جامعة أبو بكر بلقايد
كلية التكنولوجيا
UNIVERSITY OF TLEMCEN
Faculty of Technology

Department of Civil Engineering

Soil Mechanics (2)
SEG 5.2

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Sept. 2025

Chapter 0: Review of soils mechanics 1

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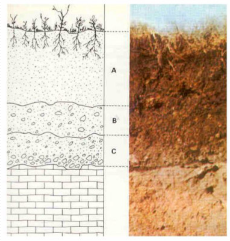


Subject 1:
Soil Mechanics 2

Third-year Civil Engineer (3IngGC))

Teaching objectives:

- *Acquire fundamental knowledge of soil mechanics.*
- *Master methods for calculating total stresses, pore pressures, and effective stresses in soils based on the loads applied.*
- *Predict settlement values over time under a given load.*
- *Understand the principles of water movement in soil.*

Previous knowledge recommended:
Core subjects from Semesters 1, 2,3 and 4

Chapter 0:
Review of soil mechanics1

- 1 Purpose of soil mechanics
- 2 Soil structure
- 3 Soil Composition
- 4 Soil Classification
- 5 Weight–Volume Relation ships

Soils are aggregates of mineral particles, and together with air and/or water in the void spaces, they form three-phase systems. A large portion of the earth's surface is covered by soils, and they are widely used as construction and foundation materials. Soil mechanics is the branch of engineering that deals with the engineering properties of soils and their behavior under stress.

Braja M. Das (2008)
Advanced Soil Mechanics
Taylor & Francis

Soil Mechanics 2
SEG 5.2
(3Eng GC)

Semester: 5
Teaching unit: UEF 4.1
WHyV: 45h00 (Course: 1h30, T: 1h30, HA:1h30)
Credits: 5 Coefficient: 3

Contents:
CHAPTER 1: ***SOIL CONSTRAINTS*** (5 WEEKS)
CHAPTER 2: ***SOIL SETTLEMENT AND CONSOLIDATION*** (5 WEEKS)
CHAPTER 3: ***SOIL HYDRAULICS*** (3 WEEKS)

Assessment: Tutorial 20%,
Hands On Works 20%,
Final examination: 60%.

TW & HW : Dr. F. AYAD

Soil Mechanics

Soil mechanics is a branch of engineering mechanics that describes the behavior of soils. It differs from fluid mechanics and solid mechanics in the sense that soils consist of a heterogeneous mixture of fluids (usually air and water) and particles (*usually clay, silt, sand, and gravel*) but soil may also contain organic solids, liquids, and gasses and other matter.

Simple things go wrong

All were due to basic errors

Basic Geotechnical Engineering Skills What Can Graduated Do? (First John Burland Lecture) ISSMGE (2016)

Geotechnical engineering is not a stand-alone discipline:

There is common ground between:

- geology and engineering
- structures and foundations and slopes
- seepage and hydraulics

It is all physics; and a little chemistry.

Basic Geotechnical Engineering Skills What Can Graduated Do?
(First John Burland Lecture) ISSMGE (2016)

Geology and Geotechnical Engineering

Geology – describes the past | Engineering – predicts the future

How did this material become like it is? | How much will these foundations settle when loaded?

Basic Geotechnical Engineering Skills What Can Graduated Do?
(First John Burland Lecture) ISSMGE (2016)

Is the basis of geotechnical engineering:

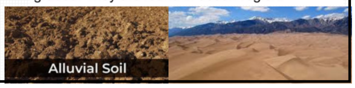
1 Empirical:
Soils in the ground are variable and there can be no universal soil mechanics theory; ground engineering has to be empirical.

2 Theoretical:
Soils are collections of mineral grains unbonded or only weakly bonded; there are simple universal theories.

Basic Geotechnical Engineering Skills What Can Graduated Do?
(First John Burland Lecture) ISSMGE (2016)

Different types of soils (1)

- **Alluvial soil** :~ Deposited by running water and are found in river banks and river beds. They are generally poorly graded and uniformly graded.
- **Aeolian soil** :~ Deposited by wind, they are mainly coarse grained particles and poorly graded. They are found in desert region.



Different types of soils (2)

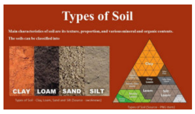
- **Lacustrine soil (lake soil)** :~ Soil particles carried by flowing water and deposited in lakes are called lake soil. These are highly compressible and have high void ratio but shear strength is less.
- **Marine soil** :~ Soil particles carried by flowing water and deposited in oceans and seas are called marine soil. Its shear strength is better than lake soil and have high void ratio.

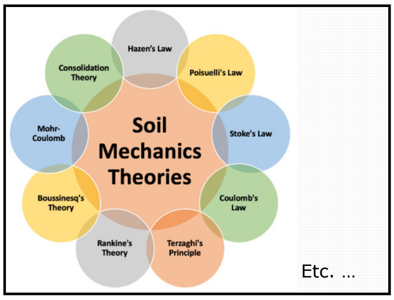
Different types of soils (3)

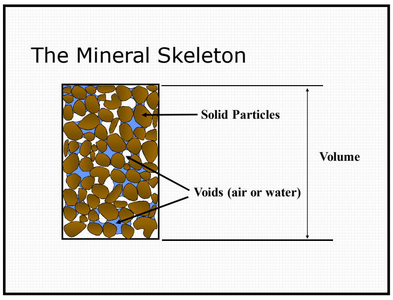
- **Glacial soil (Drift)** :~ These are the mixture of stone pieces, silts, sands and clay which are formed glaciers, these are generally well grade soil.
- **Colluvial soil** :~ Soil is transported and deposited by gravity are called colluvial soil.
- **Cohesive soil** :~ Soils in which the absorbed water and particle attraction act such that it deforms plastically at different water contents are known as cohesive soils or clay.

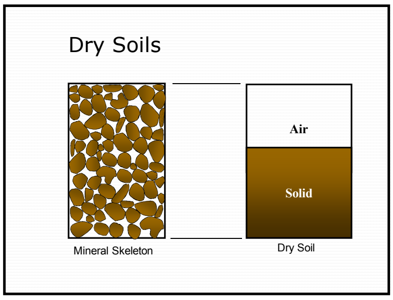
Different types of soils (4)

- **Black cotton soil** :~ *In* its high percentage of the clay mineral. It has very low bearing capacity. It posses high swelling and shrinkage property.
- **Loam** :~ It is a mixture of sand, clay and silt.

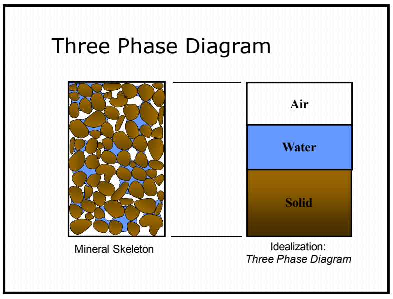


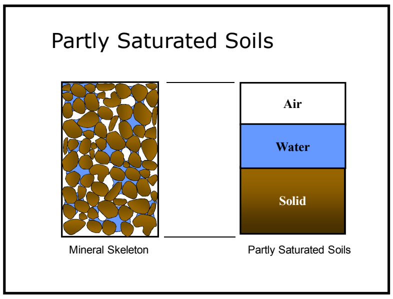

Soil Mechanics Theories
Hazen's Law
Poisuelli's Law
Stoke's Law
Coulomb's Law
Terzaghi's Principle
Rankine's Theory
Boussinesq's Theory
Mohr-Coulomb
Consolidation Theory
Etc. ...

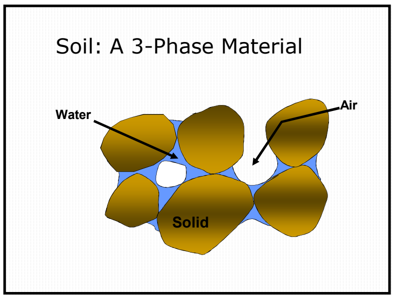

The Mineral Skeleton
Solid Particles
Volume
Voids (air or water)

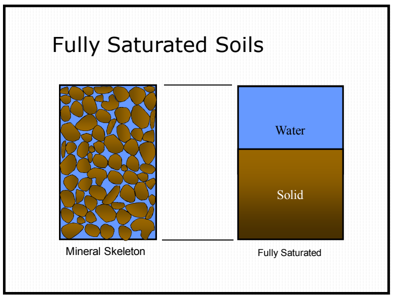

Dry Soils
Air
Solid
Mineral Skeleton
Dry Soil

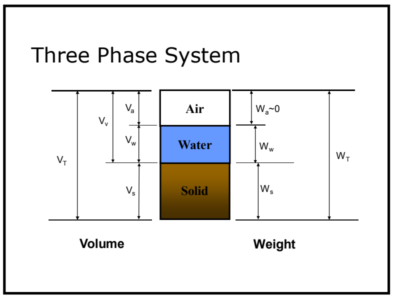

Noncohesive
Cohesive
Fine
Soil
grained
Coarse


Three Phase Diagram
Air
Water
Solid
Mineral Skeleton
Idealization:
Three Phase Diagram


Partly Saturated Soils
Air
Water
Solid
Mineral Skeleton
Partly Saturated Soils


Soil: A 3-Phase Material
Water
Air
Solid


Fully Saturated Soils
Water
Solid
Mineral Skeleton
Fully Saturated


Three Phase System
Air
Water
Solid
V_a
V_v
V_w
V_T
V_s
W_a~0
W_w
W_s
W_T
Volume
Weight

Weight Relationships

- Weight Components:
  - Weight of Solids = W_s
  - Weight of Water = W_w
  - Weight of Air ~ 0

$$Water\ Content,\ w(\%) = \frac{W_w}{W_s} \times 100\%$$

درجة التشبُّع

Degree of Saturation (S)

Degré de Saturation (Sr)

$$0 \le S \le 1$$

Dry Soil ≤ Partly Saturated Soils ≤ Fully Saturated

Mass density

- density $\rho = \frac{M}{V}$
- dry density $\rho_d = \frac{M_s}{V}$

where :

ρ : density of soil (kg/m³)

ρ_d : dry density of soil (kg/m³)

M : total mass of the soil sample (kg)

Ms : mass of soil solids in the sample (kg)

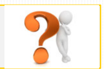
Volumetric Relationships

- Volume Components:
  - Volume of Solids = V_s
  - Volume of Water = V_w
  - Volume of Air = V_a
  - Volume of Voids = $V_a + V_w = V_v$

$$Void\ Ratio,\ e = \frac{V_v}{V_s}$$

$$Porosity,\ n(\%) = \frac{V_v}{V_T} \times 100\%$$

Mass, Weight, Density, Gravity



density, n--mass per unit volume.
density--the mass of a unit volume of a material at a specified temperature.
density--weight per unit volume, usually expressed in pounds per cubic inch, pounds per cubic foot, or kilograms per cubic meter.
mass, n--the quantity of matter in a body. (See also weight.)
weight, n--the force exerted on a body by gravity.
weight, vt--to determine the mass of a material.

