **is the relative depth of the neutral axis**

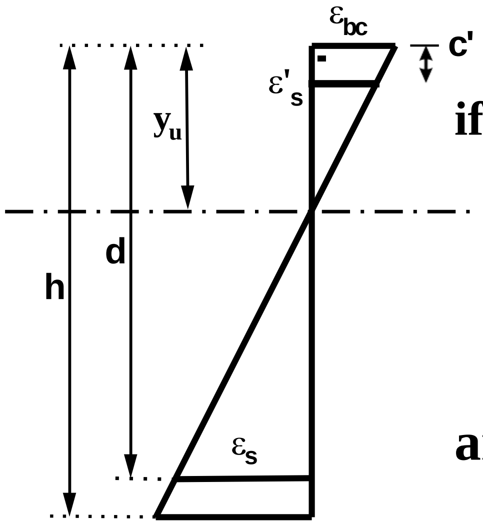
➢ Equilibrium equations :

$$0.8\, \alpha_u\, bd\, f_{bu} + A'_u\, \sigma'_s - A_u\, \sigma_s = 0$$
$$\mu_u\, bd^2\, f_{bu} + A'_u\, \sigma'_s\, (d - c') = M_u$$

What are the unknowns in this system of equations?

if the deformation diagram is known, the parameters y_u, ε_{bc}, ε_s, ε'_s, σ_s et σ'_s are known.

In fact :



The deformation diagram gives:

$$\frac{\varepsilon_{bc}}{y_u} = \frac{\varepsilon_s}{d-y_u} = \frac{\varepsilon'_s}{y_u-c'}$$

if the deformation line passes through the pivot A we have :

$$\varepsilon_s = 10\ ‰ \ ; \quad \varepsilon_{bc} = \frac{y_u}{d-y_u}10‰ \quad \text{et} \quad \varepsilon'_s = \frac{y_u-c'}{d-y_u}10‰$$

and if the deformation line passes through the pivot B we have :

$$\varepsilon_{bc} = 3.5‰ ; \quad \varepsilon_s = \frac{d-y_u}{y_u}3.5‰ \quad \text{et} \quad \varepsilon'_s = \frac{y_u-c'}{y_u}3.5‰$$

Once the deformations are known, the corresponding stresses can be deduced.

Now let's assume that the dimensions of the section of concrete (b,h)are known

and take approximate values for d and c' to be

$d=0.9h$ et $c'=d/9$

This leaves a number of unknowns:

- **the y_u position of the deformation diagram**
- **and reinforcement sections A_u, A'_u**

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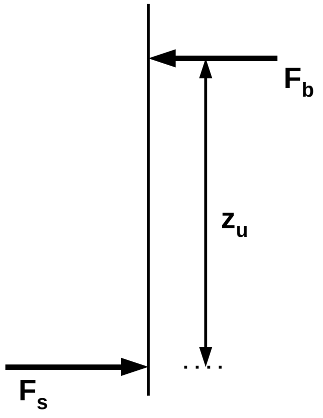
As the steel reinforcement is primarily intended to absorb the tensile stress, we will first assume that : $A'_u = 0$

a) Section without compressed reinforcement

if $A'_u = 0$ the equilibrium of the moments gives:

$$\mu_u = \frac{M_u}{bd^2 f_{bu}} \qquad (F_b z_u = M_u)$$



or $\mu_u = 0.8\, \alpha_u (1 - 0.4\, \alpha_u)$

So α_u is the lower race of the equation:

$$0.4\, \alpha_u^{\,2} - \alpha_u + 1.25\, \mu_u = 0$$

soit

$$\alpha_u = \frac{1-\sqrt{1-2\,\mu_u}}{0.8} = 1.25(1-\sqrt{1-2\,\mu_u})$$

Writing the equilibrium of the moments with respect to the point of application of F_b, we obtain :

$$M_u = F_s\, z_u = A_u\, \sigma_s\, z_u$$

Resulting :

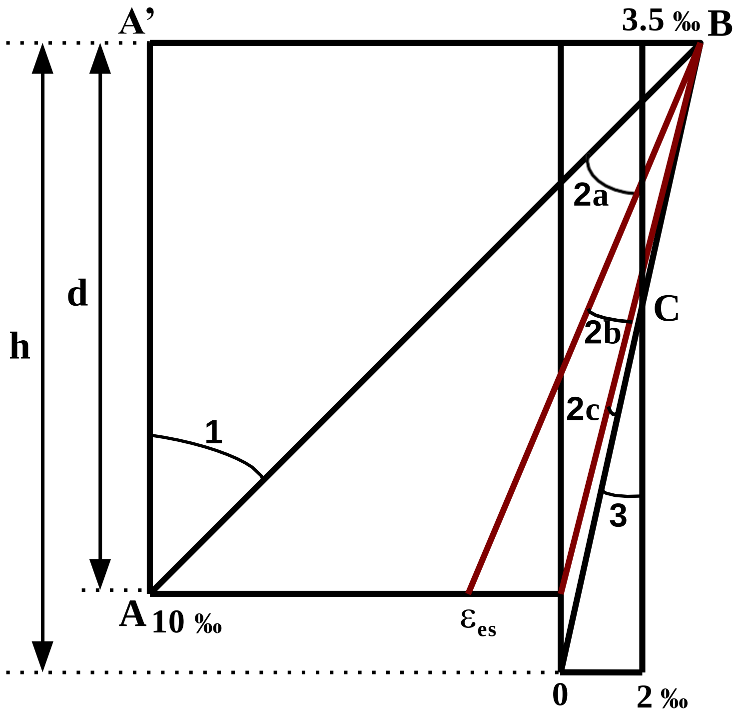
$$A_u = \frac{M_u}{\sigma_s\, z_u} \quad \text{avec } z_u = d(1-0.4\alpha_u) \quad \text{et } \alpha_u = 1.25(1-\sqrt{1-2\mu_u})$$

b) Position of the deformation diagram

To place the deformation diagram within the domain
possible, possible, we compare the reduced moment of calculation μ_u

$$\mu_u = \frac{M_u}{bd^2 f_{bu}}$$

with the reduced moments corresponding to the limits of the different domains.



- **domain 1** $\mu_u \leq 0.186$ $\alpha_u \leq \alpha_{AB} = 0.259$
- **domain 2a** $0.186 < \mu_u \leq \mu_\ell$ $0.259 < \alpha_u \leq \alpha_\ell$
- **domain 2b** $\mu_\ell < \mu_u \leq 0.48$ $\alpha_\ell < \alpha_u \leq 1$
- **domain 2c** $0.48 < \mu_u \leq \mu_{BC}$ $1 < \alpha_u \leq \alpha_{BC} = h/d$
- **domain 3** $\mu_{BC} < \mu_u$ $\alpha_{BC} < \alpha_u$