Chapter 3: Sequence, Series and Trigonometry

I. Arithmetic Sequence (Porgression) An arithmetic sequence is a sequence of the form $a, a+d, a+2d, \ldots$ The number a is the first term, and d is the common difference of the sequence. The difference between two consecutive terms is d

$$a_n - a_{n-1} = d\,.$$

- The nth Term of an Arithmetic Sequence $a, a+d, a+2d, \ldots$ is $a + (n-1)\,d$
- a_1, a_2, a_3 are the first three terms.
- $a(n+1)/2$ is called middle term.
- a_n and a_{n+1} are two consecutive terms
- For arithmetic sequence $a, a+d, a+2d$, ..., the nth partial sum

$$S_n = (n/2)(2a + (n-1)d) \text{ or } S_n = (n/2)(a + a_n)$$

Examples

In arithmetic sequence 1,4,7,10,13,..., the first term is 1, the difference is 3. So, the formula for n-th term is

$$1 + (n-1)3 = 3n - 2.$$

Applying the formula we can say the 100-th term of the sequence is 298. The partial sum of the sequence is

$$(n/2)(1 + 3n - 2) = (3/2)n2 - (n/2).$$

II. Geometric Sequence $a, ar, ar^2, ar^3, \ldots$ is a geometric sequence.

- $a_1 = a$ is the first term.
- $a_n = ar^{n-1}$ is the n-th term of the geometric sequence.
- r is the common ratio.
- Partial Sum of Geometric Sequence $S_n = a + ar + ar^2 + \ldots + ar^{n-1} = a\frac{|r^n-1|}{|r-1|}$

Examples

In arithmetic sequence 2,4,8,16,32,..., the first term is 2, the ratio is also 2. So, the formula for n-th term is

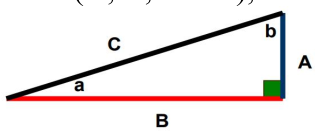
$$2\,2^{n-1} = 2^n.$$

Applying the formula we can say the 10-th term of the sequence is 1024. The partial sum of the sequence is

$$2(2^n - 1)/(2-1) = 2^{n+1} - 2.$$

III. Serises The sum of finite or infinite sequence $\Sigma a_n = a_1 + a_2 + \ldots$ is called series.

III. Trigonometry Trigonometry is the study of angle measurement. When you have a right triangle there are 5 things we can know about it: the lengths of the sides (A, B, and C), and the measures of the acute angles (a and b)



- The hypotenuse is always the longest side, and opposite from the right angle.
- The opposite side is the side directly across from the angle you are considering (angle a).
- The adjacent side is the side next to angle you are considering.

*The Trigonometric Functions sin (sine), cos (cos; cosine), tan (tan; tangent), sec (sec), csc (cosec),cot (cotangent) :

- The trigonometric functions are periodic. The sine and cosine functions have the period 2π (360°); the tangent and cotangent functions have the period π (180°).
- Sin x = Opposite side/Hypotenuse
- Cos x = Adjacent side/Hypotenuse
- Tan x = Opposite side/Adjacent side
- Cot x = adjacent side/opposite side
- Sec x = hypotenuse/adjacent side
- Csc x=hypotenuse/opposite side

A way to remember how to make the 3 basic Trig Ratios is to use the mnemonic

Silly Old Hen, Cackles And Howls, Till Old Age.

This relationship can be summarized:

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

- All functions have positive values for angles in Quadrant I,
- Sine and Cosecant have positive values for angles in Quadrant II,
- Tangent and Cotangent have positive values for angles in Quadrant III,
- Cosine and Secant have positive values for angles in Quadrant IV.