ASTM DICTIONARY OF ENGINEERING SCIENCE & TECHNOLOGY 2005

ATTENTION

Pay attention to units

Example: Volumetric Ratios

- Determine void ratio, porosity and degree of saturation of a soil core sample

Data:

- Weight (mass) of soil sample = 1013g
- Vol. of soil sample = 585.0cm³
- Specific Gravity, G_s = 2.65
- Dry weight (mass) of soil = 904.0g

Volumetric Relationships

- Volume Components:
  - Volume of Solids = V_s
  - Volume of Water = V_w
  - Volume of Air = V_a
  - Volume of Voids = $V_a + V_w = V_v$

$$Degree\ of\ Saturation,\ S(\%) = \frac{V_w}{V_V} \times 100\%$$

ATTENTION

$$Degree\ of\ Saturation, S(\%) = Degré\ de\ Saturation, Sr(\%)$$

درجة التشبُّع

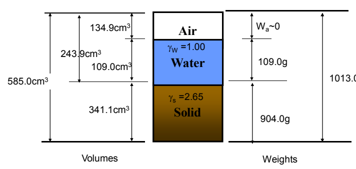
Specific Gravity

$$Specific\ Gravity = \frac{Weight\ of\ a\ Substance}{Weight\ of\ an\ Equal\ Volume\ of\ Water}$$

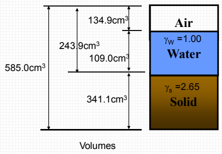
$$Specific\ Gravity = \frac{Unit\ Weight\ of\ a\ Substance}{Unit\ Weight\ of\ Water}$$

- Unit weight of Water, γ_w
  - ρ_w = 1.0 g/cm³ (strictly accurate at 4° C)
  - γ_w = 9.81 kN/m³

Example



Example

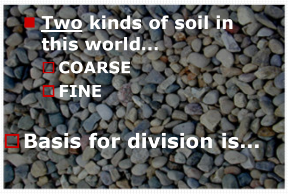


$$e = \frac{V_v}{V_s} = \frac{243.9}{341.1} = 0.72$$

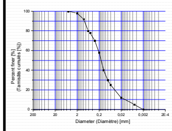
$$n(\%) = \frac{V_v}{V_T} \times 100\% = \frac{243.9}{585.0} \times 100 = 41.7\%$$

$$S(\%) = \frac{V_w}{V_v} \times 100\% = \frac{109.0}{243.9} \times 100 = 44.7\%$$

TWO KINDS of Soil…



Grain Size Distribution Curves



D_{10} : diameter corresponding to 10%
D_{30} : diameter corresponding to 30%
D_{60} : diameter corresponding to 60%

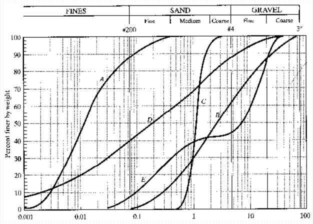
Uniformity coefficient (C_u): $C_u = \frac{D_{60}}{D_{10}}$

Coefficient of gradation (C_c): $C_c = \frac{D_{30}^2}{D_{60} \times D_{10}}$

Soil Unit weight (kN/m³)

- Bulk (or Total) Unit weight
  $\gamma = W_T / V_T$
- Dry unit weight
  $\gamma_d = W_s / V_T$
- Buoyant (submerged) unit weight
  $\gamma_b = \gamma - \gamma_w$

Grain Size Distribution Curves



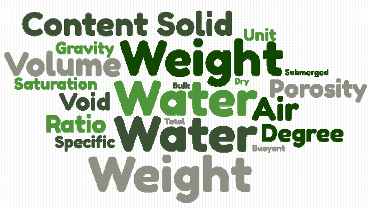
Engineering Characterization of Soils

Soil Properties that Control its Engineering Behavior

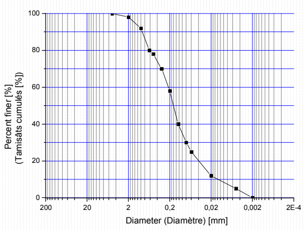
Particle Size

coarse-grained
- Particle/Grain Size Distribution
- Particle Shape

fine-grained
- Soil Plasticity



Grain Size Distribution Curves



Classification of Soils for Engineering Purposes
USCS (Unified Soil Classification System)

COARSE-GRAINED SOILS (More than 50 % > 0,08 mm)

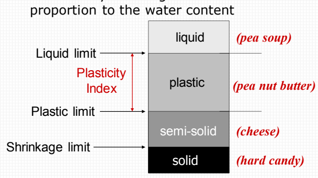
Definitions			Symbols	Conditions	Designation
Gravels	More than 50 % of coarse fraction retained > 2mm	Clean Gravels Less than 5 % fines < 0,08 mm	GW	Cu > 4 & 1< Cc < 3	Well-graded grave
			GP	One of GW's conditions is not satisfied	Poorly graded gravel
		Gravels with Fines More than 12 % fines < 0,08 mm	GM	Atterberg limits below the A line	Silty gravel
			GC	Atterberg limits above the A line	Clayey gravel
Sands	More than 50 % of coarse fraction retained < 2mm	Clean Sands Less than 5 % fines < 0,08 mm	SW	Cu > 6 & 1< Cc < 3	Well-graded sand
			SP	One of SW's conditions is not satisfied	Poorly graded sand
		Sands with Fines More than 12 % fines < 0,08 mm	SM	Atterberg limits below the A line	Silty sand
			SC	Atterberg limits above the A line	Clayey sand

When the fine particle content (< 0.08 mm) is between 5% and 12%, dual symbols is used.

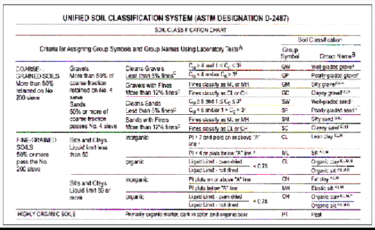
For example : SW-SM

Atterberg Limits

- Consistency of fine-grained soil varies in proportion to the water content



USCS Classification Chart



Chapter 1: Soil constraints

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Oct. 2025

Plasticity Index (*PI*, I_P)

- $PI = LL – PL$

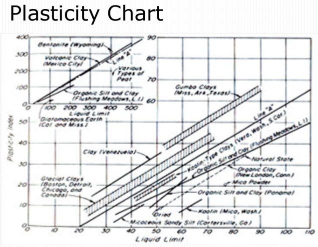
  or $I_P = w_L - w_P$

- Note: These are *water contents*, but the percentage sign is not typically shown.

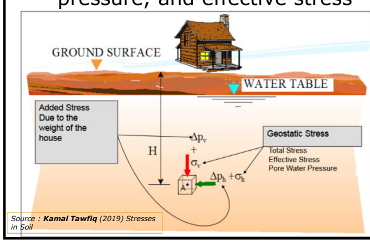
CHAPTER 1: SOIL CONSTRAINTS (5 WEEKS)

- Concepts of total stress, pore pressure, and effective stress
- Terzaghi's postulate
- Calculation of stress in overburdened soil

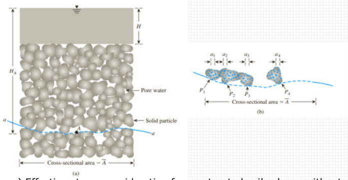
Plasticity Chart



- Concepts of total stress, pore pressure, and effective stress

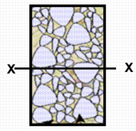


Stresses in Saturated Soil without Seepage

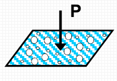


a) Effective stress consideration for a saturated soil column without seepage;
(b) forces acting at the points of contact of soil particles at the level of point A

Stresses in Soil Masses



Soil Unit

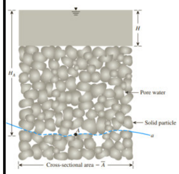


Area = A

σ = P/A

Assume the soil is fully saturated, all voids are filled with water.

Source : **Priyantha Jayawickrama** (2010) Soil Mechanics, Texas Tech University

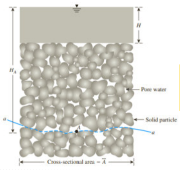


$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat}$$

$$\sigma' = [H\gamma_w + (H_A - H)\gamma_{sat}] - H_A\gamma_w$$
$$= (H_A - H)(\gamma_{sat} - \gamma_w)$$
$$= (\text{Height of the soil column}) \times \gamma'$$

where $\gamma' = \gamma_{sat} - \gamma_w$ equals the submerged unit weight of soil. Thus, we can see that the effective stress at any point A is independent of the depth of water, H, above the submerged soil.

