Chapter 1: Dimensional Analysis and Uncertainty Calculation

Part 1: Dimensional analysis:

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1.1 Introduction

The observation of physical phenomena is incomplete if it does not lead to quantitative information, which is the measurement of physical quantities. To study a physical phenomenon, one must examine the important variables; the mathematical relationship between these variables constitutes a physical law. This is possible in certain cases, but for other cases, it is necessary to use a modeling method such as dimensional analysis.

1.2 Definition of Dimensional Analysis

It is a theoretical tool for interpreting problems based on the dimensions of the involved physical quantities: length, time, mass, and so on. Dimensional analysis allows for:

- Verifying the validity of dimensioned equations.
- Investigating the nature of physical quantities.
- Exploring the homogeneity of physical laws.
- Determining the unit of a physical quantity based on fundamental units (meter, second, kilogram, etc..).

1.3 Physical Quantity

A physical quantity is an observable and measurable property through a specifically designed instrument. Mechanics acknowledges seven fundamental physical quantities: length, time, mass, electric current, temperature, quantity of material, and luminous intensity. Other physical quantities, known as derived quantities, are expressed in terms of these three fundamental quantities, such as velocity, acceleration, force, and more. . . .. Note : In general, for first-year students in Mathematics and Computer Science (MI), Mathematics (M), and Computer Science (I), the focus is primarily on the first three fundamental quantities: length, time, and mass.

1.4 International System of Units

The value of a physical quantity is given in relation to a standard known as a "unit." The first four fundamental units constitute the MKSA International System (Meter, Kilogram, Second, Ampere). Using these fundamental units, derived units can be constructed: area (m^2), velocity (m/s), force (kg m/s^2)...

Fundamental quantities	Units (in the international system MKSA)	Symbols
Length	Metre	(m)
Mass	Kilogramme	(kg)
Time	Seconde	(s)
Current intensity	Ampere	(A)
Temperature	Kelvin	(K)
Light intensity	Candela	(Cd)
Quantity of material	Mole	(mol)

Table 1: International System of Units

There are specific units such as N (Newton) for force, Hz (Hertz) for frequency, Watt for power, Pascal (Pa) for pressure... Note: There are two systems of units:

- The International System (SI) known as MKSA (Meter, Kilogram, Second, Ampere), which is the most widely used system.
- The CGS system (Centimeter, Gram, Second), which is less commonly used.

1.5 Dimensional Equations

Dimension represents the nature of a physical quantity. A physical quantity has only one possible dimension. The dimension of a quantity G is denoted by: $[G] = L$
By denoting M, L, and T as the dimensions of the fundamental quantities mass, length, and time, we can express the dimensions of other derived quantities in terms of these three. The resulting equations are the dimension equations for these physical quantities.

The Fundamental Quantities	Symbols	Dimensions	Units (International System (SI))
Length	l	$[l] = L$	Meter(m)
Mass	m	$[m] = M$	Kilogramme (kg)
Time	t	$[t] = T$	Seconde (s)
Current intensity	I	$[I] = I$	Ampere (A)
Temperature	T	$[T] = \theta$	Kelvin (K)
Light intensity	j	$[j] = J$	Candela (Cd)
Quantity of material	n	$[n] = N$	Mole (mol)

Table 2: International System of Units

Example :

- $$[speed] = [v] = \frac{[length]}{[time]} = \frac{L}{T} = L.T^{-1} \quad (1)$$
  and the unit of speed is (m/s)

- $$[acceleration] = [a] = \frac{[speed]}{[[time]} = \frac{L.T^-1}{T} = L.T^{-2} \quad (2)$$
  and the unit of acceleration is (m/s^2)

- $$[Force[= [F] = [mass]\,.\,[acceleration] = M.L.T^(-2) = M.L.T^{-2} \quad (3)$$
  and the unit of force is Newton or $(kg.m/s^2)$.

Notes :

- The dimension of constants is always equal to 1; we say they are dimensionless.
- Angles and functions like sin, cos, tan, exp, ln, and log are dimensionless functions.$[Numericvalue] = 1, [angle] = 1, [cos\alpha] = [sin\alpha] = [tan\alpha] = [cot\alpha] = [lnx] = [ex] = 1$.

1.6 Homogeneity of Dimensional Equations

The two sides of a dimension equation must have the same dimensions since they represent quantities of the same nature. G is a physical quantity:

- $$G = A \pm B \Rightarrow [G] = [A] = [B] \quad (4)$$

- $$G = A.B \Rightarrow [G] = [A] \, . \, [B] \quad (5)$$

- $$G = \frac{A}{B} \Rightarrow [G] = \frac{[A]}{[B]} \quad (6)$$

- $$G = A^n \Rightarrow [G] = [A]^n \quad (7)$$

Note :

- A heterogeneous (non-homogeneous) equation is necessarily False.
- A homogeneous equation is not necessarily true.
- Dimensions cannot be added (or subtracted).

Conclusions :

- Dimensional analysis serves the following purposes:
- Verification of the homogeneity of physical formulas.
- Determination of the nature of a physical quantity.
- Exploration of the general form of physical laws.