Stresses in Saturated Soil without Seepage



$$\sigma = H\gamma_w + (H_A - H)\gamma_{sat}$$

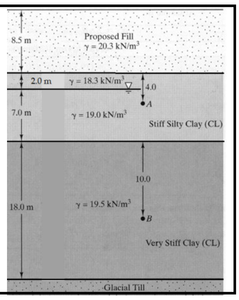
where σ = total stress at the elevation of point A
γ_w = unit weight of water
γ_{sat} = saturated unit weight of the soil
H = height of water table from the top of the soil column
H_A = distance between point A and the water table

Effective Stress

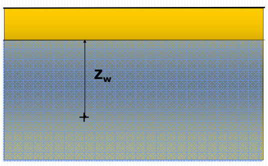
- From the standpoint of the soil skeleton, the water carries some of the load. This has the *effect* of lowering the stress level for the soil.
- *Therefore, we may define*
  effective stress = total stress minus pore pressure

σ' = σ - u where, σ' = effective stress
σ = total stress
u = pore pressure

Ex. 1.2



Groundwater



U = pore water pressure = $\gamma_w Z_w$

Source : **Priyantha Jayawickrama** (2010) Soil Mechanics, Texas Tech University

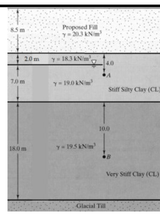
Effective Stress

σ' = σ - u

- The effective stress is the force carried by the soil skeleton divided by the total area of the surface.
- The effective stress controls certain aspects of soil behavior, notably, compression & strength.

Source : **Priyantha Jayawickrama** (2010) Soil Mechanics, Texas Tech University

Ex. 1.2

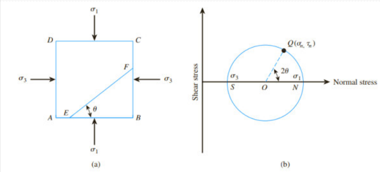


	Proposed Fill	Silty clay	Silty clay Sat	Point A	Silty clay Sat	Very Stiff Clay	Point B
Total stress	172,55	36,6	38	247,15	95	195	537,15
Pore pressure	0	0	20	20	50	100	170
Effective stress				227,15			367,15

Terzaghi's postulate

Karl von Terzaghi (2 October 1883 – 25 October 1963) was an Austrian mechanical engineer, geotechnical engineer, and geologist known as the "father of soil mechanics and geotechnical engineering".

- (a) Soil element with AB and AD as major and minor principal planes;
- (b) Mohr's circle for soil element shown in (a)



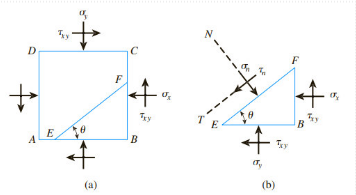
Coefficient of earth pressure at-rest

For coarse-grained soils, the coefficient of earth pressure at rest can be estimated by using the empirical relationship (Jaky, 1944)

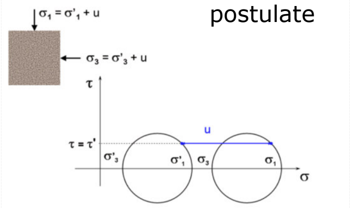
$$K_o = 1 - \sin \phi'$$

where ϕ' = drained friction angle.

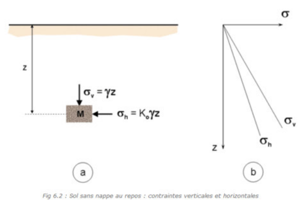
- (a) A soil element with normal and shear stresses acting on it; (b) free body diagram of EFB as shown in (a)



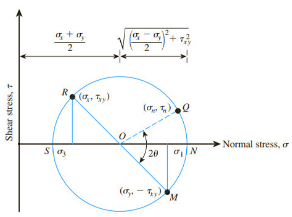
Terzaghi's postulate



- Soil without water table: vertical and horizontal stresses



- Principles of the Mohr's circle



Coefficient of earth pressure at-rest

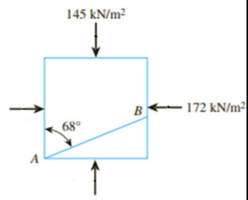
Case of soil mass in a state of static equilibrium —that is, if it does not move either to the right or to the left of its initial position, In that case $K_0 = \frac{\sigma_h}{\sigma_v}$

Where K_o : Coefficient of earth pressure at-rest

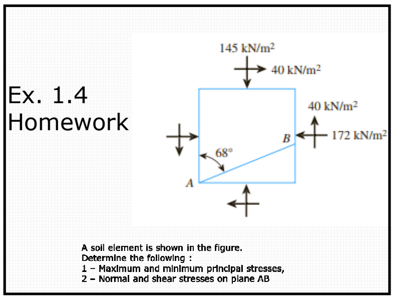
Soil	K_o
Dense sand	0.35
Loose sand	0.6
Normally consolidated clays	0.5 – 0.6
Clay, OCR = 3.5	1.0
Clay, OCR = 2.0	2.8

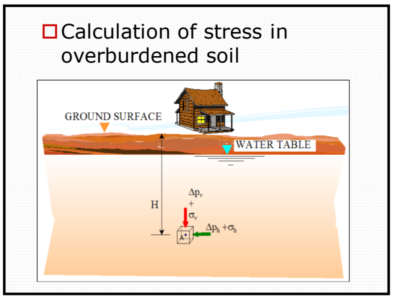
Review _ Mohr's circle
https://web.mit.edu/16.unified/www/FALL/materials/documents/HO-M-6(Mohrs)(08).pdf

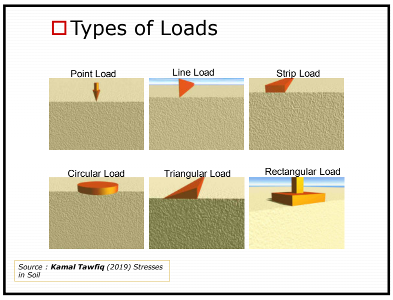
Ex. 1.3

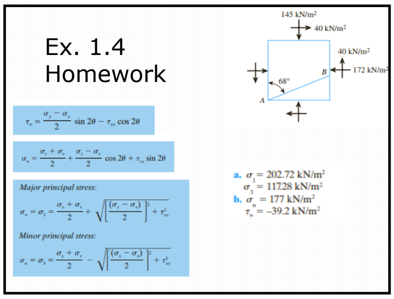


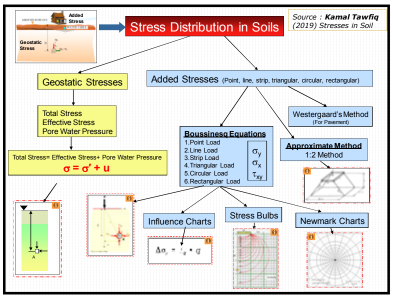
A soil element is shown in the figure.
Determine the following :
1 – Maximum and minimum principal stresses,
2 – Normal and shear stresses on plane AB

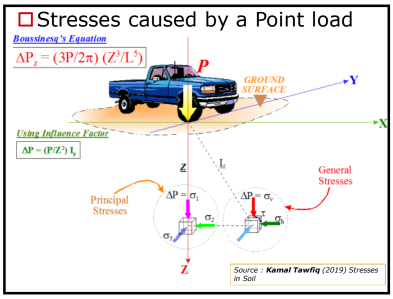

Ex. 1.4
Homework
145 kN/m²
40 kN/m²
40 kN/m²
172 kN/m²
68°
A
B
A soil element is shown in the figure.
Determine the following :
1 – Maximum and minimum principal stresses,
2 – Normal and shear stresses on plane AB

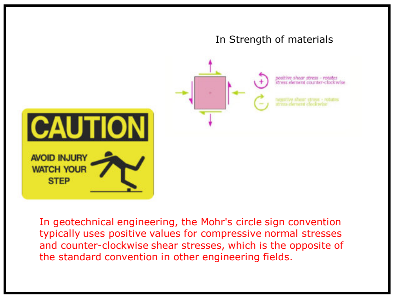

Calculation of stress in overburdened soil
GROUND SURFACE
WATER TABLE
H

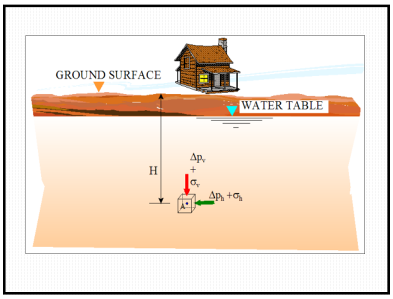

Types of Loads
Point Load
Line Load
Strip Load
Circular Load
Triangular Load
Rectangular Load
Source : Kamal Tawfiq (2019) Stresses in Soil

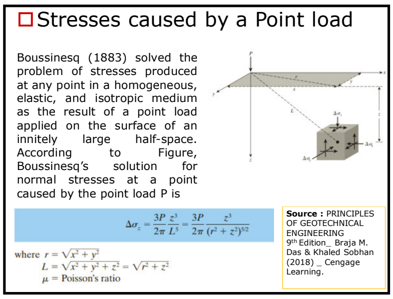

Ex. 1.4
Homework
145 kN/m²
40 kN/m²
40 kN/m²
172 kN/m²
Major principal stress:
Minor principal stress:
a. σ1 = 202.72 kN/m²
σ3 = 117.28 kN/m²
b. σn = 177 kN/m²
τn = −39.2 kN/m²


Stress Distribution in Soils
Source : Kamal Tawfiq (2019) Stresses in Soil
Geostatic Stresses
Added Stresses (Point, line, strip, triangular, circular, rectangular)
Total Stress
Effective Stress
Pore Water Pressure
Total Stress= Effective Stress+ Pore Water Pressure
σ = σ' + u
Westergaard's Method (For Pavement)
Boussinesq Equations
1.Point Load
2.Line Load
3.Strip Load
4.Triangular Load
5.Circular Load
6.Rectangular Load
Approximate Method
1:2 Method
Influence Charts
Stress Bulbs
Newmark Charts


Stresses caused by a Point load
Boussinesq's Equation
ΔPz = (3P/2π) (Z³/L⁵)
P
GROUND SURFACE
Y
X
Using Influence Factor
ΔP = (P/Z²) Iz
Z
L
Principal Stresses
General Stresses
Z
Source : Kamal Tawfiq (2019) Stresses in Soil


In Strength of materials
CAUTION
AVOID INJURY
WATCH YOUR
STEP
In geotechnical engineering, the Mohr's circle sign convention typically uses positive values for compressive normal stresses and counter-clockwise shear stresses, which is the opposite of the standard convention in other engineering fields.


GROUND SURFACE
WATER TABLE
H


Stresses caused by a Point load
Boussinesq (1883) solved the problem of stresses produced at any point in a homogeneous, elastic, and isotropic medium as the result of a point load applied on the surface of an innitely large half-space. According to Figure, Boussinesq's solution for normal stresses at a point caused by the point load P is
Δσz = 3P z³ / 2π L⁵ = 3P / 2π · z³ / (r² + z²)^5/2
where r = √(x² + y²)
L = √(x² + y² + z²) = √(r² + z²)
μ = Poisson's ratio
Source : PRINCIPLES OF GEOTECHNICAL ENGINEERING 9th Edition_ Braja M. Das & Khaled Sobhan (2018) _ Cengage Learning.

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

where

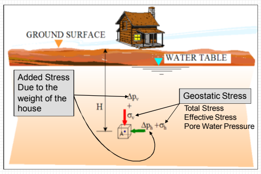
$$I_1 = \frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}$$

r/z	I_1	r/z	I_1	r/z	I_1
0	0.4775	0.36	0.3521	1.80	0.0129
0.02	0.4770	0.38	0.3408	2.00	0.0085
0.04	0.4765	0.40	0.3294	2.20	0.0058
0.06	0.4723	0.45	0.3011	2.40	0.0040
0.08	0.4699	0.50	0.2733	2.60	0.0029
0.10	0.4657	0.55	0.2466	2.80	0.0021
0.12	0.4607	0.60	0.2214	3.00	0.0015
0.14	0.4548	0.65	0.1978	3.20	0.0011
0.16	0.4482	0.70	0.1762	3.40	0.00085
0.18	0.4409	0.75	0.1565	3.60	0.00066
0.20	0.4329	0.80	0.1386	3.80	0.00051
0.22	0.4242	0.85	0.1226	4.00	0.00040
0.24	0.4151	0.90	0.1083	4.20	0.00032
0.26	0.4050	0.95	0.0956	4.40	0.00026
0.28	0.3954	1.00	0.0844	4.60	0.00021
0.30	0.3849	1.20	0.0513	4.80	0.00017
0.32	0.3742	1.40	0.0317	5.00	0.00014
0.34	0.3632	1.60	0.0200		

Source : PRINCIPLES OF GEOTECHNICAL ENGINEERING 9th Edition_ Braja M. Das & Khaled Sobhan (2018)

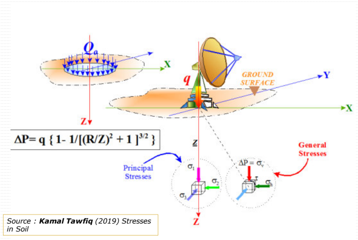
Cengage Learning.

There are two types of lateral stresses in soil.



*Source : **Kamal Tawfiq** (2019) Stresses in Soil*

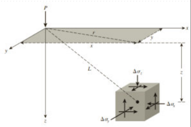
Circular Load



*Source : **Kamal Tawfiq** (2019) Stresses in Soil*

Ex 1.5

Consider a point load P = 5 kN. Calculate the vertical stress increase $\Delta\sigma_Z$ at z = 0, 2 m, 4 m, 6 m, 10 m, and 20 m. Given x = 3 m and y = 4 m



Source : PRINCIPLES OF GEOTECHNICAL ENGINEERING 9th Edition_ Braja M. Das & Khaled Sobhan (2018)

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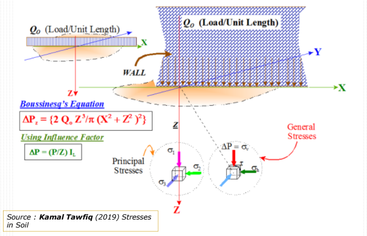
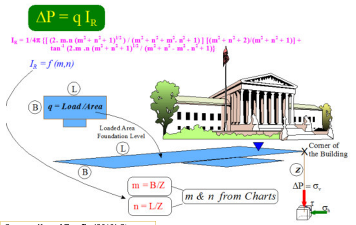
Stresses caused by a Line Load



*Source : **Kamal Tawfiq** (2019) Stresses in Soil*

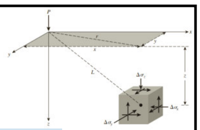
Rectangular Load



*Source : **Kamal Tawfiq** (2019) Stresses in Soil*

Ex 1.5

Consider a point load P = 5 kN. Calculate the vertical stress increase $\Delta\sigma_Z$ at z = 0, 2 m, 4 m, 6 m, 10 m, and 20 m. Given x = 3 m and y = 4 m



Solution

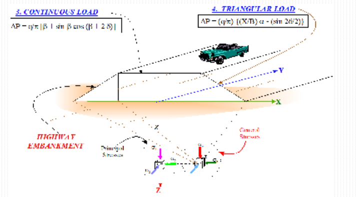
$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = 5\text{ m}$$

The following table can now be prepared.

r (m)	z (m)	$\frac{r}{z}$	I_1	$\Delta\sigma_z = \left(\frac{P}{z^2}\right)I_1$ (kN/m²)
5	0	∞	0	**0**
	2	2.5	0.0034	**0.0043**
	4	1.25	0.0424	**0.0133**
	6	0.83	0.1295	**0.0180**
	10	0.5	0.2733	**0.0137**
	20	0.25	0.4103	**0.0051**

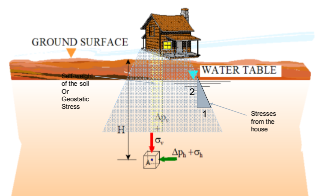
Source : PRINCIPLES OF GEOTECHNICAL ENGINEERING 9th Edition Braja M. Das & Khaled Sobhan (2018) _ Cengage Learning.

Continuous & Triangular Loads



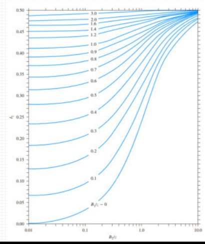
*Source : **Kamal Tawfiq** (2019) Stresses in Soil*

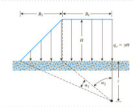
Geostatic Stresses
&
Stress Distribution in Soils



*Source : **Kamal Tawfiq** (2019) Stresses in Soil*

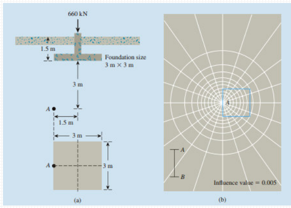
- Osterberg's chart for determination of vertical stress due to embankment loading





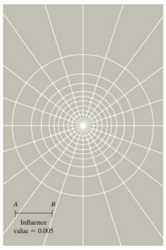
Source : PRINCIPLES OF GEOTECHNICAL ENGINEERING 9th Edition
Braja M. Das & Khaled Sobhan (2018) _ Cengage Learning.

- Ex 1.6



Source : PRINCIPLES OF GEOTECHNICAL ENGINEERING 9th Edition
Braja M. Das & Khaled Sobhan (2018) _ Cengage Learning.

- Newmark's Charts



The procedure for obtaining vertical pressure at any point below a loaded area is as follows:

Step 1. Determine the depth z below the uniformly loaded area at which the stress increase is required.
Step 2. Plot the plan of the loaded area with a scale of z equal to the unit length of the chart (AB).
Step 3. Place the plan (plotted in step 2) on the inuence chart in such a way that the point below which the stress is to be determined is located at the center of the chart.
Step 4. Count the number of elements (M) of the chart enclosed by the plan of the loaded area.

Solution

Point A is located at a depth 3 m below the bottom of the foundation. The plan of the square foundation has been replotted to a scale of AB = 3 m and placed on the influence chart (Figure) in such a way that point A on the plan falls directly over the center of the chart. The number of elements inside the outline of the plan is about 48.5. Hence,

$$\Delta\sigma_z = (IV)qM = 0.005\left(\frac{660}{3 \times 3}\right)48.5 = \mathbf{17.78\ kN/m^2}$$

Chapter 2: Soil settlement and consolidation

Habib TROUZINE, Ph.D.
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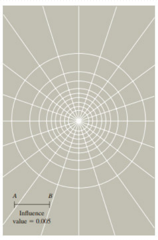
Oct./Nov. 2025



The increase in the pressure at the point under consideration is given by

$$\Delta\sigma_z = (IV)qM$$

where IV = influence value
q = pressure on the loaded area

Chapter 2: Soil settlement and consolidation

- Definitions of settlement, compressibility, and consolidation
- Settlement of granular soils and saturated fine soils
- Terzaghi's consolidation theory
- Laboratory study of compressibility

Settlement

A stress increase caused by the construction of foundations or other loads compresses soil layers. The compression is caused by (a) deformation of soil particles, (b) relocations of soil particles, and (c) expulsion of water or air from the void spaces. In general, the soil settlement caused by loads may be divided into three broad categories:

1. *Elastic settlement* (or *immediate settlement*), which is caused by the elastic deformation of dry soil and of moist and saturated soils without any change in the moisture content. Elastic settlement calculations generally are based on equations derived from the theory of elasticity.
2. *Primary consolidation settlement*, which is the result of a volume change in saturated cohesive soils because of expulsion of the water that occupies the void spaces.
3. *Secondary consolidation settlement*, which is observed in saturated cohesive soils and organic soil and is the result of the plastic adjustment of soil fabrics. It is an additional form of compression that occurs at constant effective stress.

Elastic Settlement in sand

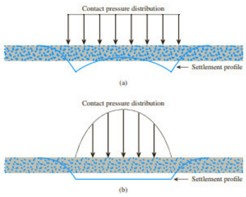


Figure 11.2 Elastic settlement profile and contact pressure in sand: (a) flexible foundation; (b) rigid foundation

I_s = shape factor (Steinbrenner, 1934)

$$= F_1 + \frac{1-2\mu_s}{1-\mu_s}F_2$$

$$F_1 = \frac{1}{\pi}(A_0 + A_1)$$

$$F_2 = \frac{n'}{2\pi}\tan^{-1}A_2$$

$$A_0 = m' \ln \frac{(1+\sqrt{m'^2+1})\sqrt{m'^2+n'^2}}{m'(1+\sqrt{m'^2+n'^2+1})}$$

$$A_1 = \ln \frac{(m'+\sqrt{m'^2+1})\sqrt{1+n'^2}}{m'+\sqrt{m'^2+n'^2+1}}$$

$$A_2 = \frac{m'}{n'\sqrt{m'^2+n'^2+1}}$$

Settlement

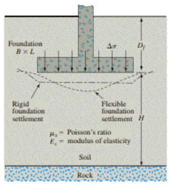
The total Settlement of a foundation can be given as

$$S_T = S_e + S_c + S_s$$

Where S_T = total settlement
S_c = primary consolidation settlement
S_s = primary consolidation settlement
S_e = elastic settlement

Elastic Settlement Calculation

Figure shows a shallow foundation subjected to a net force per unit area equal to Δσ. Let the Poisson's ratio and the modulus of elasticity of the soil supporting it be μs and Es, respectively. Theoretically, if the foundation is perfectly fexible, the settlement may be expressed as



$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$$

I_f = depth factor (Fox, 1948) $= f\left(\frac{D_f}{B}, \mu_s, \text{ and } \frac{L}{B}\right)$

- For calculation of settlement at the *center* of the foundation:

$$\alpha = 4$$
$$m' = \frac{L}{B}$$
$$n' = \frac{H}{\left(\frac{B}{2}\right)}$$

- For calculation of settlement at a *corner* of the foundation:

$$\alpha = 1$$
$$m' = \frac{L}{B}$$
$$n' = \frac{H}{B}$$

Elastic Settlement in clay

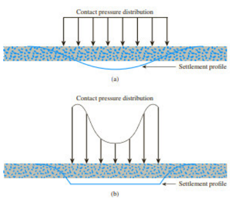


Figure 11.1 Elastic settlement profile and contact pressure in clay: (a) flexible foundation; (b) rigid foundation

Elastic Settlement Calculation

$$S_e = \Delta\sigma(\alpha B')\frac{1-\mu_s^2}{E_s}I_s I_f$$

where $\Delta\sigma$ = net applied pressure on the foundation
μ_s = Poisson's ratio of soil
E_s = average modulus of elasticity of the soil under the foundation measured from $z = 0$ to about $z = 5B$
B' = $B/2$ for center of foundation
= B for corner of foundation
I_s = shape factor (Steinbrenner, 1934)

I_f = depth factor (Fox, 1948)

α = factor that depends on the location on the foundation where settlement is being calculated

The elastic settlement of a *rigid foundation* can be estimated as

$$S_{e(\text{rigid})} \approx 0.93 S_{e(\text{flexible, center})}$$

Due to the nonhomogeneous nature of soil deposits, the magnitude of E_s may vary with depth. For that reason, Bowles (1987) recommended using a weighted average value of E_s in Eq. (11.1) or

$$E_s = \frac{\sum E_{s(i)}\Delta z}{\bar{z}}$$

where $E_{s(i)}$ = soil modulus of elasticity within a depth Δz
$\bar{z}$ = H or $5B$, whichever is smaller

□ Consolidation

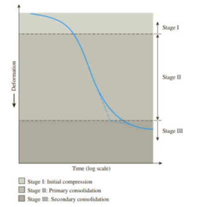
Soil Consolidation refers to the process in which the volume of a saturated (partially or fully) soil decreases due to an applied stress. The term was introduced by Karl von Terzaghi also known as the "father of soil mechanics and geotechnical engineering". Terzaghi established the one-dimensional consolidation theory and changed the definition of the term since it was previously associated (and still is, in geosciences) with the compaction of clay sediments that formed shales.

□ Consolidation

The consolidation procedure is commonly separated into 3 stages:

1. Initial consolidation: A quick volume loss of the soil mass associated with the application of external stress that compresses the air inside the soil's voids.
2. Primary consolidation: Soil settlement during which the excess pore water pressure is transferred to the soil's skeleton
3. Secondary consolidation: A subsequent settlement procedure that occurs after primary consolidation and is associated with internal changes in the soil's structure while subjected to nearly constant load. This process is commonly referred to as creep

□ **Time–deformation plot during consolidation for a given load increment**



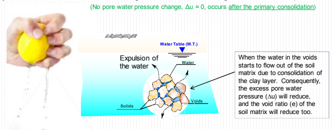
Consolidation Settlement (Time Dependent Settlement)

* Consolidation settlement occurs in cohesive soils due to the expulsion of the water from the voids.
* Because of the soil permeability the rate of settlement may varied from soil to another.
* Also the variation in the rate of consolidation settlement depends on the boundary conditions.

$$S_{Consolidation} = S_{primary} + S_{secondary}$$

Primary Consolidation — Volume change is due to reduction in pore water pressure

Secondary Consolidation — Volume change is due to the rearrangement of the soil particles (No pore water pressure change, $\Delta u = 0$, occurs after the primary consolidation)



□ **The Oedometer Test**

The simplest case of consolidation examined is the one-dimensional consolidation. In this case, the lateral strain of the soil mass is neglected. The testing procedure to quantify the critical soil properties associated with soil consolidation is the Oedometer Test. The term "Oedometer" derives from the Ancient Greek language and means "to swell". The test is one of the most commonly conducted, and important, laboratory tests in geotechnical engineering. The Oedometer Test aims at measuring the vertical displacement of a cylindrical, saturated soil sample subjected to a vertical load while it is radially constrained. In the subsequent test, the incremental loading consolidation test is described. Note that there is also a constant rate of strain (CRS) test, that nowadays is becoming more popular

□ **Void Ratio–Pressure Plots**

Step 1. Calculate the height of solids, H_s, in the soil specimen using the equation

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w}$$

where W_s = dry weight of the specimen
M_s = dry mass of the specimen
A = area of the specimen
G_s = specific gravity of soil solids
γ_w = unit weight of water
ρ_w = density of water

Step 2. Calculate the initial height of voids as

$$H_v = H - H_s$$

where H = initial height of the specimen.

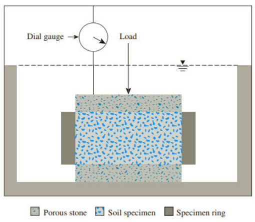
Step 3. Calculate the initial void ratio, e_o, of the specimen, using the equation

$$e_o = \frac{V_v}{V_s} = \frac{H_v}{H_s}\frac{A}{A} = \frac{H_v}{H_s}$$

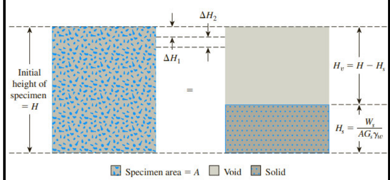
□ Consolidation

When a load is applied in a low permeability soil, it is initially carried by the water that exists in the porous of a saturated soil resulting in a rapid increase of pore water pressure. This excess pore water pressure is dissipated as water drains away from the soil's voids and the pressure is transferred to the soil skeleton which is gradually compressed, resulting in settlements. The consolidation procedure lasts until the excess pore water pressure is dissipated.

□ **Schematic diagram of a consolidometer**



□ **Change of height of specimen in one-dimensional consolidation test**



Void Ratio–Pressure Plots (2)

Step 4. For the first incremental loading, σ_1 (total load/unit area of specimen), which causes a deformation ΔH_1, calculate the change in the void ratio as

$$\Delta e_1 = \frac{\Delta H_1}{H_s}$$

(ΔH_1 is obtained from the initial and the final dial readings for the loading).

It is important to note that, at the end of consolidation, total stress σ_1 is equal to effective stress σ'_1.

Step 5. Calculate the new void ratio after consolidation caused by the pressure increment as

$$e_1 = e_o - \Delta e_1$$

For the next loading, σ_2 (*note:* σ_2 equals the cumulative load per unit area of specimen), which causes additional deformation ΔH_2, the void ratio at the end of consolidation can be calculated as

$$e_2 = e_1 - \frac{\Delta H_2}{H_s}$$

At this time, σ_2 = effective stress, σ'_2. Proceeding in a similar manner, one can obtain the void ratios at the end of the consolidation for all load increments.

The effective stress σ' and the corresponding void ratios (e) at the end of consolidation are plotted on semilogarithmic graph paper. The typical shape of such a plot is shown in Figure

Example 2.1

Following are the results of a laboratory consolidation test on a soil specimen obtained from the field: Dry mass of specimen = 128 g, height of specimen at the beginning of the test = 2.54 cm, G_s = 2.75, and area of the specimen = 30.68 cm².

Effective pressure, σ' (kN/m2)	Final height of specimen at the end of consolidation (cm)
0	2.540
53.62	2.488
107.25	2.465
214.50	2.431
429.01	2.389
858.01	2.324
1716.03	2.225
3432.6	2.115

Make necessary calculations and draw an e versus log σ' curve.

Normally Consolidated and Overconsolidated Clays

The overconsolidation ratio (OCR) for a soil can now be defined as

$$OCR = \frac{\sigma'_c}{\sigma'}$$

where σ'_c = preconsolidation pressure of a specimen
σ' = present effective vertical pressure

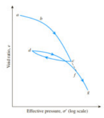


Figure 11.16 Plot of e against log σ' showing loading, unloading, and reloading branches

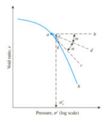
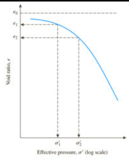
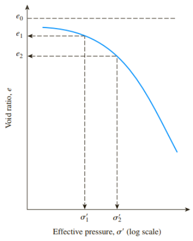


Figure 11.17 Graphic procedure for determining preconsolidation pressure



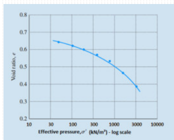
Typical plot of e against $\log \sigma'$



Solution

$$H_s = \frac{W_s}{AG_s\gamma_w} = \frac{M_s}{AG_s\rho_w} = \frac{128 \text{ g}}{(30.68 \text{ cm}^2)(2.75)(1 \text{g/cm}^3)} = 1.52 \text{ cm}$$

Sigma' (kN/m²)	H (cm)	Hv (cm)	e
0	2,540	1,023	0,674
53.62	2,488	0,971	0,640
107.25	2,465	0,948	0,625
214.50	2,431	0,914	0,603
429.01	2,389	0,872	0,575
858.01	2,324	0,807	0,532
1716.03	2,225	0,708	0,467
3432.06	2,115	0,598	0,394



Preconsolidation pressure

In the literature, some empirical relationships are available to predict the preconsolidation pressure. Some examples are given next.

- Stas and Kulhawy (1984):

$$\frac{\sigma'_c}{p_a} = 10^{[1.11 - 1.62(LI)]} \qquad (11.27)$$

where p_a = atmospheric pressure (≈100 kN/m²)
LI = liquidity index

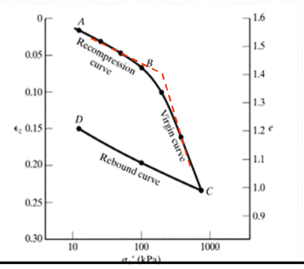
- Hansbo (1957)

$$\sigma'_c = \alpha_{(VST)}\, c_{u(VST)} \qquad (11.28)$$

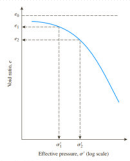
where $\alpha_{(VST)}$ = an empirical coefficient = $\frac{222}{LL(\%)}$
$c_{u(VST)}$ = undrained shear strength obtained from vane shear test (Chapter 12)

In any case, these above relationships may change from soil to soil. They may be taken as an initial approximation.

Test result



Normally Consolidated and Overconsolidated Clays



Casagrande (1936) suggested a simple graphic construction to determine the preconsolidation pressure σ'_c from the laboratory e-log σ' plot. The procedure is as follows (see Figure 11.17):

Step 1. By visual observation, establish point a, at which the e-log σ' plot has a minimum radius of curvature.
Step 2. Draw a horizontal line ab.
Step 3. Draw the line ac tangent at a.
Step 4. Draw the line ad, which is the bisector of the angle bac.
Step 5. Project the straight-line portion gh of the e-log σ' plot back to intersect line ad at f. The abscissa of point f is the preconsolidation pressure, σ'_c.

The overconsolidation ratio (OCR) for a soil can now be defined as

Example 2.2

Following are the results of a laboratory consolidation test.

Pressure, σ' (kN/m²)	Void ratio, e
50	0.840
100	0.826
200	0.774
400	0.696
800	0.612
1000	0.528

Using Casagrande's procedure, determine the preconsolidation pressure σ'_c.

Solution

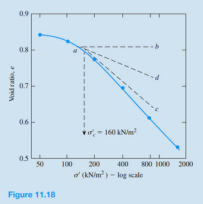


Figure 11.18 shows the e-log σ' plot. In this plot, a is the point where the radius of curvature is minimum. The preconsolidation pressure is determined using the procedure shown in Figure 11.17. From the plot, σ'_c = 160 kN/m².

For normally consolidated clays that exhibit a linear e-log σ' relationship (see Figure 11.20),

$$\Delta e = C_c[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o] \quad (11.34)$$

where C_c = slope of the e-log σ' plot and is defined as the compression index. Substitution of Eq. (11.34) into Eq. (11.33) gives

$$S_c = \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) \quad (11.35)$$

In overconsolidated clays (see Figure 11.21), for $\sigma'_o + \Delta\sigma' \le \sigma'_c$, field e-log σ' variation will be along the line hj, the slope of which will be approximately equal to that for the laboratory rebound curve. The slope of the rebound curve C_s is referred to as the *swell index*; so

$$\Delta e = C_s[\log(\sigma'_o + \Delta\sigma') - \log \sigma'_o] \quad (11.36)$$

From Eqs. (11.33) and (11.36), we obtain

$$S_c = \frac{C_s H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_o}\right) \quad (11.37)$$

If $\sigma'_o + \Delta\sigma' > \sigma'_c$, then

$$S_c = \frac{C_s H}{1+e_o} \log\frac{\sigma'_c}{\sigma'_o} + \frac{C_c H}{1+e_o} \log\left(\frac{\sigma'_o + \Delta\sigma'}{\sigma'_c}\right) \quad (11.38)$$

However, if the e-log σ' curve is given, one can simply pick Δe off the plot for the appropriate range of pressures. This number may be substituted into Eq. (11.33) for the calculation of settlement, S_c.

Secondary Consolidation Settlement

$$C_\alpha = \frac{\Delta e}{\log t_2 - \log t_1} = \frac{\Delta e}{\log(t_2/t_1)}$$

where C_α = secondary compression index
Δe = change of void ratio
t_1, t_2 = time

The magnitude of the secondary consolidation can be calculated as

$$S_s = C'_\alpha H \log\left(\frac{t_2}{t_1}\right)$$

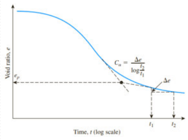
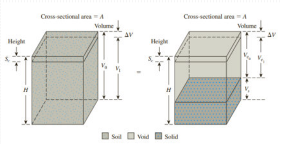


Figure : *Variation of e with log t under a given load increment and definition of secondary consolidation index*

Calculation of Settlement from One-Dimensional Primary Consolidation



Correlations for Compression Index, Cc*

Equation	Reference	Region of applicability
$C_c = 0.007(LL - 7)$	Skempton (1944)	Remolded clays
$C_c = 0.01 w_N$		Chicago clays
$C_c = 1.15(e_o - 0.27)$	Nishida (1956)	All clays
$C_c = 0.30(e_o - 0.27)$	Hough (1957)	Inorganic cohesive soil: silt, silty clay, clay
$C_c = 0.0115 w_N$		Organic soils, peats, organic silt, and clay
$C_c = 0.0046(LL - 9)$		Brazilian clays
$C_c = 0.75(e_o - 0.5)$		Soils with low plasticity
$C_c = 0.208 e_o + 0.0083$		Chicago clays
$C_c = 0.156 e_o + 0.0107$		All clays

*After Rendon-Herrero, 1980. With permission from ASCE.
Note: e_o = in situ void ratio; w_N = in situ water content.

Time Rate of Consolidation

Terzaghi (1925) proposed the 1st theory to consider the rate of one-dimensional consolidation for saturated clay soils. The mathematical derivations are based on the following six assumptions (also see Taylor, 1948):

1. The clay–water system is homogeneous.
2. Saturation is complete.
3. Compressibility of water is negligible.
4. Compressibility of soil grains is negligible (but soil grains rearrange).
5. The flow of water is in one direction only (that is, in the direction of compression).
6. Darcy's law is valid.

Rate of outflow of water − Rate of inflow of water = Rate of volume change

Thus,

$$\left(v_z + \frac{\partial v_z}{\partial z} dz\right) dx\, dy - v_z\, dx\, dy = \frac{\partial V}{\partial t}$$

where V = volume of the soil element
v_z = velocity of flow in z direction

Let us consider a saturated clay layer of thickness H and cross-sectional area A under an existing average effective overburden pressure, σ'_o. Because of an increase of effective pressure, $\Delta\sigma'$, let the primary settlement be S_c. Thus, the change in volume (Figure 11.22) can be given by

$$\Delta V = V_0 - V_1 = HA - (H - S_c)A = S_c A \quad (11.29)$$

where V_0 and V_1 are the initial and final volumes, respectively. However, the change in the total volume is equal to the change in the volume of voids, ΔV_v. Hence,

$$\Delta V = S_c A = V_{v_0} - V_{v_1} = \Delta V_v \quad (11.30)$$

where V_{v_0} and V_{v_1} are the initial and final void volumes, respectively. From the definition of void ratio, it follows that

$$\Delta V_v = \Delta e V_s \quad (11.31)$$

where Δe = change of void ratio. But

$$V_s = \frac{V_0}{1+e_o} = \frac{AH}{1+e_o} \quad (11.32)$$

where e_o = initial void ratio at volume V_0. Thus, from Eqs. (11.29) through (11.32),

$$\Delta V = S_c A = \Delta e V_s = \frac{AH}{1+e_o} \Delta e$$

or

$$S_c = H \frac{\Delta e}{1+e_o} \quad (11.33)$$

Correlations for Swell Index (Cs)

The swell index is appreciably smaller in magnitude than the compression index and generally can be determined from laboratory tests. In most cases,

$$C_s \simeq \frac{1}{5} \text{ to } \frac{1}{10} C_c$$

The swell index was expressed by Nagaraj and Murty (1985) as

$$C_s = 0.0463\left[\frac{LL(\%)}{100}\right] G_s \quad (11.45)$$

Based on the modified Cam clay model, Kulhawy and Mayne (1990) have shown that

$$C_s \approx \frac{PI}{370} \quad (11.46)$$

Time Rate of Consolidation (2)

$$T_v = \frac{c_v t}{H_{dr}^2} = \text{time factor}$$

The time factor is a nondimensional number.

Because consolidation progresses by the dissipation of excess pore water pressure, the degree of consolidation at a distance z at any time t is

$$U_z = \frac{u_o - u_z}{u_o} = 1 - \frac{u_z}{u_o} \quad (11.63)$$

where u_z = excess pore water pressure at time t.

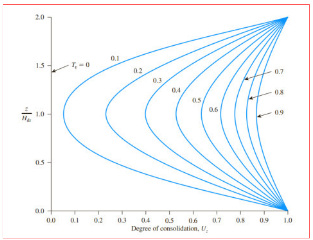
Equations (11.61) and (11.63) can be combined to obtain the degree of consolidation at any depth z. This is shown in Figure 11.29.

The average degree of consolidation for the entire depth of the clay layer at any time t can be written from Eq. (11.63) as

$$U = \frac{S_{c(t)}}{S_c} = 1 - \frac{\left(\frac{1}{2H_{dr}}\right)\int_0^{2H_{dr}} u_z\, dz}{u_o} \quad (11.64)$$

where U = average degree of consolidation
$S_{c(t)}$ = settlement of the layer at time t
S_c = ultimate settlement of the layer from primary consolidation

Variation of U_z with T_v and z/H_{dr}



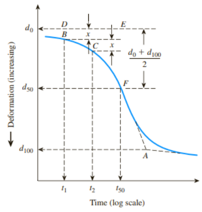
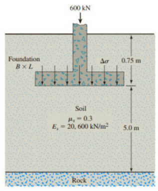


Figure: Logarithm of time method for determining coefficient of consolidation

☐ Example 2.3

A vertical column load, $P = 600$ kN, is applied to a rigid concrete foundation with dimensions $B = 1$ m and $L = 2$ m, as shown in Figure . The foundation rests at a depth $D_f = 0.75$ m on a uniform dense sand with the following properties: average modulus of elasticity, $E_s = 20{,}600$ kN/m², and Poisson's ratio, $\mu_s = 0.3$. Estimate the elastic settlement due to the net applied pressure, $\Delta\sigma$, on the foundation. Given: $H = 5$ m.



Elastic settlement: 12.1 mm

Variation of T_v with U

For $U = 0$ to 60%, $T_v = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2$

For $U > 60\%$, $T_v = 1.781 - 0.933 \log (100 - U\%)$

U (%)	T_v	U (%)	T_v	U (%)	T_v	U (%)	T_v
0	0	26	0.0531	52	0.212	78	0.529
1	0.00008	27	0.0572	53	0.221	79	0.547
2	0.0003	28	0.0615	54	0.230	80	0.567
3	0.00071	29	0.0660	55	0.239	81	0.588
4	0.00126	30	0.0707	56	0.248	82	0.610
5	0.00196	31	0.0754	57	0.257	83	0.633
6	0.00283	32	0.0803	58	0.267	84	0.658
7	0.00385	33	0.0855	59	0.276	85	0.684
8	0.00502	34	0.0907	60	0.286	86	0.712
9	0.00636	35	0.0962	61	0.297	87	0.742
10	0.00785	36	0.102	62	0.307	88	0.774
11	0.0095	37	0.107	63	0.318	89	0.809
12	0.0113	38	0.113	64	0.329	90	0.848
13	0.0133	39	0.119	65	0.340	91	0.891
14	0.0154	40	0.126	66	0.352	92	0.938
15	0.0177	41	0.132	67	0.364	93	0.993
16	0.0201	42	0.138	68	0.377	94	1.055
17	0.0227	43	0.145	69	0.390	95	1.129
18	0.0254	44	0.152	70	0.403	96	1.219
19	0.0283	45	0.159	71	0.417	97	1.336
20	0.0314	46	0.166	72	0.431	98	1.500
21	0.0346	47	0.173	73	0.446	99	1.781
22	0.0380	48	0.181	74	0.461	100	∞
23	0.0415	49	0.188	75	0.477		
24	0.0452	50	0.197	76	0.493		
25	0.0491	51	0.204	77	0.511		

Square-root-of-time method

In the square-root-of-time method, a plot of deformation against the square root of time is made for the incremental loading Other graphic constructions required are as follows:

Step 1. Draw a line AB through the early portion of the curve.
Step 2. Draw a line AC such that $\overline{OC} = 1.15\overline{OB}$. The abscissa of point D, which is the intersection of AC and the consolidation curve, gives the square root of time for 90% consolidation ($\sqrt{t_{90}}$).
Step 3. For 90% consolidation, $T_{90} = 0.848$, so

$$T_{90} = 0.848 = \frac{c_v t_{90}}{H_{dr}^2}$$

or

$$c_v = \frac{0.848 H_{dr}^2}{t_{90}}$$

H_{dr} in Eq. (11.73) is determined in a manner similar to that in the logarithm-of-time method.

☐ Determination of Coefficient of Consolidation

Logarithm-of-time method

For a given incremental loading of the laboratory test, the specimen deformation against log-of-time plot is shown in Figure The following constructions are needed to determine c_v.

Step 1. Extend the straight-line portions of primary and secondary consolidations to intersect at A. The ordinate of A is represented by d_{100}—that is, the deformation at the end of 100% primary consolidation.
Step 2. The initial curved portion of the plot of deformation versus log t is approximated to be a parabola on the natural scale. Select times t_1 and t_2 on the curved portion such that $t_2 = 4t_1$. Let the difference of specimen deformation during time $(t_2 - t_1)$ be equal to x.
Step 3. Draw a horizontal line DE such that the vertical distance BD is equal to x. The deformation corresponding to the line DE is d_0 (that is, deformation at 0% consolidation).
Step 4. The ordinate of point F on the consolidation curve represents the deformation at 50% primary consolidation, and its abscissa represents the corresponding time (t_{50}).
Step 5. For 50% average degree of consolidation, $T_v = 0.197$

$$T_{50} = \frac{c_v t_{50}}{H_{dr}^2}$$

or

$$c_v = \frac{0.197 H_{dr}^2}{t_{50}}$$

where H_{dr} = average longest drainage path during consolidation.

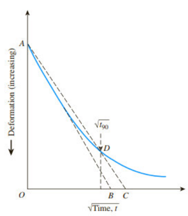


Figure : Square-root-of-time fitting method

Chapter 3: Soil Hydraulics

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Dec. 2025

3.1 Water in soil

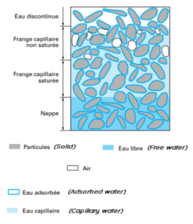

Figure 3.2. Different states of water in the soil. (Jean-Pierre MAGNAN (2020) L'eau dans le sol ©Techniques de l'Ingénieur, C 212)

3.2 Permeability

Soils are permeable due to the existence of interconnected voids through which water can flow from points of high energy to points of low energy. The study of the flow of water through permeable soil media is important in soil mechanics. It is necessary for estimating the quantity of underground seepage under various hydraulic conditions, for investigating problems involving the pumping of water for underground construction, and for making stability analyses of earth dams and earth-retaining structures that are subject to seepage forces.

Chapter 3: Soil hydraulics

- Definitions of hydraulic gradient, hydraulic head, and velocity of water in soil
- Darcy's law
- Measurement of permeability coefficient in the laboratory and in situ
- Permeability of stratified soils
- Soil flow network: streamlines, equipotential lines

3.4 Bernoulli's Equation

Total head h :

$$h = \frac{u}{\gamma_w} + \frac{v^2}{2g} + Z$$

Pressure head, Velocity head, Elevation head

where h = total head
u = pressure
v = velocity
g = acceleration due to gravity
γ_w = unit weight of water

Daniel BERNOULLI (1700-1782)

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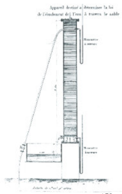
3.3 Darcy
and his sand column experiment

Darcy's Law (1856)

$V = k.i$
ou
$q = A.k.i$



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3.1 Water in soil

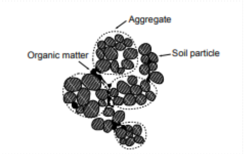
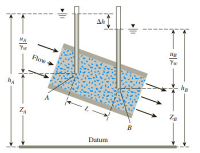


Figure 3.1.Structure of soil aggregate. (After Nakano, M., Transport Phenomena in Soils, University of Tokyo Press, Tokyo (1991)

3.4 Bernoulli's Equation (2)

If Bernoulli's equation is applied to the flow of water through a porous soil medium, the term containing the velocity head can be neglected because the seepage velocity is small, and the total head at any point can be adequately represented by :

$$h = \frac{u}{\gamma_w} + Z$$



$V = k.i$ ou $q = A.k.i$

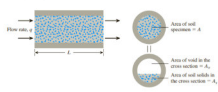
where
V : discharge velocity, which is the quantity of water flowing in unit time through a unit gross cross-sectional area of soil at right angles to the direction of flow
k : hydraulic conductivity (otherwise known as the coefficient of permeability)

$$q = vA = A_v v_s$$

where v_s = seepage velocity
A_v = area of void in the cross section of the specimen

However,

$$A = A_v + A_s$$



Hydraulic Conductivity

- Hydraulic conductivity is generally expressed in cm/sec or m/sec in SI units
- The hydraulic conductivity of soils depends on several factors: fluid viscosity, pore-size distribution, grain-size distribution, void ratio, roughness of mineral particles, and degree of soil saturation.
- In clayey soils, structure plays an important role in hydraulic conductivity.

Hydraulic Conductivity

- Other major factors that affect the permeability of clays are the ionic concentration and the thickness of layers of water held to the clay particles.
- The hydraulic conductivity of unsaturated soils is lower and increases rapidly with the degree of saturation.
- The hydraulic conductivity of a soil is also related to the properties of the fluid

Hydraulic Conductivity

$$k = \frac{\gamma_w}{\eta}\overline{K}$$

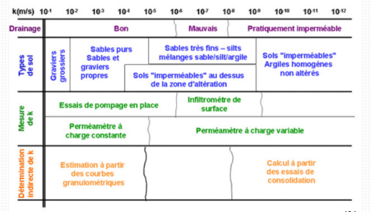
where γ_w = unit weight of water
η = dynamic viscosity of water
$\overline{K}$ = absolute permeability

The *absolute permeability* $\overline{K}$ is expressed in units of L^2 (that is, cm²,

Table 7.1 Typical Values of Hydraulic Conductivity of Saturated Soils

Soil type	k: cm/sec	k: ft/min
Clean gravel	100–1.0	200–2.0
Coarse sand	1.0–0.01	2.0–0.02
Fine sand	0.01–0.001	0.02–0.002
Silty clay	0.001–0.00001	0.002–0.00002
Clay	<0.000001	<0.000002

Drainage & Permeability



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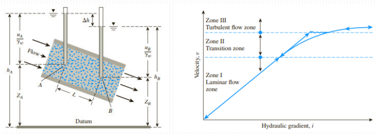
- Hydraulic gradient



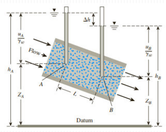
The loss of head between two points, A and B, can be given by

$$\Delta h = h_A - h_B = \left(\frac{u_A}{\gamma_w} + Z_A\right) - \left(\frac{u_B}{\gamma_w} + Z_B\right)$$

The head loss, Δh, can be expressed in a nondimensional form as

$$i = \frac{\Delta h}{L}$$

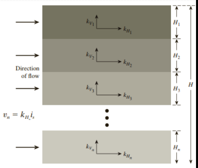
- Hydraulic gradient



$$i = \frac{\Delta h}{L}$$

where i = hydraulic gradient
L = distance between points A and B—that is, the length of flow over which the loss of head occurred

Equivalent Hydraulic Conductivity in Stratified Soil —horizontal flow



$$v = k_{H(eq)} i_{eq};\quad v_1 = k_{H_1} i_1;\quad v_2 = k_{H_2} i_2;\quad v_3 = k_{H_3} i_3;\ \ldots\ v_n = k_{H_n} i_n$$

$$k_{H(eq)} = \frac{1}{H}(k_{H_1}H_1 + k_{H_2}H_2 + k_{H_3}H_3 + \cdots + k_{H_n}H_n)$$

Source : Braja Das

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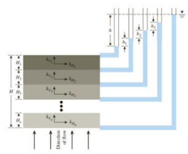
Equivalent Hydraulic Conductivity in Stratified Soil — vertical flow

$$h = H_1 i_1 + H_2 i_2 + H_3 i_3 + \cdots + H_n i_n$$

$$k_{v(eq)} = \frac{H}{\left(\frac{H_1}{k_{v_1}}\right) + \left(\frac{H_2}{k_{v_2}}\right) + \left(\frac{H_3}{k_{v_3}}\right) + \cdots + \left(\frac{H_n}{k_{v_n}}\right)}$$



Source : Braja Das

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Permeability of stratified soils

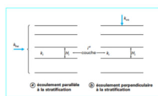
– pour un écoulement parallèle à la stratification (figure **7a**), le coefficient de perméabilité équivalent k_{he} :

$$k_{he} = \frac{\sum_{i=1}^{n} k_i H_i}{\sum_{i=1}^{n} H_i}$$

– pour un écoulement perpendiculaire à la stratification (figure **7b**), le coefficient de perméabilité équivalent k_{ve} :

$$k_{ve} = \frac{\sum_{i=1}^{n} H_i}{\sum_{i=1}^{n} (H_i/k_i)}$$

On peut montrer que, dans tous les cas, $k_{ve} \le k_{he}$.



Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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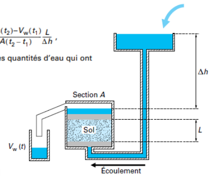
☐ Constant-head test

$$k = \frac{\text{vitesse d'écoulement}}{\text{gradient}} = \frac{V_w(t_2) - V_w(t_1)}{A(t_2 - t_1)} \frac{L}{\Delta h},$$

où $V_w(t_1)$ et $V_w(t_2)$ sont respectivement les quantités d'eau qui ont traversé l'éprouvette aux temps t_1 et t_2.



Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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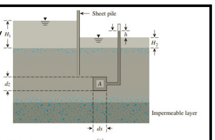
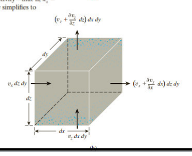
Laplace's Equation of Continuity



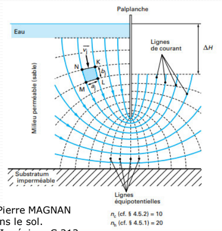
$$k_x \frac{\partial^2 h}{\partial x^2} + k_z \frac{\partial^2 h}{\partial z^2} = 0$$

If the soil is isotropic with respect to the hydraulic conductivity—that is, $k_x = k_z$—the preceding continuity equation for two-dimensional flow simplifies to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$



Flow around a sheet pile



Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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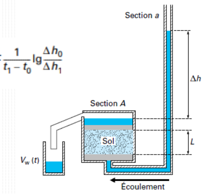
☐ Falling - head test

$$k = \frac{aL}{A}\frac{1}{t_1 - t_0}\ln\frac{\Delta h_0}{\Delta h_1} = 2{,}3\frac{aL}{A}\frac{1}{t_1 - t_0}\lg\frac{\Delta h_0}{\Delta h_1}$$



Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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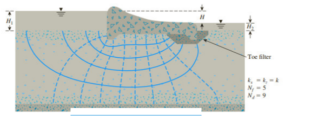
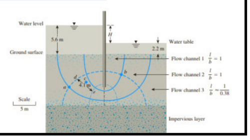
Flow line / Equipotential line /Flow Nets

- ☐ A flow line is a line along which a water particle will travel from upstream to the downstream side in the permeable soil medium. *(Nf number of flow channels)*
- ☐ An equipotential line is a line along which the potential head at all points is equal. *(Nd : number of potential drops)*
- ☐ A combination of a number of flow lines and equipotential lines is called a flow net.

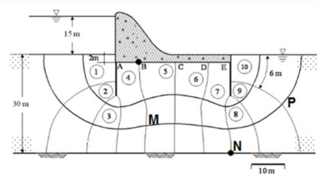


$$q = k\frac{HN_f}{N_d}$$

$$q = kH\left(\frac{N_f}{N_d}\right)n$$



Flow under the base of a concrete dam



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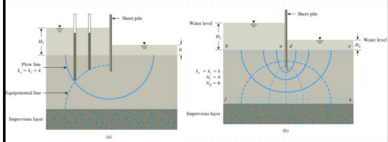
Flow line / Equipotential line /Flow Nets

- ☐ Flow nets are constructed for the calculation of groundwater flow and the evaluation of heads in the media.
- ☐ The equipotential lines intersect the flow lines at right angles. *(Nf number of flow channels) (Nd : number of potential drops)*
- ☐ The flow elements formed are approximate squares.



Pumping Test (or Aquifer Test)

The pumping test is designed to estimate the overall or "large-scale" permeability coefficient of the soil, as well as the storage factor and the pumping range. It is carried out in awell with a diameter large enough to accommodate a pump or strainer. Piezometers are placed around the well. Pumping is then carried outat a constant flow rate and the lowering of the water table n the well and in the piezometers is monitored. The test can be carried out in transient or steady-state conditions.

Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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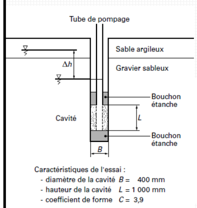
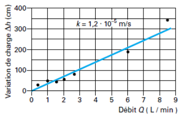
Lefranc test

The principle involves injecting or pumping water at a constant flow rate into a cavity bounded laterally and at the bottom by the borehole wall and, at the top, by a watertight plug.





Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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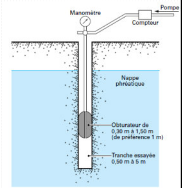
Lugeon test

In compact and fissured formations, the point test is carried out under pressure in a section of borehole limited either by the bottom and a plug, or by two plugs. This is known as the Lugeon test. Lugeon testing is a delicate operation. Ground permeability is expressed in Lugeon units. It characterizes above all the state of fissuring of the massif and the possibility of water circulation.



Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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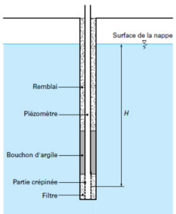
Interstitial pressure measurement

Open Piezometer

Soil pore pressures are measured using piezometers. The simplest piezometer consists of a tube, the lower part of which is cored (perforated) to allow water to enter the tube. The cored part of the tube must be isolated from the rest of the water table by a watertight plug, in order to limit the size of the zone where pressure is measured.



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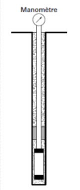
Closed piezometers



When the head or pressure varies over time, thewater level in the tube varies after water exchange with the ground.If the ground is highly permeable (typically, for a permeability coefficient greater than 10–5 m/s), these variations are instantaneous.Otherwise, they require a certain amount of time, known as the piezometer response time, which must be limited if rapid variations in water pressure are to be measured. The response time can be reduced in two ways: either by reducing the diameter of the measuring tube, or by using closed piezometers, whose measuring cavity is limited to a few cubic centimeters.
There are several types of closed piezometers:
— hydraulic piezometers
— membrane piezometers

Source : Jean-Pierre MAGNAN (2020) L'eau dans le sol. Techniques de l'Ingénieur C 212

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Closed hydraulic piezometer
Piézomètre fermé de type hydraulique

Lecture de la NA 5249-2 (2017)

Reconnaissance et essais géotechniques - Essais géo hydrauliques - Partie 2 : essai de perméabilité à l'eau dans un forage en tube ouvert
ISO 22282-2: 2012

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